# Nonlinear dynamics of magnetic reconnection in collisionless plasmas

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Magnetic reconnection (MR) consists of a rearrangement of the connection between magnetic field lines and plasma elements and is a process of great relevance for both astrophysical and laboratory plasmas. Indeed MR is believed to play a key role in phenomena such as solar flares, coronal mass ejections, sawtooth oscillations in tokamaks and magnetic relaxation in reversed field pinches [1, 2]. MR can take place only if the condition of freezing of the magnetic field into the plasma is violated. In high temperature plasmas, where collisions are infrequent, such violation can be caused by the electron inertia, which prevents the plasma from behaving as a perfect conductor.

A fluid four-field model for MR in collisionless plasmas has been derived in Refs [3, 4] and the corresponding dimensionless model equations in a Cartesian cordinate system $(x,y,z)$ are given by

$$\frac{\partial(\psi - d_e^2\nabla^2\psi)}{\partial t} + [\varphi, \psi - d_e^2\nabla^2\psi] - d_\beta[\psi, Z] = 0, \tag{1}$$

$$\frac{\partial Z}{\partial t} + [\varphi, Z] - c_\beta[v, \psi] - d_\beta[\nabla^2\psi, \psi] = 0, \tag{2}$$

$$\frac{\partial \nabla^2\varphi}{\partial t} + [\varphi, \nabla^2\varphi] + [\nabla^2\psi, \psi] = 0, \tag{3}$$

$$\frac{\partial v}{\partial t} + [\varphi, v] - c_\beta[Z, \psi] = 0. \tag{4}$$

where $\mathbf{B} = \nabla\psi \times \hat{\mathbf{z}} + (B^{(0)} + c_\beta Z)\hat{\mathbf{z}}$ is the magnetic field and $\mathbf{v} = -\nabla\varphi \times \hat{\mathbf{z}} + v\hat{\mathbf{z}}$ is the plasma average velocity. $B_0$ is the intensity of the constant guide field and $c_\beta$ is a constant defined as $c_\beta = \sqrt{\beta/(1+\beta)}$ where $\beta = (5/3)P^{(0)}/{B^{(0)}}^2$, with $P_0$ indicating a constant background pressure. The parameter $d_\beta$ is given by $d_\beta = d_i c_\beta$, with $d_{i,e}$ indicating the ion and electron skin depth, respectively. Notice that in (1), which originates from the electron momentum equation, it is the presence of a term proportional to $d_e^2$, i.e. to the electron inertia, that allows MR to take place.

The system (1)-(4) has been shown to admit a non-canonical Hamiltonian formulation [5, 6] and the corresponding Poisson bracket has been shown to possess four infinite independent families of Casimirs. The knowledge of such invariants made it possible to reformulate the system as

$$\frac{\partial D}{\partial t} = -[\varphi, D], \tag{5}$$

$$\frac{\partial \omega}{\partial t} = -[\varphi, \omega] + (d_e^2 + d_i^2)^{-1}[D, \psi], \tag{6}$$

$$\frac{\partial T_+}{\partial t} = -[\varphi_+, T_+], \tag{7}$$

$$\frac{\partial T_-}{\partial t} = -[\varphi_-, T_-]. \tag{8}$$

In the above equations $D = \psi - d_e^2\nabla^2\psi + d_i v$ is the ion canonical momentum (up to corrections of order $d_e^2/d_i^2$), $\omega = \nabla^2\varphi + Z/(d_\beta + c_\beta d_e^2/d_i)$ is a "generalized" vorticity, whereas the fields $T_\pm = D - (d_i + d_e^2/d_i)v \mp d_e\sqrt{1 + d_e^2/d_i^2}Z$ are scalar fields advected by velocity fields obtained from the generalized stream functions $\varphi_\pm := \varphi \pm c_\beta\sqrt{1 + d_i^2/d_e^2}\psi$. The above formulation makes it possible to see that the system possesses the three Lagrangian invariants $D$, $T_+$ and $T_-$, whose contour lines are preserved during the time evolution of the system. The model equations (1)-(4) have been solved over a rectangular domain consisting of 1024×1024 gridpoints and imposing double periodic boundary conditions. The adopted initial condition corresponds to the equilibrium $\psi_0(x) = 1/\cosh^2(x), \phi_0 = 0, Z_0 = 0, v_0 = 0$, which is unstable to reconnecting perturbations. The evolution of the system is triggered by means of a perturbation on the current density of the form $\delta j(x,y) = \delta j(x)\cos(2\pi y/L_y)$, where $\delta j(x)$ is a function localized around the resonant surface $x = 0$ and $L_y$ is the length of the simulation domain in the $y$ direction. After the perturbation has been turned on the system evolves, allowing reconnection to take place, which leads to the formation of magnetic islands. In this contribution, rather than on the magnetic structures, we focus in particular on the identification of vorticity structures that appear as a consequence of the reconnection process.

