

Role of temperature and density in stationary solutions of the Vlasov-Maxwell system

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Abstract

Stationary selfconsistent solutions of the Vlasov Maxwell system in a magnetized plasma (so called Vlasov equilibria) with both density and temperature gradients are investigated.

Studies of collisionless magnetic reconnection, in particular numerical studies, will require the knowledge of appropriate Vlasov equilibrium configurations as starting points. Vlasov equilibria corresponding to plasma configurations with temperature gradients (in addition to density gradients), to force free magnetic configurations or to magnetic fields with non-aligned plasma flows need be investigated. In fact, stationary solutions (equilibria) of the Vlasov-Maxwell system based on Jeans' theorem provide a convenient starting point in the investigation of the non-linear dynamics of electromagnetic plasmas in collisionless regimes and of stellar systems (with the gravitational potential replacing the electromagnetic potentials). In particular, isothermal equilibria with a nonuniform density are frequently considered because, among other reasons, they may be expected to be more resilient to the long term dissipative effects of particle collisions. In addition, they lead to physical models that are relatively simple to solve algebraically, although such models are often affected by divergences (as e.g., in isothermal stellar systems) or by unphysical boundary conditions. On the other hand plasma equilibria with nonuniform temperature distributions are of great interest as temperature gradients are known to affect the dynamics of magnetically confined plasmas, giving rise to new instabilities.

The well known static Harris pinch equilibrium describes a purely magnetic (i.e., fully neutral and with no plasma flows), isothermal one-dimensional stationary plasma configuration embedded in a magnetic field of the form $\mathbf{B}(x) = B_y(x)\mathbf{e}_y + B_z\mathbf{e}_z$, with B_z constant, possibly zero. The particle kinetic energy $\epsilon_j \equiv m_j v^2/2$ and the z component of the canonical momentum $p_{jz} = m_j v_z + Z_j e A_z/c$ are integrals of the particle motion. $j = e, i$ particle species Z_j, m_j particle charge number and mass. Any distribution function of the form $f_j(x, \mathbf{v}) = F_j(\epsilon_j, p_{jz})$ that satisfies the appropriate positivity and integrability conditions is a stationary solution of Vlasov's equation. The set of the Vlasov-Maxwell equations is then closed by calculating the current density along

z and by solving Ampère's equation $\nabla^2 A_z(x) = -4\pi \sum_j [Z_j e \int d^3v v_z F_j(\boldsymbol{\varepsilon}_j, p_z)]/c$, for the unknown vector potential, after imposing that the particle densities $n_j = \int d^3v F_j(\mathbf{v}^2, p_z)$ be equal, i.e., that the configuration is charge neutral. This allows us to find the spatial dependence of the magnetic field selfconsistently.

Obtaining Vlasov equilibria with nonuniform temperatures poses some conceptual and technical problems even in the 1-D case.

From the technical point of view it turns out that imposing the neutrality condition is algebraically more involved¹ than in the isothermal case unless, for the sake of simplicity, we assume a cold ion distribution $f_i(\mathbf{x}, \mathbf{v}) = n_e(\mathbf{x}) \delta^3(\mathbf{v})$, where the particle position is now a constant of the motion. Such an ion distribution provides charge neutrality but does not contribute to the plasma current.

From the conceptual point of view we are confronted by a wide variety of physically viable functional dependence of the electron distribution function on the particle energy ε and canonical momentum p_z . A guideline can be set by looking for solutions where the entropy minimization is performed at fixed canonical momentum in which case the Lagrange multipliers related to the particle and energy conservation become functions of the canonical momentum. On this basis we introduce two arbitrary functions $h(p_z)$ and $g(p_z)$ and consider electron distribution functions of the form [1]

$$F_e(\varepsilon, p_z) = \frac{n_0 g^2(p_z)}{(2\pi T_0/m_e)^{3/2}} \exp[-h^2(p_z) \varepsilon/T_0]. \quad (1)$$

By integrating over $v_{x,y}$ and by defining $l(p_z) \equiv g^2(p_z)/h^2(p_z)$, we obtain $\mathcal{F}_e(v_z^2, p_z) = [n_0 l(p_z)/(2\pi T_0/m_e)^{1/2}] \exp[-h^2(p_z) m_e v_z^2/(2T_0)]$, with

In a number of investigations on plasma stability it would be of interest to start from stationary Vlasov solutions with inhomogeneous temperature but uniform density, so as to separate the effects of temperature and density gradients. Such configurations are not available within the class of solutions described by Eq.(1), unless we resort to a perturbative approach where the dependence of the electron distribution on p_z is assumed to be weak. In this case if we set $l(p_z) = h(p_z) = 1 + \eta \tilde{h}(p_z)$, where $\eta > 0$ is a small parameter such that $\eta |\tilde{h}| \ll 1$ for all x and for all values of v_z within the main body of the distribution function, *the density is uniform up to $\mathcal{O}(\eta^2)$ terms* provided $\tilde{h}(p_z)$ is a first or second order polynomial in p_z .

¹Actually local neutrality need not be imposed in order to find physically interesting equilibria, but the inclusion of the electrostatic potential leads to a nonlinear system where Poisson's and Ampère's equations are coupled.

In the interesting case $\tilde{h}(p_z) = p_z^2$, to first order in η , we find

$$n_e/n_0 \sim 1 - \eta, \quad j_z/(en_0v_{the}) \sim -2\eta A_z, \quad (2)$$

and $\Pi_{xx}/(n_0T_0) = \Pi_{yy}/(n_0T_0) \sim 1 - 2\eta(1 + A_z^2)$, $\Pi_{zz}/(n_0T_0) \sim 1 - 2\eta(3 + A_z^2)$, where Π is the anisotropic pressure tensor. Solving Ampère's equation we find an oscillatory magnetic field configuration with $A_z = \cos[(2\eta)^{1/2}x]$ and an oscillatory pressure tensor.

