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Nonlinear kinetic modelling of Stimulated Raman Scattering

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Our work is motivated by the need to provide an estimate for the laser reflectivity due to Raman scattering in the Laser MegaJoule (LMJ) [1] or the National Ignition Facility (NIF) [2]. In these facilities, the electron temperature T_e and density will be such that the electron plasma wave (EPW) generated by stimulated Raman scattering (SRS) will have a rather slow phase velocity v_φ. Then, as this wave grows, it significantly modifies the bulk of the electron distribution function : nonlinear kinetic effects are important. In particular, the non collisional damping rate, ν, of the EPW is reduced compared to its linear value, which enhances SRS. Moreover, the real frequency of the EPW nonlinearly shifts,which entails a phase shift $\delta\varphi$ between the EPW and the laser drive, and this quenches SRS. One of the key points to quantify these effects is the derivation of the electron susceptibility, χ, which we now address.

We assume that the total electric field is $\vec{E}_{\mathrm{tot}} = E_p \sin(\varphi)\hat{x} + [E_l \sin(\varphi_l) + E_s \cos(\varphi_s)]\,\hat{y}$, where E_p, E_l, and E_s are slowly-varying envelopes for, respectively, the plasma, laser, and scattered waves. The wave numbers and frequencies are given by $k_{\,,l,s} = \partial_x \varphi_{\,,l,s}$ and $\omega_{\,,l,s} = -\partial_t \varphi_{\,,l,s}$. The phase shift between the laser drive and the plasma wave is $\delta\varphi \equiv \varphi + \varphi_s - \varphi_l$. Then, the total longitudinal force, due to the electrostatic and electromagnetic waves, results from an effective electrostatic field : $E_{\mathrm{eff}} \equiv E_0 \sin(\psi)$, where E_0 and ψ can be found in [4]. Writing the electron density as $\rho = \rho_0 e^{i\psi} + c.c$, we define the electron susceptibility by $\chi \equiv i\rho_0/(\varepsilon_0 k E_0)$. Then, for immobile ions, Gauss law *exactly* gives the two following equations

$$[1+\mathrm{Re}(\chi)]E_p = [\mathrm{Re}(\chi)\sin(\delta\varphi) + \mathrm{Im}(\chi)\cos(\delta\varphi)]E_d \tag{1}$$

$$\mathrm{Im}(\chi)E_p - k^{-1}\partial_x E_p = -[\mathrm{Re}(\chi)\cos(\delta\varphi) + \mathrm{Im}(\chi)\sin(\delta\varphi)]E_d \tag{2}$$

where $E_d \equiv ekE_lE_s/[2m\omega_l\omega_s]$.

In order to make clearer the derivation of χ, we first consider uniform wave amplitudes. In this case, as shown in Ref. [4], $\mathrm{Re}(\chi) \propto \langle\cos(\psi)\rangle$ and $\mathrm{Im}(\chi) \propto \langle\sin(\psi)\rangle$, where $\langle\cdot\rangle$ stands for the statistical average over velocities and over one wavelength. Moreover, the longitudinal dynamics is defined by the Hamiltonian $H = v^2/(2m) - (E_0/k)\cos(\psi)$ whose phase portrait is shown in Fig. 1. It can be seen in this figure that, when the wave amplitude slowly changes with time, the electron orbits are nearly symmetric with respect to the v-axis (they would be perfectly symmetric if the wave amplitude were constant). It is then possible to find a good estimate of $\langle\cos(\psi)\rangle$ by replacing these orbits by perfectly symmetric ones, which amounts to making the adiabatic approximation. However, since sine is an odd function, the adiabatic estimate of $\langle\sin(\psi)\rangle$ would just yield 0. Moreover, it is clear from Fig. 1 that the orbits of deeply trapped electrons are much more symmetric with respect to the v-axis than those close to the virtual separatrix. As a result, deeply trapped electrons will give a negligible contribution of $\langle\sin(\psi)\rangle$ and therefore to $\mathrm{Im}(\chi)$. This is the key point to calcuate $\mathrm{Im}(\chi)$.

