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THE INVESTIGATION OF RESISTIVE WALL MODES IN A DIVERTED TOKAMAK CONFIGURATION

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Introduction

The "advanced tokamak" concept is economically attractive if the ideal external kink beta limit is raised substantially. This is possible only if the resistive wall mode (RWM) is stabilized by either passive (close fitting conducting wall) and active (feedback coils) stabilization, or/and rapid plasma rotation. One solving method of the RWMs could be to found a new basis of orthogonal eigenvectors to ensure the self-adjointness property of the energy operator - the normal mode approach [1-2], or to replace a perfect conducting wall with one of finite conductivity and to deduce a modifield energy principle [3]. Numerical MHD stability studies in the presence of toroidal rotation, viscosity, resistive walls and current holes, by using the CASTOR FLOW code are presented in Ref. [4]. Modelling of RWMs has been made by using the VALEN code [5] with an equivalent surface current model for the plasma. Extensive study and theoretical development has also been presented by Bondeson and his co-workers [6,7] by using the MARS code.

In our approach, after writing the expression for the potential energy in terms of the perturbation of the flux function, and performing an Euler minimization, a system of ordinary differential equations in that perturbation has been obtained [8]. This system of equations describes an external kink mode if the resonance surface is situated at the plasma boundary. It is known that the external modes are stabilized by the presence of a close-fitting perfectly conducting wall but become destabilized when the wall is assumed to have finite resistivity. A general toroidal geometry with a separatrix has been considered. Natural boundary conditions for the perturbed flux function, just at the plasma boundary have been determined by using the concept of a surface current [9], replacing the vanishing boundary conditions at infinity. By adding the wall, new boundary conditions for the external kink mode, now a resistive wall mode, due to the field produced by the eddy currents induced in the wall and due to feedback coils have been determined. An influence matrix has been deduced where the complex growth rate of the RWM is found by finding the zeros of a complex analytical function representing the vanishing determinant of that matrix.

Determination of the wall answer to an external kink mode

Considering the wall thickness smaller than the other dimensions of the wall, the eddy currents can be represented by surface currents. With other words, the characteristic wall time $\tau_{skin} \simeq \mu_0 \sigma d^2$, with d the wall thickness, has to be sufficiently small compared with the variation time of the exciting magnetic fied. The magnetic permeability of the wall has been taken as $\mu = \mu_0$.

In an orthogonal curvilinear coordinate system (u, v, w) (w =const corresponding to the wall surface), with h_u and h_v the Lamé coefficients, the eddy current, with a line density $\mathbf{j}$, produced in the wall by the magnetic field $\mathbf{B}$, can be described by a scalar potential U, defined such that

$$\mathbf{j} = \nabla U \times \mathbf{n} \ \ [\text{A/m}] \tag{1}$$

where $\mathbf{n}$ is the external normal to the wall surface. With

$$j_u = \frac{1}{h_v}\frac{\partial U}{\partial v}, \quad j_v = -\frac{1}{h_u}\frac{\partial U}{\partial u}, \tag{2}$$

Thus, the current density vector are satisfying the continuity condition of the current automatically $\nabla \cdot \mathbf{j} = 0$.

By multiplying the 2nd Maxwell equation $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ with $\mathbf{n}$ and making use of the equation $\mathbf{j} = \sigma_s \mathbf{E}$, with $\sigma_s = \sigma_v d$ $[\Omega^{-1}]$, where σ_s is the surface conductivity of the wall, while σ_v is the volume conductivity, after some calculations, the diffusion equation for the eddy current stream function $U(u, v, t)$ in a thin wall has been obtained

$$\frac{1}{h_u h_v}\left[\frac{\partial}{\partial u}\left(\frac{h_v}{h_u}\frac{\partial U}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_u}{h_v}\frac{\partial U}{\partial v}\right)\right] - \frac{1}{d}\mu\sigma_s\frac{\partial U}{\partial t} = \sigma_s \frac{\partial B_n^{ext}}{\partial t}, \tag{3}$$