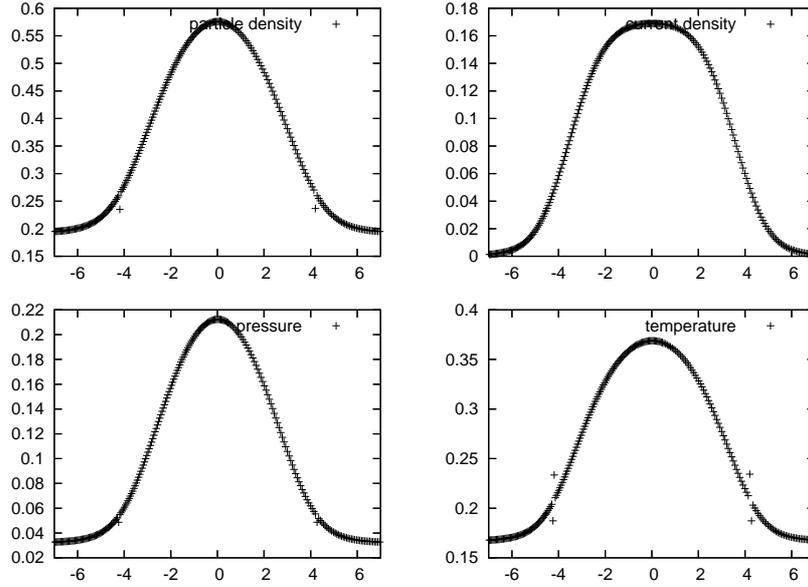


Figure 1: Particle and current densities, pressure and temperature profiles.

We search for solutions with temperature and density distributions (and thus with magnetic field) that are *asymptotically constant*, and with density and temperature gradients near the null lines of the magnetic field. In particular we are interested in configurations where, in contrast to Harris solution, the *density at large x does not vanish*. It is worth noting that the systems described here are intrinsically *anisotropic* due to the asymmetric dependence on the three components of \mathbf{v} of the distribution function. The only nonzero components of the pressure tensor are the diagonal ones, and they read

$$\Pi_{xx} = \Pi_{yy} = \frac{n_0 T_0}{\sqrt{\pi}} \int_{-\infty}^{+\infty} dv_z \frac{g^2(p_z)}{h^4(p_z)} e^{-h^2(p_z)v_z^2},$$

$$\Pi_{zz} = \frac{2n_0 T_0}{\sqrt{\pi}} \int_{-\infty}^{+\infty} dv_z (v_z - V_{ez})^2 \frac{g^2(p_z)}{h^2(p_z)} e^{-h^2(p_z)v_z^2}.$$

The difference between Π_{zz} and the other two components gives an estimate of the anisotropy of the system. This property is important because the reconnection instabilities are strongly

affected by anisotropy. Since the equilibrium rests only on the xx component of the pressure tensor, we define the temperature as $T/T_0 = \Pi_{xx}/n_e$.

As a relevant example we consider the distribution function

$$F_e(p_z, \mathbf{v}^2) = [n_0/(\pi^{3/2}v_{the}^3)] \exp[-(\alpha + \tanh p_z) \mathbf{v}^2], \quad (3)$$

which is obtained with the choice $g(p_z) = 1$, $h^2(p_z) = \alpha + \tanh p_z$, $\alpha > 1$.

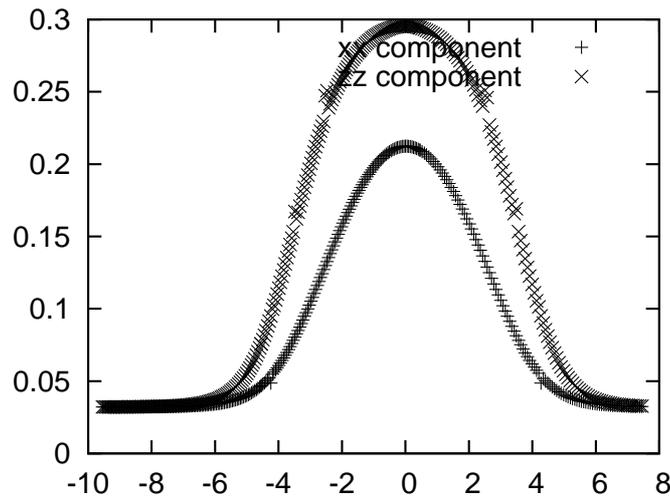


Figure 2: Diagonal components of the pressure tensor.

The particle and current density, pressure and temperature profiles are shown in Fig.(1). The current sheet typical of the Harris pinch can be recognized. The anisotropy of the system is shown in in Fig.(2): Π_{xx} and Π_{zz} have almost the same behaviour, except that the latter is bigger. This distribution function is single-peaked in v_z . The magnetic field has an hyperbolic tangent shape, resembling the Harris pinch configuration. As in Harris pinch the particle density, pressure and temperature have a maximum at $x = 0$, corresponding to the zero of the magnetic field, and are asymptotically constant (but not zero) at infinity.

An important feature to be stressed in that the introduction of a temperature inhomogeneity is accompanied by the appearance of temperature anisotropy. This is due to the dependence of the distribution function on one component of the canonical momentum which does not simply lead to a shift in velocity space, as is the case for the Harris pinch.

References

- [1] C. Montagna, F. Pegoraro. *Phys. Plasmas*, **14**, 042103 (2007) .