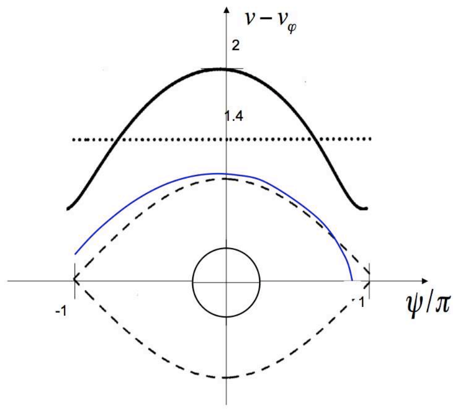

Figure 1: Phase portrait of Hamiltonian H for a slowly-varying wave amplitude. The dashed line shows the virtual separatrix.

The derivation of $\mathrm{Im}(\chi)$ is first explained when the waves amplitudes are uniform and then generalized to 3-D space variations. A first estimate, $\mathrm{Im}(\chi)_{\mathrm{pert}}$, is obtained from a first order perturbation analysis where the contribution of the deeply trapped electrons is removed. This estimate however breaks down for large values of $\int \omega_B dt$, where $\omega_B \equiv \sqrt{eE_0k/m}$ is the bounce frequency for the effective electrostatic field.

Now, if $\mathrm{Im}(\chi)$ were analytic, we could use the formula $\mathrm{Im}(\chi)(\omega+i\gamma) = \mathrm{Im}(\chi)(\omega+i0)+\gamma\partial_\omega \mathrm{Re}(\chi)$ where γ is the EPW growth rate. $\mathrm{Im}(\chi)(\omega+i0)$ is that part of $\mathrm{Im}(\chi)$ which accounts the non collisional damping rate, ν, of the EPW. As is well known (see Ref. [5]) ν vanishes as $\int \omega_B dt$ increases. Since $\int \omega_B dt \sim \omega_B/\gamma$, when the bounce frequency is large enough compared to the wave growth rate, one expects $\mathrm{Im}(\chi)(\omega+i\gamma) \approx \gamma\partial_\omega \mathrm{Re}(\chi)$. However, as noticed previously, the contribution to $\mathrm{Im}(\chi)$ of deeply trapped electrons is negligible so that, when ω_B/γ is large, the actual value of $\mathrm{Im}(\chi)$ is $\mathrm{Im}(\chi) \approx \gamma\partial_\omega \mathrm{Re}(\chi)_{\mathrm{u}}$ where the expression of $\mathrm{Re}(\chi)_{\mathrm{u}}$, which only includes the contribution of untrapped electrons, can be found in [4]. Then, in order to have an expression of $\mathrm{Im}(\chi)$ valid whatever the wave amplitude we connect the perturbative and non perturbative expressions

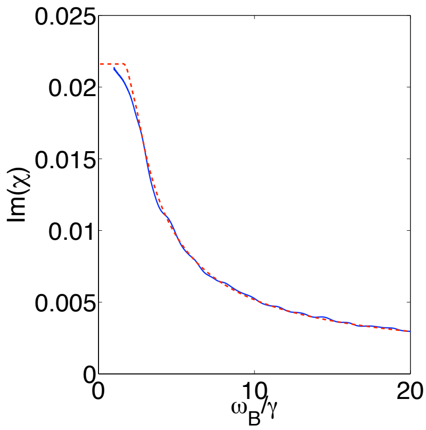

Figure 2: $\mathrm{Im}(\chi)$ calculated theoretically (red dashed line) and from a test particles simulation (blue solid line) when $\gamma = 10^{-2}\omega_{pe}$ and $v_\varphi = 3\sqrt{T_e/m}$.

