

## **Triangularity and Ellipticity effects on low-vorticity non-linear collisional diffusion in tokamaks**

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### **Abstract**

Normal average flow around elliptic magnetic surfaces with triangularity has been calculated for tokamak plasmas in the collisional regime considering non-linear flows and low-vorticity approximations. Density variations around a tokamak surface has been derived and included in our analysis. A kind of tokamak coordinates described in previous works has been used in the present paper. In order to simplify the non-linear analysis a low vorticity approximation has been used as in a previous work. Our results show the dependence of collisional diffusion with the ellipticity and triangularity. Triangularity is a very important parameter in our diffusion analysis. Numerical results for positive and negative triangularities are shown.

### **Theoretical treatment**

The effects of diffusion in tokamaks can change significantly the usual theoretical analysis, when nonlinear flows are included. When flows are considered the convective non-linear terms in the momentum equations can not be neglected. In this case the pressure is not anymore constant around a magnetic surface. A simplify treatment can be performed in the H-mode operation, where due to the internal barriers, there is low turbulence and vorticity in the plasma, and in some way this can be neglected as we assume in this work. In this paper, a non-linear magnetohydrodynamics simplified analysis for confinement and diffusion in axisymmetric tokamaks has been performed using a new kind of generalized coordinates system, recently published [1-5].

The normal diffusion flow around a magnetic surface is given by

$$\langle \rho \vec{v} \cdot \hat{N} \rangle = \langle \rho v_{\sigma} \rangle = \frac{\oint \rho \vec{v} \cdot (\hat{N}) dS}{\oint dS}, \quad (1)$$

where  $\hat{N}$  is the normal unitary vector to the surface and  $\rho$  is the plasma density function. As in previous works [1-4], the starting point is the steady state magnetohydrodynamics equation (MHD)

$$\rho \vec{v} \cdot \nabla \vec{v} = \frac{1}{c} \vec{j} \times \vec{B} - \nabla p + \rho \nu \nabla^2 \vec{v}. \quad (2)$$

From here it is obtained

$$\rho \nabla F = \frac{1}{c} \vec{j} \times \vec{B} - \vec{v} \times \vec{\omega} - \nu \nabla \times \vec{\omega}, \quad (3)$$

where  $\vec{\omega} = \nabla \times \vec{v}$  is the vorticity, and the auxiliary function  $F$  were already defined as [3]

$$F(\vec{v}, \rho, T) = \frac{v^2}{2} + W(\rho, T), \quad (4)$$

being  $W(\rho, T)$  the enthalpy given as

$$W(\rho, T) = \int^\rho \frac{1}{\rho} \frac{\partial p}{\partial \rho} d\rho. \quad (5)$$

This integral is performed along a magnetic line, where  $T$  can be considered constant, since as is well known, the heat transference along a magnetic lines is very fast. Now if the vorticity is neglected then Eq.(3) becomes

$$\rho \nabla F = \frac{1}{c} \vec{j} \times \vec{B}, \quad (6)$$

and from here  $\vec{B} \cdot \nabla F = 0$ . The perpendicular velocity  $v_\sigma$  in each point is obtained using the Eq.(6) and the Ohm's law as follows. First, it is known that  $\vec{E} + \vec{v} \times \vec{B} = \eta \vec{j}$ , giving

$$\vec{v}_\sigma = v_\sigma \hat{N} = \frac{1}{B^2} \left( E_p B_\phi - E_\phi B_p - \eta_\perp \rho \left( \frac{\partial F}{\partial \sigma} \right) \right), \quad (7)$$

now  $E_p$  is related to the parallel component of the current through the Ohm's law  $\vec{E} \cdot \vec{B} = \eta_\parallel j_\parallel B$ , and by using the continuity equation  $\nabla \cdot \vec{j} = 0$ , as well as the univaluation for the electric potential  $\phi(\vec{r}, t)$ , the value of  $E_p$  can be obtained as (See Eq.[52] of Ref.[1])

$$E_p = -\eta_\parallel \rho \frac{B_\phi}{B_p^2} \frac{\partial F}{\partial \sigma} + \eta_\parallel \frac{B^2}{B_p} \left[ \oint \frac{\rho B_\phi}{B_p^2} \frac{\partial F}{\partial \sigma} ds + \frac{1}{\eta_\parallel} \oint \frac{E_\phi B_\phi}{B_p} ds \right] \left( \oint \frac{B^2}{B_p} ds \right). \quad (8)$$

On the other hand, using the coordinates  $(\tilde{\sigma}, \tilde{s}, \phi)$  described in previous works [1-4], the equation (6) can be reduced to

$$\rho \frac{\partial F(\tilde{\sigma})}{\partial \sigma} = j_p B_\phi - j_\phi B_p. \quad (9)$$

