

MAST Plasma Response Investigations using DINA-CH

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Introduction: At low aspect ratio, the radial field necessarily tends towards zero on the inboard side, so that both the radial field from the vertical position control coils, and to a lesser extent the decay index $-\frac{R}{B_z} \frac{\partial B_z}{\partial R}$, become inhomogeneous. As a result, the response of the plasma to a vertical perturbation, and thus the vertical control, is suspected to be more non-linear than for conventional R/a tokamaks.

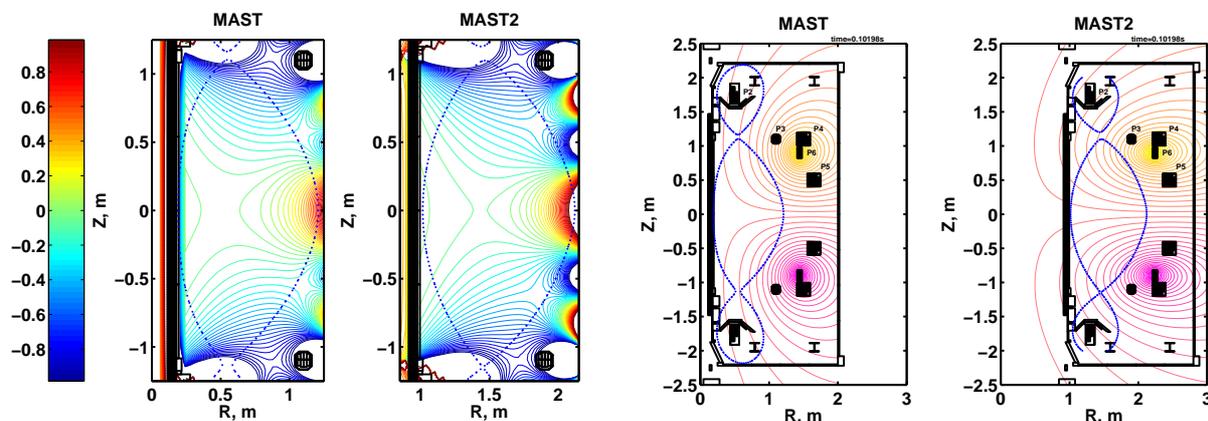


Figure 1: Poloidal cross-section of the vessels for MAST and MAST2, showing the decay index (left-hand plots) and the shape of the field due to the vertical control (P6) coil (right-hand plots). The initial plasma equilibrium for each is superimposed in blue, and the poloidal field coils are marked in black.

This in turn raises issues for controller designers, namely: If there is a non-linear effect, should it be taken into consideration for controller design? How different is the ST to conventional tokamaks? Good feedback controllers in tokamaks are essential as they are required to keep the plasma in a stable equilibrium and prevent or minimise the severity of disruptions such as the Vertical Displacement Event (VDE), where halo currents flow through the vessel structure generating $\mathbf{j} \times \mathbf{B}$ forces that have the potential to be very damaging when scaled up to ITER-size devices. Non-linearity in the system would place extra demands on the feedback and require higher bandwidth performance of the amplifiers.

Process of the Investigation: DINA-CH[1][2], a non-linear, time dependent equilibrium evolution code was used to compare the variation in response to a vertical perturbation between spherical and conventional tokamaks. MAST has been analysed alongside a second, hypothetical tokamak of conventional aspect ratio, dubbed MAST2 and identical in design except for aspect ratio R/a. All MAST coordinates were shifted radially outwards by 0.8m, giving MAST2 R/a=3 where MAST has R/a=1.3 (Figure 1). The plasma shape, and hence the vertical stability, vary with aspect ratio even if the currents in the PF coils are held constant, so to compensate for this the current in the ‘divertor’ coils (P2) has been increased until the growth rate in MAST2