In Fig. 1 contour plots of the vorticity field $U = \nabla^2\varphi$ are shown at different times in a very low-$\beta$ regime but with a finite value of $d_\beta$ (note that for $\beta \to 0$ then $d_\beta$ tends to $\rho_s$, which is the Larmor radius of ions with electron temperature). From the plots it emerges that the vorticity field tends to form filamented structures with increasingly small spatial scales. This behavior can be explained by noting that in the $c_\beta \to 0$ limit, with finite $d_\beta$, the four-field model essentially reduces to the two-field model derived by Schep et al. [7]. Indeed the vorticity filamentation process was already observed and explained in the context of this two-field model [8]. By making use of the same argument adopted for the two-field model we can then conclude that the filamentation process is due to the stretching of the Lagrangian invariants $T_\pm$, which are advected by the fields $\varphi_\pm$, whose contour lines evolve in time rotating in opposite directions.

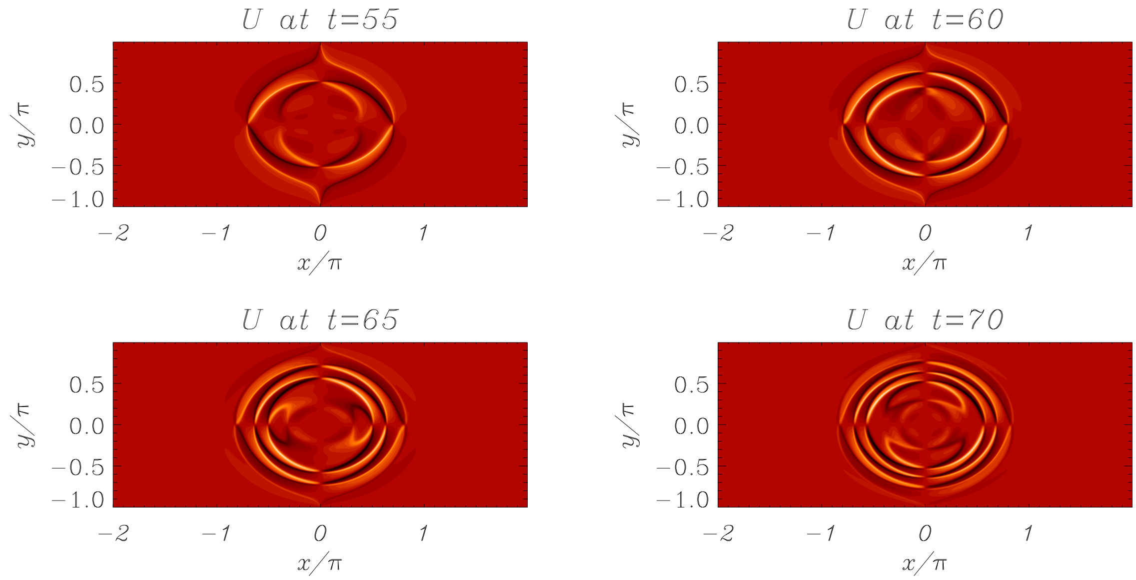


Figure 1: Contour plots of the vorticity $U$ at different time steps for $c_\beta = 0.001$, $d_i = 240$, $d_e = 0.24$. The amplitude of the field point from pale yellow to black.

