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Effect of a runaway electron current on tearing modes

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Following tokamak disruptions the Ohmic current is sometimes replaced by a current of runaway electrons, which is likely to be smaller but more peaked in the center of the discharge than the pre-disruption current. In such a situation the question of the resistive stability of the post-disruption plasma, where the equilibrium current is entirely carried by the runaway electrons while the cold background plasma is relatively resistive, becomes crucial. The problems of the linear stability and nonlinear properties of the classical tearing mode in such discharges have recently been addressed in the framework of a simplified plane slab model in which the current profile is symmetric about the reconnection surface [1]. Linear growth was found to be very similar to that in a plasma at the same temperature with a normal inductive current. However the saturated island size was larger than in a conventional discharge and displayed non-linear instabilities at quite moderate values of Δ'.

The numerical results presented in ref. [1] were obtained for a periodic equilibrium magnetic field, corresponding to the equilibrium magnetic flux function $\psi_{eq} = \cos(x)$. This fact implied the presence of magnetic islands at the boundaries of the integration domain, $x = \pm\pi$, as well as the island at $x = 0$. Here we extend this analysis to a more realistic geometry for tokamak application. First of all, we consider a different equilibrium which permits us to avoid the use of periodic boundary conditions; secondly, using the new equilibrium, we will consider the saturation of asymmetric tearing modes, which are more relevant to the tokamak situation. The analytic theory of [1] has been extended for this purpose and compared with numerical results using an Harris type equilibrium with a superimposed strong guide field. Hence, we assume $\psi_{eq}(x) = \log[\cosh(x)]$ where the magnetic field $B = B_0\mathbf{e}_z + \nabla\psi(x,y,z)\times\mathbf{e}_z$, with ψ the poloidal flux function and B_0 the strong guide field.

We point out that for the non-symmetric problem considered here, ψ depends on the variables y and z only through the linear combination $y_* \equiv y + (k_z/k_y)z$, where k_y and k_z are respectively the wave vectors along y and z. It is convenient to introduce a transformation, $(x,y) \rightarrow (x,y_*)$, leading to a new magnetic flux function with one ignorable coordinate:

$$\psi_* \equiv \psi(x,y_*) + \alpha x, \qquad (1)$$

where $\alpha = k_z/k_y$. This transformation corresponds to a rotation by an angle α in the (y,z) plane. X and O points of the magnetic island structure correspond to extrema of ψ_*, defined by $\nabla\psi_* = 0$. In this way, the two-dimensional structure of the problem is recovered. For the equilibrium chosen here the rational surface is located where $\tanh(x_s) = \alpha$. We also assume a plasma flow represented by $\mathbf{v} = \mathbf{e_z} \times \nabla\phi$, with ϕ the stream function. Following the procedure of [1], this MHD model can be reduced to three equations for Ψ_*, ϕ and J_R, the current density of the runaway electrons. The first equation is the plasma vorticity equation:

$$\frac{\partial U}{\partial t} + [\phi, U] = [J, \Psi_*], \tag{2}$$

where $U = \nabla^2\phi$ is the vorticity component along the z-direction. The second equation is the z-component of the resistive Ohm law,

$$\frac{\partial \Psi_*}{\partial t} + [\phi, \Psi_*] = \eta(J - J_R) \tag{3}$$

and the third equation describes the evolution of the runaway current, J_R:

$$\frac{\partial J_R}{\partial t} - \frac{c}{v_A}[J_R, \psi_*] + [\phi, J_R] = 0, \tag{4}$$

where J is the total current, J_R is the current carried by the runaway electrons at the velocity of light, c and v_A is the Alfvèn speed.

These equations were used in ref. [1] for symmetric modes with $k_z = 0$. In this case, the analytic prediction for the saturated island width, w, is:

$$\Delta' = 0.27bw, \tag{5}$$

where Δ' is the standard tearing instability parameter defined in [2] and $b = -J''_{eq}/J_{eq}$ evaluated at the rational surface $x = 0$. For this problem, numerical simulations obtained with $\psi_{eq} = \cos(x)$ gave a good agreement for small values of Δ', but got progressively worse at larger values of Δ'. Actually, the results in ref. [1], show that the numerical data are well fitted by a formula including higher order corrections proportional to w^3, according to:

$$\Delta' = 0.27bw - 0.013w^3 \tag{6}$$

The main implication of eq. 6 concerns the presence of a critical $\Delta' \sim 0.47$ above which no saturation should occur.

In this paper, we first present the results obtained with the new equilibrium in the symmetric case and then the extension to the asymmetric case will be given.

In fig. 1 the results for symmetric tearing modes, where $\alpha = 0$, are shown. We see that the agreement between the theoretical prediction (dashed line) and the numerical data is pretty good at small Δ' values and gets worse at higher values. This is not surprising since the calculations on which the derivation of eq. 5 is based assume that the island width is small, *i.e.* $w \sim \Delta' << 1$. In this case no higher order correction is necessary to fit the data. This leads to the conclusion that the results obtained for the $\cos(x)$ equilibrium were influenced by the presence of the periodic island at the boundary of the integration domain.