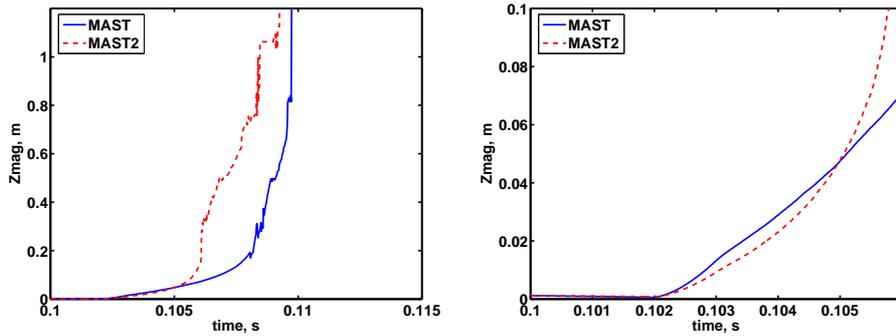


Figure 2: The change in the Z position with time for both tokamaks, showing first a wider range and then a close-up to the Z range of interest.

approximately matches that in MAST, at least for the early phase, see Figure 2. A PD controller keeps the plasma in equilibrium and restoring voltage is applied to the P6 coils, shown in Figure 1. It should be noted that the feedback system used here is not representative of a full plasma control system since no voltage saturation is assumed.

A sinusoidal perturbation was applied to the system in “packets” of five oscillations at the same amplitude A ($A\sin\omega_0 t$ for five periods). This was applied as a voltage into the feedback loop, so the actual voltage on the P6 coils ($V(P6)$) was the sum of the sinusoidal perturbation and any restoring feedback voltage. The amplitude A was increased at discrete intervals until the plasma excursion became too large to be held by the feedback system. Four frequencies were simulated—50Hz, 100Hz, 200Hz and 400Hz—but only 50Hz and 100Hz were considered since the others are significantly above the characteristic frequencies of the P6 coil casings (160Hz and 80Hz for MAST and MAST2 respectively). The responses in $I_p Z$ and $V(P6)$, were analysed. $I_p Z$ is the product of plasma current, I_p , and vertical displacement of the plasma with respect to the vessel centre, Z , and is the quantity that is detected experimentally. Recently, a similar method of analysing the system response to a sinusoidal perturbation has been used by control engineers to diagnose the non-linearity in simulated systems with friction effects[3][4].

To analyse the non-linearity in the response of the plasma to the applied perturbation, the response signals were decomposed into their lower order harmonics. Any strong odd harmonics present would point to non-linearity. The even harmonics are not of interest here as their pres-

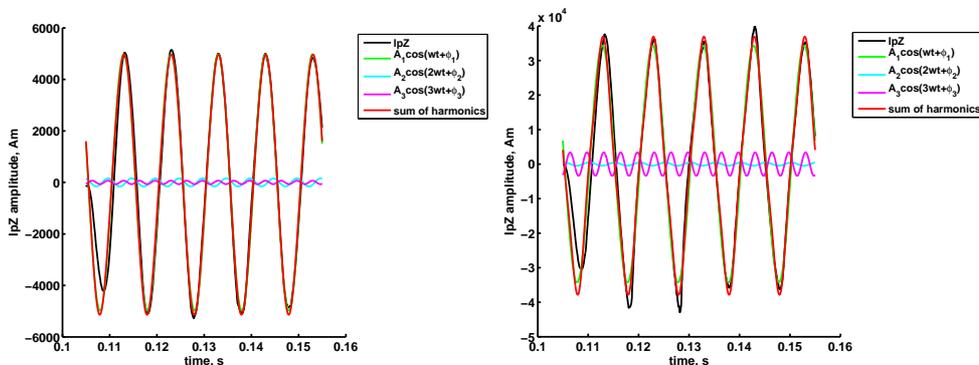


Figure 3: Figures showing the original response signal, $I_p Z$, and how the calculated 3 harmonics compare to it when summed. Left—low perturbation amplitude; right—high perturbation amplitude.

