

## On Kinetic Theory and Hall MHD Description of the $q=1$ Inertial Layer in Fishbone Modes

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**1. Introduction:** An oscillatory “fishbone” instability with toroidal mode number  $n=1$  and radial plasma displacement similar to the “top-hat” structure of the internal  $n=1$  kink mode was discovered on the PDX tokamak with neutral beam injection (NBI) [1] and was then observed on many tokamaks. First theoretical interpretations [2, 3] of the fishbone instability established the resonant wave particle interaction as a key drive of the instability and identified two characteristic radial scales of a fishbone mode: an “outer” region comparable to the  $q=1$  radius where the energetic particle drive comes from, and an “inertial” layer in the vicinity of the  $q=1$  surface, in which a small radial scale is determined by the timescale of the instability. In the inertial layer, continuum damping balances the drive and determines the instability threshold [2]. It was later found in [4] that the inertial layer of the fishbone mode [2] has a two-layer structure, with a splitting between the two layers of

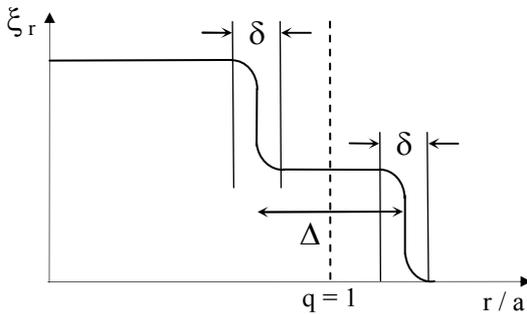


Fig 1: Double-layer structure of fishbone

$\Delta \sim r_1 R \omega / S_1 V_A$ , and the inner width of each layer,  $\delta \sim (\gamma / \omega) \Delta$ , as Figure 1 shows. Here,  $r_1$  is the radius of the  $q=1$  surface,  $R$ ,  $S_1$ ,  $V_A$  are major radius, magnetic shear at  $r=r_1$  and Alfvén velocity,  $\omega$  and  $\gamma$  are the mode frequency and growth rate.

The  $n=1$  fishbone instability is often observed in NBI-heated plasmas of spherical tokamaks NSTX [5] and MAST [6], where the tight aspect ratio and high values of  $\beta \sim 10\text{-}40\%$  make description of the fishbone drive non-trivial [5, 7]. In this paper, we note that one-fluid MHD description of the inertial layer of the fishbone also may break down in STs, as the ion Larmor radius,  $\rho_i = V_{Ti} / \omega_{Bi} \approx 1.5$  cm, typically satisfies  $\rho_i / \delta \sim S_1 / (\gamma / \omega) \sim 10 \gg 1$  and may become comparable to  $\Delta$ . Here, typical MAST parameters  $B_0 \sim 0.4$  T,  $n \sim 5 \cdot 10^{19} \text{ m}^{-3}$ ,  $r_1 \sim a / 3 \sim 0.2$  m,  $R \sim 0.9$  m,  $S_1 \sim 0.1$  and fishbone frequency of  $\omega \approx 1.3 \cdot 10^5 \text{ sec}^{-1}$  with a net growth rate  $\gamma / \omega \sim 1\%$  were used. In the regime with  $k_{\perp} \rho_i \geq 1$ , an adequate description of the fishbone layer requires a kinetic approach, similar to the  $n=1$  kink mode with full-order finite Larmor radius (FLR) effect [8].

**2. Kinetic description of plasma perturbations:** We consider the precessional fishbone mode [2] in the frequency range  $\omega_{*i} \ll \omega \ll \omega_{Bi}$ , in the hot-ion regime,  $\rho_i / \Delta \geq 1$ , with  $|d/dr|^{-1} \sim \delta < \Delta \ll r_1$ . The response of ion distribution function to electric field, e.g. in the form  $E \equiv E_x = E_0 e^{-i\omega t + ikx}$ , is found from Vlasov kinetic equation as a function of integrals of the unperturbed ion motion [9]. In a slab geometry, with  $B_0$  in the  $z$  direction, the unperturbed ion motion is given by:

$$\mathbf{v} = V(\mathbf{e}_x \cos \phi + \mathbf{e}_y \sin \phi), \quad \phi = \alpha - \omega_{Bi} t; \quad x = X - (V / \omega_{Bi}) \sin \phi, \quad y = Y + (V / \omega_{Bi}) \cos \phi,$$

where  $V$ ,  $\alpha$ ,  $X$  and  $Y$  are constants of integration. The electric field of the fishbone perturbs the velocities and coordinates of ions by  $\delta \mathbf{v}$  and  $\xi$ , which are given by:

$$d(\delta \mathbf{v}) / dt = [\delta \mathbf{v} \times \mathbf{b}] \cdot \omega_{Bi} + e_i \mathbf{E} / m_i; \quad d\xi / dt = \delta \mathbf{v}.$$