The following conditions can also be derived [1-4]

$$\frac{\partial F(\tilde{\sigma})}{\partial \sigma} = \left( \frac{\partial F(\tilde{\sigma})}{\partial \sigma} \right)_1, \quad \mu(\lambda, \theta) = \exp \left[ - \int_0^s \kappa_\sigma ds \right], \quad (10)$$

$$B_\phi = \frac{B_{\phi_1} R_1}{R}, \quad j_p = \frac{j_{p1} R_1 \mu}{R}, \quad B_p = \frac{B_{p1} R_1 \mu}{R}. \quad (11)$$

Here as usual the subindex 1 denotes the outmost point in the magnetic surface, and  $\kappa_\sigma$  is the curvature of the orthogonal curves, to the magnetic surfaces. In the treatment performed here,

the resistivities  $\eta_{\parallel}$  and  $\eta_{\perp}$  are considered to be independent of the plasma shape. Using the Eq.(12) and (13), the equation for  $\rho$  becomes

$$\rho \left( \frac{\partial F(\tilde{\sigma})}{\partial \sigma} \right)_1 = \frac{R_1^2 B_{\varphi_1} j_{\varphi_1}}{R_1^2} - j_{\varphi} \frac{B_{p_1} R_1}{R}, \quad (12)$$

where  $j_{\varphi}$  has been obtained from the Ampere law(See Eq.(19) of Ref.[3]), giving

$$j_{\varphi} = \frac{c}{4\pi} \left[ \kappa_s B_p + \left( \frac{\partial B_p}{\partial \sigma} \right)_{\tilde{s}} \right]. \quad (13)$$

The dimensionless equations for the magnetic surfaces are taken to be [5]

$$\hat{R}(\lambda, \theta) = \frac{R}{R_m} = 1 + \tilde{a} \lambda \cos \theta - \lambda^2 \left( \tilde{\Delta} + \frac{\tilde{a} T(a)}{2} \sin^2 \theta \right), \quad (14)$$

$$\hat{z}(\lambda, \theta) = \frac{z}{z_m} = \tilde{a} \lambda E(a) \left( \sin \theta - \frac{\lambda T(a)}{4} \sin 2\theta \right), \quad (15)$$

where  $\tilde{a} = a/R_m$ ,  $\tilde{\Delta} = \Delta/R_m$ , and  $E(a)$  and  $T(a)$  are the ellipticity and triangularity distortion, respectively, and  $\Delta(a)$  is the Shafranov shift. In this set of equations  $\lambda$  denotes the corresponding magnetic surfaces and its values are in the interval  $(0, 1)$ . In this way after a long cumbersome treatment, the dimensionless equation to be used for  $\rho$  is finally  $\rho(\lambda, \theta) = G(\lambda, \theta) \rho_1(\lambda)$ , with

$$G(\lambda, \theta) = \frac{R_1^2}{R^2} \left[ 1 + \frac{\mu \tilde{\alpha} \tilde{\gamma}_1 R_1}{4\pi(1 - \tilde{\gamma}_1 \tilde{g})} \left( \kappa_s - \frac{\kappa_{s1}}{\mu} + \frac{\partial}{\partial \sigma} \left( \ln \left( \frac{R_1}{R} \right) \right) + \frac{\partial \ln \mu}{\partial \sigma} \right) \right], \quad (16)$$

whit the dimensionless quantities  $\tilde{\alpha} = (c B_{p_1})/(R_1 j_{p_1})$ ,  $\tilde{g} = j_{\varphi_1}/j_{p_1}$  and  $\tilde{\gamma}_1 = B_{p_1}/B_{\varphi_1}$ . Using all our previous analysis the normalized diffusion normal flux around a magnetic surface will be

$$\frac{\langle \rho \bar{v}_{\sigma} \rangle}{\langle \rho \bar{v}_{\sigma} \rangle_{oc}} = \frac{1}{\alpha_0 \hat{I}_0} \left[ \frac{\hat{I}_2}{\hat{R}_1^2} + \frac{\eta_{\parallel}}{\eta_{\perp}} \frac{1}{\tilde{\gamma}_1^2 \hat{R}_1} \left( \hat{I}_3 - \frac{\hat{I}_1^2}{\hat{I}_4} \right) \right] - \tilde{E}_{\varphi_1} \left[ \hat{I}_7 - \frac{\hat{I}_1 \hat{I}_6}{\hat{I}_4} + \tilde{\gamma}_1^2 \hat{I}_5 \right]. \quad (17)$$