Given the relation $U = (T_+ - T_-)/2d_e d_\beta + \omega$, it emerges that the filamented structures observed in $U$ are given by the difference between the stretched fields $T_\pm$. It is important, indeed, to point out that in this limit the contribution of the field $\omega$ is negligible. In fact, although it was not possible to show the related plots due to limitation in the size of the present paper, it can be seen that for very low $\beta$, the amplitude of the $\omega$ field only weakly deviates from the initial value $\omega_0 = 0$.

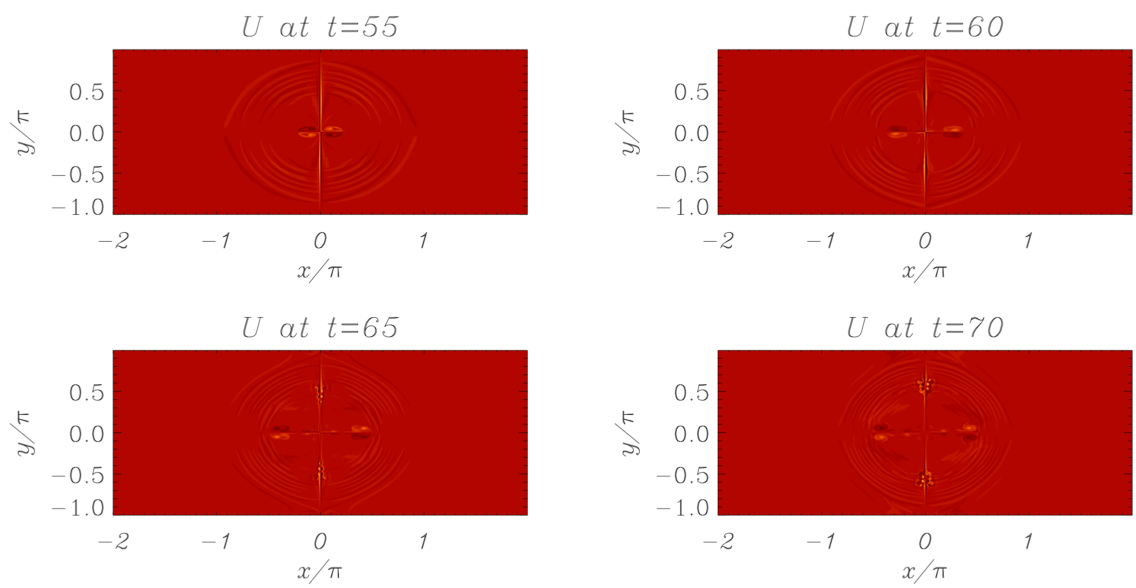


Figure 2: Contour plots of the vorticity $U$ at different time steps for $c_\beta = 0.3$, $d_i = 2.4$, $d_e = 0.24$. The amplitude of the field point from pale yellow to black.

On the other hand, Fig. 2, which refers to a higher $\beta$ case, shows qualitatively different structures in the vorticity plots. Indeed in this regime one observes the formation of two vertical jets colliding at the center of the simulation box. After the collision, two pairs of vortices are formed, which propagate in opposite directions toward the boundaries along the $y = 0$ axis. In the meantime the vertical jets undergo a Kelvin-Helmholtz-like instability due to the presence of local strong velocity gradients. Note that a filamented structures, analogous to the one observed in the very low-$\beta$ case, is also present. The coexistence of filamented structures and unstable jets can again be interpreted in terms of the fields $T_\pm$ and $\omega$. Indeed, at higher $\beta$ the field $\omega$ grows due to the presence of the source term in Eq. (6), which in this regime plays an effective role. The vorticity plots are then given by the superposition of the filamented part, due to the term $\frac{T_+ - T_-}{2d_e d_\beta}$, and of the structures originated by the jets collision, which are due to the evolution of $\omega$. Note that a Kelvin-Helmholtz-like instability following a reconnection process, was already observed in the two-field model in the limit of cold electrons [9, 10]. However, in that case filamentation was strongly suppressed due to a transition in the form of the invariants as the electron temperature goes to 0. The above results, on the other hand, show that at sufficiently high values of $\beta$, filamentation and Kelvin-Helmholtz instability can actually coexist.

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