Substituting for  $x$  in the electric field and integrating, one finds the components of the ion displacement,  $\xi_x$ ,  $\xi_y$ , correct to the zeroth and first powers of  $\cos \phi$  and  $\sin \phi$ :

$$2\xi_x = \xi_+ + \xi_-, \quad 2\xi_y = -i(\xi_+ - \xi_-); \quad \xi_{\pm} = -\frac{eE}{m_i} \sum_{n=-\infty}^{+\infty} \frac{J_n^2 + J_{n+1}(J_{n+1} e^{i\phi} + J_{n-1} e^{-i\phi})}{(\omega - n\omega_{Bi})[\omega - (n \pm 1)\omega_{Bi}]},$$

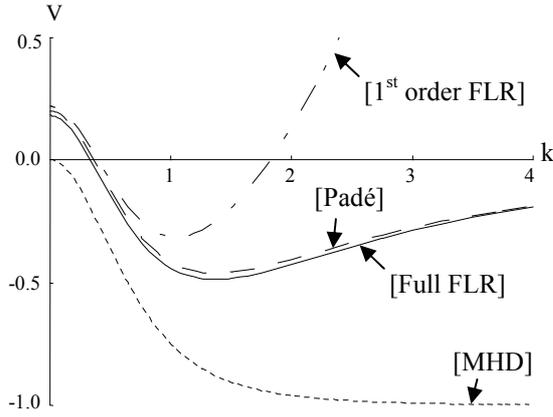
where the Bessel function  $J_n \equiv J_n(kV / \omega_{Bi})$ , allow the perturbed ion density to be calculated:

$$n'_i = \int f_{0i} \left( y - (V / \omega_{Bi}) \cos \phi - \xi_y \right) (1 - ik\xi_x) d^3 \mathbf{v} = -n_0 \frac{e\varphi}{T_i} [1 - I_0(b) e^{-b}], \quad (1)$$

where  $b_0 \equiv k^2 T_i / (m_i \omega_{Bi}^2)$ , and  $f_{0i}$  is the Maxwellian function. Similar ion response is also found in the gyro-kinetic approach [10], while the electron response obtained from the continuity equation and the drift kinetic equation is [10]:

$$n'_e = k_{\parallel} j_{\parallel} / (e_e \omega); \quad j_{\parallel} = \left( \omega^2 n_0 e^2 / c T_e k_{\parallel}^2 \right) [1 + \zeta Z(\zeta)] \left( A_{\parallel} - (ck_{\parallel} / \omega) \varphi \right). \quad (2)$$

Here,  $\zeta \equiv \omega / k_{\parallel} v_{Te}$ ,  $k_{\parallel} \equiv \mathbf{k} \cdot \mathbf{B}_0 / B_0$ ,  $Z(\zeta)$  is the plasma dispersion function and the electrostatic potential  $\varphi$  and the component of the vector potential along  $\mathbf{B}_0$ ,  $A_{\parallel} \equiv \mathbf{A} \cdot \mathbf{B}_0 / B_0$ , were introduced to describe the perturbed electromagnetic field. Charge neutrality and Ampère's law then lead to an eigenvalue equation, which in the presence of a sheared magnetic field,  $\mathbf{B} = (0, x/L_s, 1)B$ , and perturbations of the form  $\phi = \phi(x) e^{-i\omega t + ik_y y}$ , may be recast in the form of the  $E = 0$  solution of a Schrödinger equation  $d^2 \psi / dk^2 + [E + V(k)] \psi = 0$  for  $\psi = (1 + k^2)^{1/2} \tilde{\phi}$ . With  $\Gamma_0 = I_0(b) e^{-b}$ ,  $b = b_0 (1 + k^2)$ ,



**Fig 2:** Effective potential:  $\Omega=1$ ,  $\tau=1$ ,  $b_0=1/8$

$$b_0 = k_y^2 \rho_i^2 / 2, \tau = T_e / T_i, \quad k = k_x / k_y,$$

$$\Omega = \omega L_s / v_A \quad \text{and} \quad \tilde{\phi} = [\tau(\Gamma_0 - 1) - 1] \hat{\phi}, \quad \text{the}$$

effective potential is:

$$V(k) = \frac{1}{(1+k^2)^2} - \frac{\Omega^2}{(1+k^2)} \frac{1}{b_0} \frac{(1-\Gamma_0)}{(1+\tau[1-\Gamma_0])} \quad (3)$$

The impact of FLR effects to various orders may be understood upon expanding this potential in the gyroradius parameter,  $b_0$  [10].

To leading order, the continuous spectrum of MHD, associated with the Alfvén singularity, is obtained - as shown in Figure 2 the potential then does not allow bound states. Retaining terms linear in  $b_0$ , the order of the equation is increased and the singularity lifted, producing a discrete spectrum. Upon retaining full FLR effects, the singular nature of the equation returns, although with a different character to the MHD case.