The dimensionless integrals  $\hat{I}_i (i = 0 \text{ to } 7)$  are defined as

$$\begin{aligned} \hat{I}_0 &= \oint \hat{R} d\hat{s}, \quad \hat{I}_1 = \oint \frac{G(\lambda, \theta) \hat{R} d\hat{s}}{\tilde{\mu}(s)}, \quad \hat{I}_2 = \oint \frac{G(\lambda, \theta) \hat{R}^3 \tilde{\mu}(s) d\hat{s}}{(1 + \tilde{\gamma}_1^2 \tilde{\mu}^2(s))}, \\ \hat{I}_3 &= \oint \frac{G(\lambda, \theta)^2 \hat{R}^3 d\hat{s}}{\tilde{\mu}(s)(1 + \tilde{\gamma}_1^2 \tilde{\mu}^2(s))}, \quad \hat{I}_4 = \oint \frac{(1 + \tilde{\gamma}_1^2 \tilde{\mu}^2(s)) d\hat{s}}{\hat{R} \tilde{\mu}(s)}, \quad \hat{I}_5 = \oint \frac{G(\lambda, \theta) \hat{R} \tilde{\mu}(s) d\hat{s}}{(1 + \tilde{\gamma}_1^2 \tilde{\mu}^2(s))}. \\ \hat{I}_6 &= \oint \frac{d\hat{s}}{\hat{R}} \tilde{\mu}(s), \quad \hat{I}_7 = \oint \frac{G(\lambda, \theta) \hat{R} d\hat{s}}{\tilde{\mu}(s)(1 + \tilde{\gamma}_1^2 \tilde{\mu}^2(s))}. \end{aligned} \quad (18)$$

and  $\alpha_0$  is defined by

$$\alpha_0 = \frac{\hat{I}_2(0)}{\hat{R}_1^2} + \frac{\eta_{\parallel}}{\eta_{\perp}} \frac{1}{\hat{R}_1^2 \tilde{\gamma}_1^2} \left( \hat{I}_3(0) - \frac{\hat{I}_1(0)^2}{\hat{I}_4(0)} \right). \quad (19)$$

All the integrals in  $\alpha_0$  are evaluated for a circular surface with  $\kappa_{\sigma} = 0$  and no triangularity  $\delta(a) = 0$ , condition denoted by the symbol (0) [5]. Dimensionless toroidal electric field  $\tilde{E}_{\varphi_1}$  is also defined as

$$\tilde{E}_{\varphi_1} = -E_{\varphi_1} / ((\eta_{\perp}/B_{\varphi_1})(\partial p/\partial \sigma)_1) \quad (20)$$

## RESULTS AND CONCLUSION

In this paper the calculations are carried out for the JET ( ) with the plasma parameters  $R_m = 3m$ ,  $a = 0.95$ , for the major and minor radius respectively. The Shafranov shift is taken as  $\Delta(a) = a/10$ ,  $\tilde{\gamma}_1 = 0.3$ ,  $\tilde{\alpha} = 0.001$ ,  $\tilde{g} = 3$ ,  $\eta_{\parallel} = 1.65 \times 10^{-9} \left(17/T_e^{3/2}\right)$  and  $\eta_{\perp} = 1.97\eta_{\parallel}$ . In Figure 1, the dimensionless normal flow is shown as a function of the triangularity for three different values of the dimensionless electric field ( $\tilde{E}_{\varphi_1} = 2.9, 3, 3.1$ ) and for a given elongation  $\kappa(a) = 1.2$ . In this figure confinement corresponds to negative values of diffusion velocity. This figure show that for a given ellipticity the confinement is restricted to a select range of triangularities, for each value of dimensionless electric field. In this MHD model, positive triangularities produce better confinement than the negatives ones and the maximum confinement is not at zero triangularity. Confinement increases by increasing the dimensionless inductive electric field. For small values of  $\tilde{E}_{\varphi_1}$  and large values of triangularities there is not confinement. Triangularity seems to be more important for confinement than elongation.

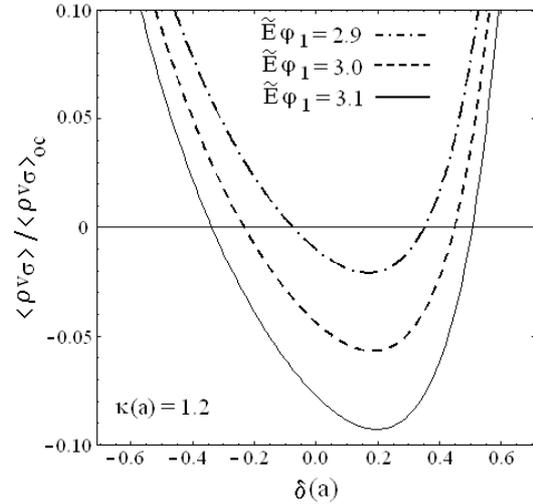


Figure 1: Dimensionless normalized diffusion flow versus triangularity, with an elongation of  $\kappa(a) = 1.2$  by three different values of dimensionless toroidal electric fields

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