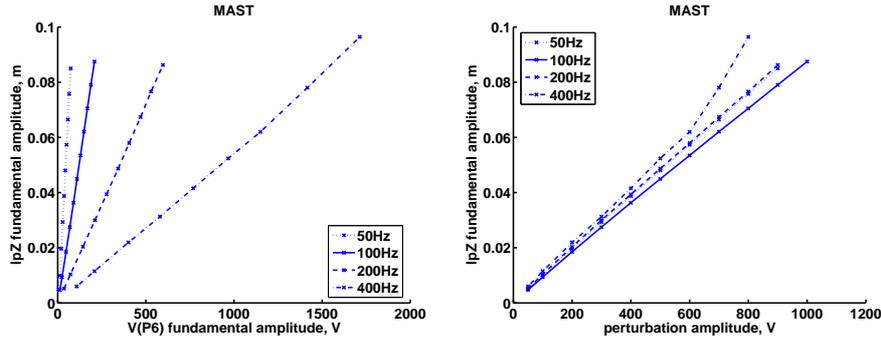


Figure 4: Effect of the feedback on plasma excursion, shown by normalised $I_p Z$. Left—as the frequency increases the Z excursion of the plasma reduces for a given $V(P6)$ (=perturbation + feedback), as the current in the coil is reduced; right—there is little variation with frequency of normalised $I_p Z$ for a given perturbation amplitude.

ence would signify an asymmetric effect, whereas in the up-down symmetric system to which a symmetric perturbation has been applied, the response is expected to be symmetric. The harmonics were calculated using Fourier analysis over varying-length windows across the signal to minimise any noise and transient effects at the start of the oscillation. The final amplitude of the harmonic was found by averaging over all of these windows. Figure 3 shows, for the first 3 harmonics, how when summed they compare to the original response signal. It is clear from the deterioration of fit that as the perturbation amplitude is increased the non-linearity increases.

Results: Figure 4 shows how the response signals vary with frequency and gives an indication of how the Z of the plasma is regulated by the feedback. On the left one sees the expected behaviour in that the excursion amplitude, shown by $I_p Z$ normalised to average I_p , ie. $I_p Z / |I_p|$, falls with increasing frequency for a given P6 voltage $V(P6)$. In the right-hand plot the excursion amplitude is approximately proportional to the perturbation amplitude independent of frequency, showing that the feedback is successfully compensating for the varying response.

Thus $I_p Z$ is shown to give an indication of non-linearity, but the effect is small because of its reduction by the feedback control. Further evidence of this can be seen by looking at the results of harmonic analysis on the $I_p Z$ signal compared with the $V(P6)$ signal, see Figure 5, which shows the 3ω harmonic relative to the fundamental at 50Hz and 100Hz for $I_p Z$ and $V(P6)$. Both signals show similar trends, but the relative amplitude of the 3ω harmonic of the $I_p Z$ signal is about an order of magnitude less than that of the $V(P6)$ signal. It is therefore more instructive to focus this analysis on the voltage on the P6 coil ($V(P6)$) to show the full extent of the non-linearity.

Now considering only the harmonics in $V(P6)$, the MAST and MAST2 tokamaks are compared at 50Hz and 100Hz, see Figure 6. There is a clear dominance in the non-linearity of MAST over MAST2 in 3ω , (a), at small amplitude. At a plasma excursion of between 2cm and 3cm, MAST shows an amplitude relative to the

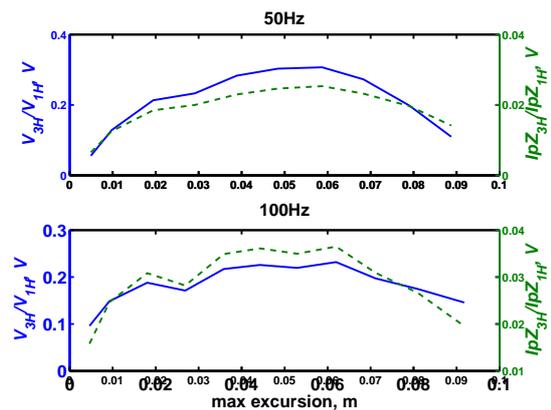


Figure 5: Comparison of the harmonics in $I_p Z$ (dashed line) and $V(P6)$, showing that $V(P6)$ is much more significant.