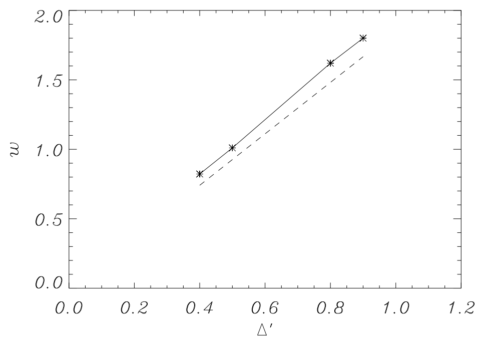

Figure 1: Island width as function of Δ' for symmetric tearing modes with $\alpha = 0$. The numerical data are represented by the asterisks, while the analytical prediction is represented by the dashed line.

Following the analytic method of ref. [1] we have derived an expression for the island width also in the asymmetric case, obtaining:

$$\Delta' = 0.27wb + 1.07wa^2 \tag{7}$$

where $a = J'_{eq}/J_{eq}$, evaluated at the rational surface $x_s = \tanh^{-1}(\alpha)$. We note that this formula predicts a bigger island, by approximately a factor of two, than the one in absence of the runaway current derived in ref. [3].

Figs. 2 and 3 summarize the results for the asymmetric cases. Three different values of the α parameter have been chosen: $0.2, 0.577, 0.8$. In all three cases the agreement with the analytical result of eq. 7 is good at small Δ'. We note that the agreement is still good for higher values of Δ'. This fact does not have any implication on our analytic theory, whose validity is still confined to small values of the island width, but confirms again the influence of the periodic boundary conditions on our previous results.

In conclusion we have analyzed the problem of tearing mode instability in a tokamak discharge where the plasma current is carried entirely by runaway electrons. First, we have revisited the symmetric case presented in ref. [1] using a different equilibrium which allows us to follow the evolution of a magnetic island until saturation for larger Δ' values. The main finding here is that the analytical theory of ref. [1] is confirmed for small Δ' but that the bifurcation behavior that was observed at larger Δ' is not recovered. We believe that our previous result was influenced by the presence of a periodic magnetic island at the boundary of the integration domain. Secondly, with the new equilibrium we analyzed the asymmetric tearing modes,

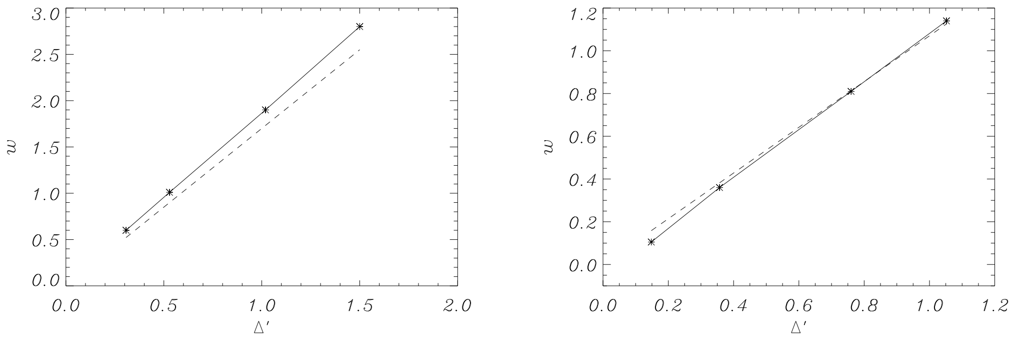

Figure 2: Island width as function of Δ' for asymmetric tering modes with $\alpha = 0.2$ (left frame) and $\alpha = 0.577$ (right frame). The numerical data are represented by the asterisks, while the analytical prediction is represented by the dashed line.

more relevant for tokamak situations. We derived an analytic formula for the saturated island width and compared it with numerical simulations. The agreement is close. A new numerical campaign to explore smaller Δ' regimes is already foreseen.

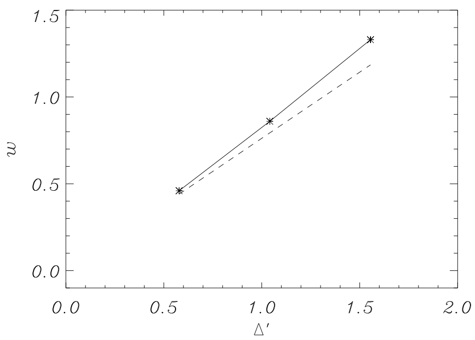

Figure 3: Island width as function of Δ' for asymmetric tearing modes with $\alpha = 0.8$. The numerical data are represented by the asterisks, while the analytical prediction is represented by the dashed line.

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