of $\mathrm{Im}(\chi)$ the following way, $\mathrm{Im}(\chi) \approx \mathrm{Im}(\chi)_{\mathrm{pert}} \times [1 - Y(\omega_B/3|\gamma|)] + \gamma\partial_\omega \mathrm{Re}(\chi)_{\mathrm{u}} \times Y(\omega_B/3|\gamma|)$, where $Y(x) \equiv \tanh^5[(e^x - 1)^3]$. This theoretical estimate is in very good agreement with the values of $\mathrm{Im}(\chi)$ derived from test particles simulation as can be seen in Fig. 2, which also shows that $\mathrm{Im}(\chi)$ remains close to its linear value provided that $\omega_B < 3|\gamma|$. $\mathrm{Im}(\chi)$ may be more conveniently written the following way : $\mathrm{Im}(\chi)E_p = \partial_\omega \mathrm{Re}(\chi)_{\mathrm{eff}}[\nu + \partial_t]E_p$, with the following explicit formula for the nonlinear, non collisional, damping rate ν

$$\nu\partial_\omega \mathrm{Re}(\chi)_{\mathrm{eff}} = -\frac{f_0'(v_\phi)}{(k\lambda_D)^2}\left[\pi - 2\tan^{-1}\left(\frac{V_{lim}}{|\gamma|}\right) + \frac{2|\gamma|V_{lim}}{\gamma^2 + V_{lim}^2}\right] \times [1 - Y(\omega_B/3|\gamma|)] \tag{3}$$

where f_0 is the unperturbed distribution function, and $V_{lim} \equiv \max[0, (4\omega_B/\pi)(1 - 3\gamma/2\omega_B)]$ accounts for the exclusion of the deeply trapped electrons when evaluating $\mathrm{Im}(\chi)$ perturbatively.

The previous formula are easily generalized to account for a 3-D space variation of the waves amplitudes (see Ref. [4]). From Eq. (2), the envelope equation for the plasma wave then is

$$\partial_t E_p + v_g\partial_x E_p + \nu E_p \approx -[\partial_\omega \mathrm{Re}(\chi)_{\mathrm{eff}}]^{-1}\mathrm{Re}(\chi)\cos(\delta\varphi)E_d \tag{4}$$

where $\nu = \int \nu_0[\gamma(v_y, v_z)]f_0(v_y, v_z)dv_y dv_z$, $\nu_0[\gamma(v_y, v_z)]$ being defined by Eq. (3) with $\gamma(v_y, v_z) \equiv E_p^{-1}\left(\partial_t E_p + v_\varphi\partial_x E_p + v_y\partial_y E_p + v_z\partial_z E_p\right)$. A similar expression holds for $\partial_\omega \mathrm{Re}(\chi)_{\mathrm{eff}}$, and $v_g \equiv v_\varphi + [k\partial_\omega \mathrm{Re}(\chi)_{\mathrm{eff}}]^{-1}[\mathrm{Re}(\chi) - 1]$. From the previous expressions, it is clear that steep transverse gradients would increase the average value of $|\gamma|$ and, as a result, ν and $\mathrm{Im}(\chi)$ would remain close to their linear values up to larger wave amplitudes. In other words, linear theory remains valid up to larger amplitudes for steeper transverse gradients.

Let us now address the derivation of $\mathrm{Re}(\chi)$ which, as shown in Ref. [4], may be evaluated adiabatically provided that $|\gamma| < \omega_{pe}/20$. In this case, at zero-order in $(kE_p)^{-1}\partial_x E_p$, Eqs. (1) and (2) yield the dispersion relation of the SRS-driven EPW