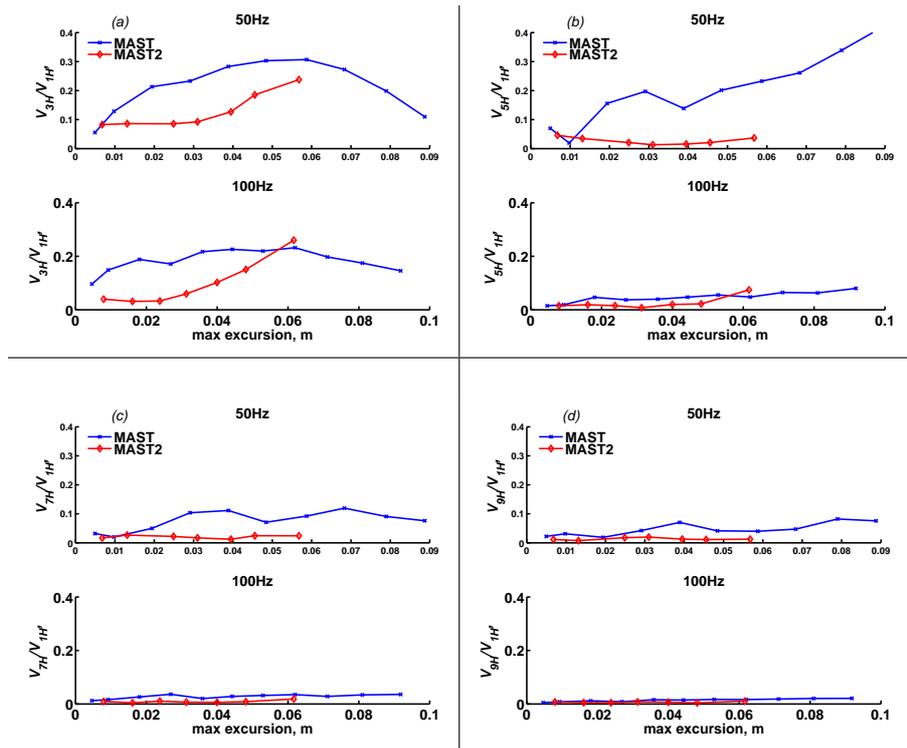


Figure 6: Ratio of the $V(P6)$ harmonics to the fundamental showing greater harmonics in MAST than MAST2 at small-amplitude. (a-d: 3ω , 5ω , 7ω and 9ω)

fundamental of, on average, a factor of 2.6 times that of MAST2 at 50Hz, and 4.8 times MAST2 at 100Hz. Since the aim of the vertical controller is to keep plasma excursion small, this small-amplitude behaviour is relevant. At 50Hz, the 5ω harmonic, (b), is also significant, increasing to more than 30%, but higher harmonics, as expected, have less amplitude.

Conclusions: The 3ω harmonics verify that the spherical tokamak, MAST does display higher non-linearity in the vertical control than does its equivalent conventional aspect ratio tokamak, MAST2, which also displays some non-linear behaviour. Although STs have better intrinsic vertical stability, this non-linearity may have implications for amplifier bandwidth requirements and is significant enough to require consideration by controller designers.

This work was funded jointly by the United Kingdom Engineering and Physical Sciences Research Council and by the European Communities under the contract of Association between EURATOM and UKAEA. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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