NUMERICAL SIMULATION FOR THE FORMATION OF A CHARGE SEPARATION AND AN ELECTRIC FIELD AT A PLASMA EDGE

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We study the problem of the generation of radial electric fields and poloidal flows to achieve radial force balance at a steep plasma gradient, in the presence of an external magnetic field. This problem is of importance for the physics of an H-mode in a tokamak. The problem is solved in a cylindrical geometry, for the case when the ion gyro-radius ρ_i is much larger than the Debye length λ_{De}. An Eulerian code is used to solve the Vlasov-Poisson system for the ions [1], two-dimensional in space (r,θ) and in velocity space (radial and poloidal), and uniform in the z direction. The constant magnetic field directed in the z-direction is of the form $B_z = B_0 /(1+ \in \cos\theta)$, where B_0 is the magnetic field in the center, and $\in = 0.2$. The plasma edge is assumed free (no floating limiter is placed in front of the plasma edge). The electrons, frozen by the magnetic field lines, have a constant density profile, which varies in space rapidly along the gradient over an ion orbit size. The frozen electrons cannot move across the magnetic field to exactly compensate the ion charge which results from the finite ions' gyro-radius along the gradient. A charge separation appears at the edge of the plasma. We compare the electric field calculated from the kinetic code with the macroscopic values calculated for the gradient of the ion pressure, and we find that these quantities balance each other along the gradient very well. The Lorentz force term is negligible along the gradient.

The pertinent equations

The inhomogeneous direction is the radial direction r, normal to a vessel surface located at $r = R$. Ions are described by 2D Vlasov equation for distribution function $f_i(r,\ \theta,\ v_r,\ v_\theta,\ t)$:

$$\frac{\partial f_i}{\partial t} + v_r \frac{\partial f_i}{\partial r} + \frac{v_\theta}{r} \frac{\partial f_i}{\partial \theta} + \left[E_r + v_\theta \omega_{ci} + \frac{v_\theta^2}{r} \right] \frac{\partial f_i}{\partial v_r} - \left(-E_\theta + \omega_{ci} v_r + \frac{v_r v_\theta}{r} \right) \frac{\partial f_i}{\partial v_\theta} = 0 \quad (1)$$

Which is coupled to Poisson's equation:

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = -(n_i - n_e) \ \ ; \ \ E_r = -\frac{\partial \phi}{\partial r} \ \ ; \ \ E_\theta = -\frac{1}{r} \frac{\partial \phi}{\partial \theta} \qquad (2)$$

In Eq.(1) time is normalized to the inverse ion plasma frequency ω_{pi}^{-1}, velocity is normalized to the acoustic velocity $c_s = \sqrt{T_e/m_i}$, and length to the Debye length $\lambda_{De} = c_s / \omega_{pi}$. T_e is the

electron temperature and m_i is the ion mass. The potential is normalized to T_e / e, and the density is normalized to the peak initial central density. ω_{ci} is the ion cyclotron frequency (normalized to ω_{pi}). The system is solved over a length $L = 150$ Debye lengths in front of the vessel surface, with an initial ion distribution function for the deuterons over the domain $r = \{R - L, R\}$ given in our normalized units, by:

$$ f_i(r, \theta, v_r, v_\theta) = n_i(r) \frac{e^{-(v_r^2 + v_\theta^2)/2 T_i}}{2\pi T_i} \qquad ; \qquad n_i(r) = 0.5(1 + \tanh((R - r - L/2)/4) \quad (3) $$

$n_e(r) = n_i(r)$ are the initial ion and electron densities respectively, $n_e(r)$ remains constant for the frozen electrons. The edge of the plasma is at $r = R - L/2$, around the center of the domain. We use also the following parameters:

$$ \frac{T_i}{T_e} = 1 ; \qquad \frac{\rho_i}{\lambda_{De}} = \sqrt{\frac{2T_i}{T_e}} \frac{1}{\omega_{ci0}/\omega_{pi}} = 10\sqrt{2} \qquad\qquad (4) $$

We have $\omega_{ci} = \omega_{cio}/(1 + \epsilon \cos \theta)$, and $\omega_{cio}/\omega_{pi} = 0.1$ in the present calculations. With the initial distribution function for the ions in Eq. (3), we have $T_{i\theta} = T_{ir} = T_i$, and the factor $2T_i$ in Eq. (4) takes into account that the perpendicular temperature is $\langle v_\perp^2 \rangle = \langle v_r^2 \rangle + \langle v_\theta^2 \rangle = \frac{2T_i}{m_i}$.

Eq.(1) is solved by a method of fractional step coupled to a method of characteristics [1].