**3. On Hall MHD description of the inertial layer:** A possibility of describing plasma perturbations of a sub-Larmor scale in a MHD-type approach is of interest, for both linear and non-linear description of the fishbone inertial layer. One of the promising approaches is single fluid Hall MHD (HMHD), equations of which in standard Alfvén units are (see, e.g. [11]):

$$n \frac{d\mathbf{V}}{dt} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p; \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[ \left( \mathbf{V} - \varepsilon \frac{\nabla \times \mathbf{B}}{n} \right) \times \mathbf{B} \right]; \quad \frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = 0. \quad (4)$$

These are obtained from two fluid theory upon neglecting resistivity and electron inertia, with the Ohm's law being re-written as a generalized induction equation. For consistency with the convective derivative, gyroviscosity should be retained in this set, but it is dropped as only the linearized set of equations is considered. Here  $\mathbf{V}$  is the mass flow velocity, normalized to the Alfvén speed, and  $n$  is the number density. Closure is provided by the assumption of an isothermal equation of state:  $P\rho^{-\gamma} = \text{constant}$ . The parameter  $\varepsilon$  is the ratio of the ion inertial length or skin depth to the representative scale length and marks the Hall term. It is equivalent to  $\omega / \omega_{Bi}$ , so inclusion of the Hall term allows for ion inertia corrections to the ideal MHD flow velocities, such as, eg the differing polarization drifts experienced by ions and electrons in the field of a shear Alfvén wave. Such corrections allow coupling of the slow magnetosonic and shear Alfvén waves, giving the shear wave a dispersive character [12]. A derivation of the generalized induction equation starting from the ion momentum equation was presented by Kuvshinov [13]. It makes it clear that the Hall term is representing a finite ion inertia term as

$$\nabla \times \frac{\mathbf{J} \times \mathbf{B}}{n_e e} = \frac{m_i c}{e} \nabla \times \frac{d\mathbf{V}_i}{dt} + \frac{c}{e} \nabla \left( \frac{1}{n} \right) \times \nabla p_i \quad (5)$$

is satisfied, where  $\mathbf{J}$  is current density. It was further shown in [13] that the linearized ion continuity equation, in which only the divergence of the transverse inertial velocity (first term in right-hand side of (5)) contributes, gives the HMHD ion response in the form:

$$n_i' = -n_0 \frac{b}{1+b} \frac{e\varphi}{T_i}. \quad (6)$$

This is in fact very close to the full-order FLR response (1), as the Padé approximation for the Bessel functions gives [8]  $1 - I_0(b)e^{-b} \approx b/(1+b)$ . The use of the ion response (6) gives an effective Padé potential, which is quite close to the full-order FLR potential as Figure 2 shows. This indicates that the sub-Larmor inertial layer of a finite-frequency  $n=1$  fishbone can be described in HMHD approach equivalent to the full-order FLR kinetic description.

**4. Conclusions:** In this paper, the fishbone inertial layer is considered in a kinetic approach and the possibility of representing the full-order FLR via the HMHD approach [11, 13] is assessed. FLR enters the eigenmode equation upon imposing quasineutrality with the ion density response to the oscillating electrostatic potential of the wave determined from a kinetic description of ions. An equivalent relation is obtained in HMHD from the ion continuity equation. In slab geometry only the inertial term contributes to the divergence of the ion velocity in the linear approximation. The Padé approximation of the true response then closely matches the HMHD response, leading to the closely related dispersion relations. Although HMHD is not a rigorous consequence of drift-ordered kinetic theory, it is seen to mimic part of the FLR response, so it provides a short-cut model that reproduces the kinetic effects well enough for linear stability analysis. Possible employment of the HMHD to nonlinear description of the fishbones [4] in the hot-ion regime is yet to be investigated.

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**References:** [1] K. McGuire et al., Phys. Rev. Lett. **50** (1983) 891; [2] L. Chen et al., Phys. Rev. Lett. **52** (1984) 1122; [3] B. Coppi, F. Porcelli, Phys. Rev. Lett. **57** (1986) 2272; [4] A. Ödholm et al., Physics of Plasmas **9** (2002) 155; [5] E. Fredrickson et al., Nucl. Fusion **43** (2003) 1258; [6] M.P. Gryaznevich, S.E. Sharapov, Plasma Phys. Control. Fusion **46** (2004) S15; [7] Ya.I. Kolesnichenko et al., Phys. Rev. Lett. **82** (1999) 3260; [8] F. Pegoraro, T.J. Schep, Plasma Phys. Control. Fusion **28** (1986) 647; [9] A.B. Mikhailovskii, Sov. Phys. JETP **16** (1963) 364; [10] J.W. Connor et al., Phys. Fluids **26** (1983) 158; [11] S. Ohsaki, S.M. Mahajan, Phys. Plasmas **11** (2004) 898; [12] A. Hasegawa, C. Uberoi, 'The Alfvén Wave', US DOE Tech. Inform. Ctr. DOE/TIC 11197 (1982); [13] B.N. Kuvshinov, Plasma Phys. Control. Fusion **36** (1994) 867.