$$1 + \alpha_d \mathrm{Re}(\chi) = 0 \tag{5}$$

where $\alpha_d \equiv [1 + 2\eta^{-1}\sin(\delta\varphi) + \eta^{-2}]/[1 + \eta^{-1}\sin(\delta\varphi)]$, and $\eta \equiv E_p/E_d$. In particular, when there is no laser drive, $E_d = 0$, $\alpha_d = 1$, and one recovers the usual electrostatic dispersion relation $1 + \mathrm{Re}(\chi) = 0$. Now, from Eq. (2), $\eta = -[\mathrm{Re}(\chi)\cos(\delta\varphi) + \mathrm{Im}(\chi)\sin(\delta\varphi)]/\mathrm{Im}(\chi)$. Since we previously showed that $\mathrm{Im}(\chi)$ decreases very quickly with the wave amplitude, so does η^{-1}. Hence, α_d quickly decreases towards unity as the wave amplitude increases, which entails an initial drop in ω that cannot be recovered from a purely electrostatic theory. By contrast, for large amplitudes ω decreases because of the nonlinear change in $\mathrm{Re}(\chi)$. Detailed calculations of α_d and $\mathrm{Re}(\chi)$ can be found in Refs. [3, 4] and, as shown in Fig. 3, there is a very good agreement between the theoretical values of $\delta\omega \equiv \omega(\Phi) - \omega(\Phi = 0)$ and those derived from 1-D simulations of SRS using the Vlasov code ELVIS [6]. Fig. 3 also shows that $|\delta\omega|$ is significantly larger than could be inferred from a purely electrostatic theory.

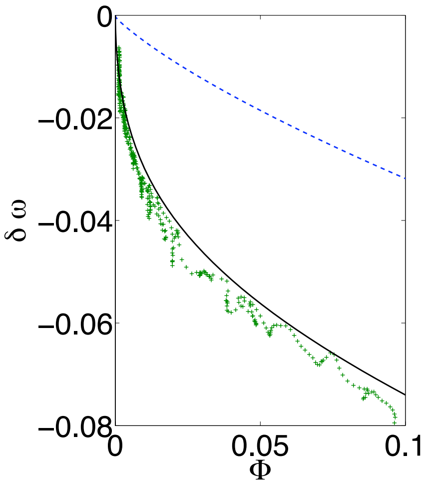

Figure 3: $\delta\omega$ calculated theoretically (black solid line) from a Vlasov simulation of SRS (green crosses) and derived from a purely electrostatic calculation (blue dashed line) when $k\lambda_D = 0.448$.

Let us now address the issue of the dephasing $\delta\varphi$. In case of Raman backscatter, the EPW amplitude initially increases with time and decreases with x (provided that the laser propagates from left to right). Since ω is a decreasing function of E_p, for short times $\partial_t \omega < 0$ and $\partial_x \omega > 0$. Then, because of the resonance condition $\omega_l = \omega_s + \omega$, $\partial_t \omega_s > 0$ and $\partial_x \omega_s < 0$ in the early stage of SRS. If the phase of the scattered wave φ_s is C^2 then, $\partial_t(\partial_x \varphi_s) = \partial_x(\partial_t \varphi_s)$, which implies $\partial_t k_s = -\partial_x \omega_s > 0$. Since $k_s < 0$, $\Delta_s \equiv (\omega_s^2 - k_s^2 c^2 - \omega_{pe}^2)/2\omega_s$ initially grows in time. Now, it is easy to prove (see Ref. [3]) that $\tan(\delta\varphi) = \Delta_s/\gamma_s$ where $\gamma_s \equiv E_s^{-1}[\partial_t E_s + (k_s c^2/\omega_s)\partial_x E_s]$. Therefore, the initial growth of Δ_s makes $\delta\varphi$ increase towards $\pi/2$ which, form Eq. (4), quenches SRS. For later times, as numerically shown in Ref. [4], k_s decreases roughly as $-\sqrt{\omega_s^2 - \omega_{pe}^2}/c$, which prevents $\delta\varphi$ to converge and remain close to $\pi/2$. Therefore, although the frequency shift does entail a phase shift which quenches SRS, this effect is mitigated by the k-shift.

In conclusion, starting from first principles, we could derive theoretically each term of the nonlinear envelope equation for the SRS-driven EPW. Hence, the only open issue to complete the nonlinear kinetic modelling of SRS is the nonlinear stability of the EPW.

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