Results

The deuterons are far away from the wall surface at $r = R$ and are not hitting the vessel surface, where the electric field E_r is zero. The electric field E_θ remained very small and negligible. Integrating Eq. (2) over the domain (R-L, R), and neglecting the azimuthal variation $\frac{1}{r}\frac{\partial \phi}{\partial \theta}$, we get the total charge σ:

$$ R E_r\big|_{r=R} - (R - L)E_r\big|_{r=R-L} = \int_{R-L}^{R}(n_i - n_e)r\, dr = \sigma \qquad\qquad (5) $$

We assume that the plasma ions are allowed to enter or leave at the left boundary. So the difference in the electric fields at the left boundary $r = R - L$ and at the wall $r = R$ in Eq. (5) must be equal to the total charge appearing in the system, and Eq. (5) must be satisfied at every time-step. In the present simulation, $E_r\big|_{r=R}$ in vacuum is set equal to zero. Then the resulting electric field at $E_r\big|_{r=R-L}$, calculated at $r = R - L$ from Eq. (2), must satisfy Eq. (5), so that $-(R - L)E_r\big|_{r=R-L} = \sigma$. We use a large value of the cylindrical plasma radius R

($R = 10000$ Debye lengths in the present calculation), so that the system behaves essentially as a Cartesian system. Fig.(1) shows the electric field at $\theta = 0$ (full curve), $\theta = \pi/2$ (broken line) and $\theta = \pi$ (dot-dash line), at time $t = 520$. The results remained symmetric from $\theta = \pi$ to $\theta = 2\pi$. The distribution of the charge (see Fig.(3)) is such that the total charge σ is zero. So from Eq.(5) the electric field at $r = R\text{-}L$ is zero, at 32 points over the 2π circle. The electric field exists only in a layer around the gradient. Fig.(2) gives at time $t = 520$ and for $\theta = 0$ the quantity $n_i E_r$ (full curve), the dash-dotted curve gives the Lorentz force, given by $- n_i <v_\theta> \omega_{ci} / \omega_{pi} = -0.1 n_i <v_\theta> /(1.+0.2)$, and the broken curve gives the pressure force ∇P_i, $P_i = 0.5 n_i (T_{ir} + T_{i\theta})$, with the following definition:

$$T_{ir,\theta}(r,\theta) = \frac{1}{n_i} \int dv_r\; dv_\theta\; (v_{r,\theta} - <v_{r,\theta}>)^2\; f_i(r,\theta,v_r,v_\theta) \tag{6}$$

$$<v_{r,\theta}> = \frac{1}{n_i} \int dv_r\; dv_\theta\; v_{r,\theta}\; f_i(r,\theta,v_r,v_\theta)\; ;\; n_i(r,\theta) = \int dv_r\; dv_\theta\; f_i(r,\theta,v_r,v_\theta) \tag{7}$$

Fig.(2) indicates a very nice agreement for the relation $n_i E_r \approx \nabla P_i$ along the gradient (the density $- n_i /12$ is also plotted for reference). The Lorentz force term remains negligible in the gradient region. The ∇P_i term is zero in the bulk at the left boundary since n_i and T_i are flat in that region.. Fig.(3) shows the charge density $n_i - n_e$ at the time $t = 520$ and $\theta = 0$ (full curve), $\theta = \pi / 2$ (broken curve) and $\theta = \pi$ (dash-dot curve). The charge is important along the gradient. Note that the spatial distribution of the charge is such that the total charge is zero. The electrons, which are frozen to the magnetic field lines, cannot compensate along the gradient the charge separation caused by the finite ion gyroradius. Fig.(4) presents at $t = 520$ the potential at $\theta = 0$ (full curve), $\theta = \pi/2$ (broken curve) and $\theta = \pi$ (dash-dot curve). Fig.(5) presents the $\vec{E} \times \vec{B} / B^2$ drift, which is written $- E_r(1+0.2 \cos\theta)/0.1$, at $\theta = 0$ (full curve), $\theta = \pi/2$ (broken curve) and $\theta = \pi$ (dash-dot curve). We plot in Fig. (6) the total poloidal current $n_i \left(\vec{E} \times \vec{B} / B^2 + \vec{v}_D \right)$, where the diamagnetic drift is $\vec{v}_D = \vec{B} \times \nabla n_i T_i / \left(n_i eB^2 \right)$ (in our units, the total poloidal current is $\left(-n_i E_r + \nabla P_i \right) \left(1 + 0.2 \cos\theta \right)/ 0.1$). This total current is essentially zero, the $\vec{E} \times \vec{B} / B^2$ drift and the diamagnetic drift are essentially equal and opposite. This is the result we get if we calculate the poloidal current $J_{i\theta} = n_i <v_\theta>$.

The absence of shear or very weak poloidal dependence of the $\vec{E} \times \vec{B} / B^2$ drift at the edge, can perhaps explain the absence of turbulence at the edge of the H-mode in a tokamak.

References

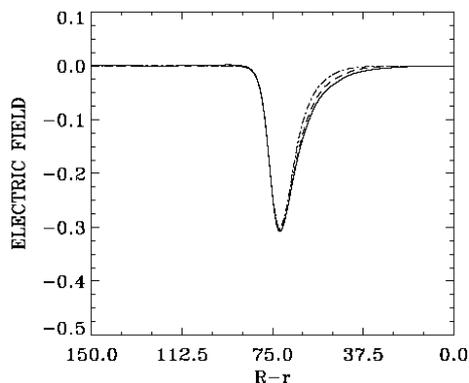[1] M. Shoucri, H. Gerhauser, K.H. Finken Comp. Phys. Comm. 164, 138 (2004)

Fig.1 Electric field E_r at θ=0, (full curve), $\theta = \pi / 2$ (dash curve), $\theta = \pi$ (dash-dot curve)
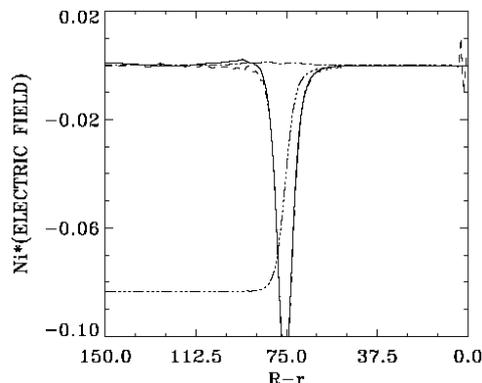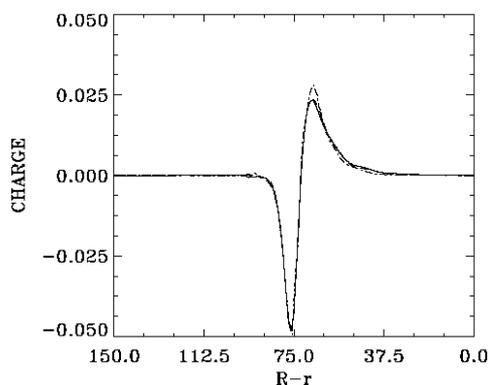
Fig.2 Plot at θ=0 of $n_i E_r$ (full curve), ∇P_i (dash curve), $-0.1 n_i < v_\theta > /(1.+0.2)$ (dash-dot curve), and $-n_i /12$ (dash-3 dots curve)

Fig.3 Charge at θ=0, (full curve), $\theta = \pi / 2$ (dash curve), $\theta = \pi$ (dash-dot curve)
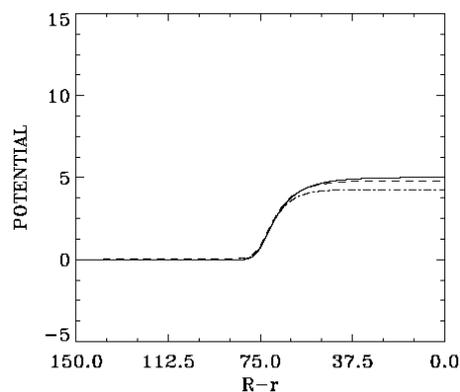
Fig.2 Potential at θ=0, (full curve), $\theta = \pi / 2$ (dash curve), $\theta = \pi$ (dash-dot curve)
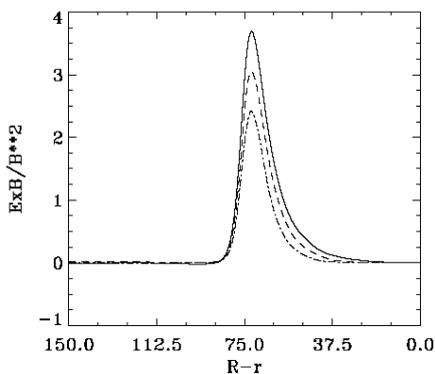
Fig.5 Drift at θ=0, (full curve), $\theta = \pi / 2$ (dash curve), $\theta = \pi$ (dash-dot curve)
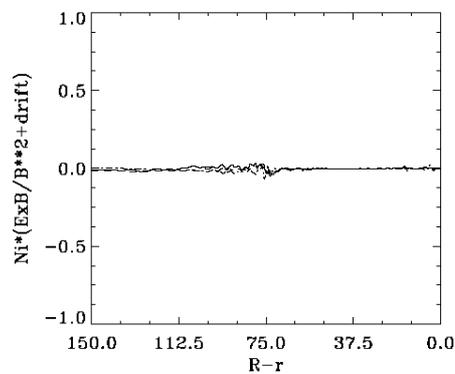
Fig.6 Poloidal current at θ=0, (full curve), $\theta = \pi / 2$ (dash curve), $\theta = \pi$ (dash-dot curve)