where $B_n = \mathbf{nB}$ is the normal component of the magnetic field at the wall surface. The initial and the boundary conditions are

$$U(u, v, 0) = 0, \quad F(u, v, U_t, U_u, U_v, t) = 0. \tag{4}$$

Possible toroidal geometries to be investigated are given in Fig. 1. For that wall, the following input data: $d = 10^{-3}$ m, $\mu = 4\pi^{-7}$ H/m, $\sigma_v = 10^7$ $1/\Omega/\text{m}$, $\sigma_s = 10^4$ $1/\Omega$, have been considered. The magnetic field produced by an external $m/n = 3/2$ kink mode and the corresponding stream function U are presented in Fig. 2. Constant $U(x, y, t)$ lines, at different time, excited by a m/n =3/2 external kink mode in a thin wall with holes are presented in Fig. 3.

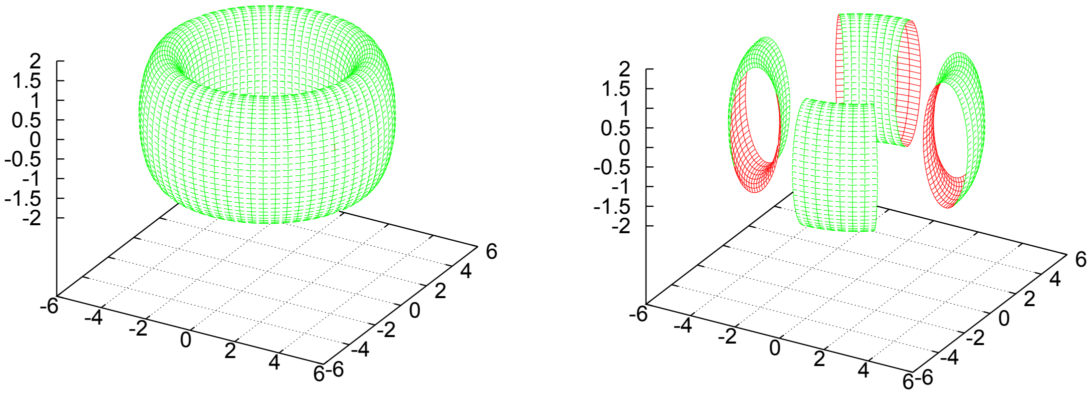

Fig. 1 Possible toroidal wall geometries.

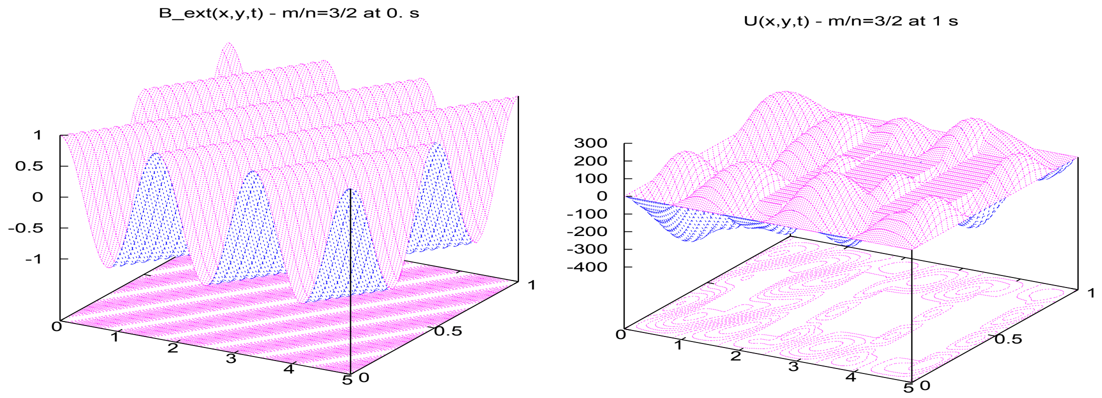

Fig. 2 Magnetic field $B_{ext}^{3/2}$ produced by an external kink mode and corresponding stream function U for a wall with holes.

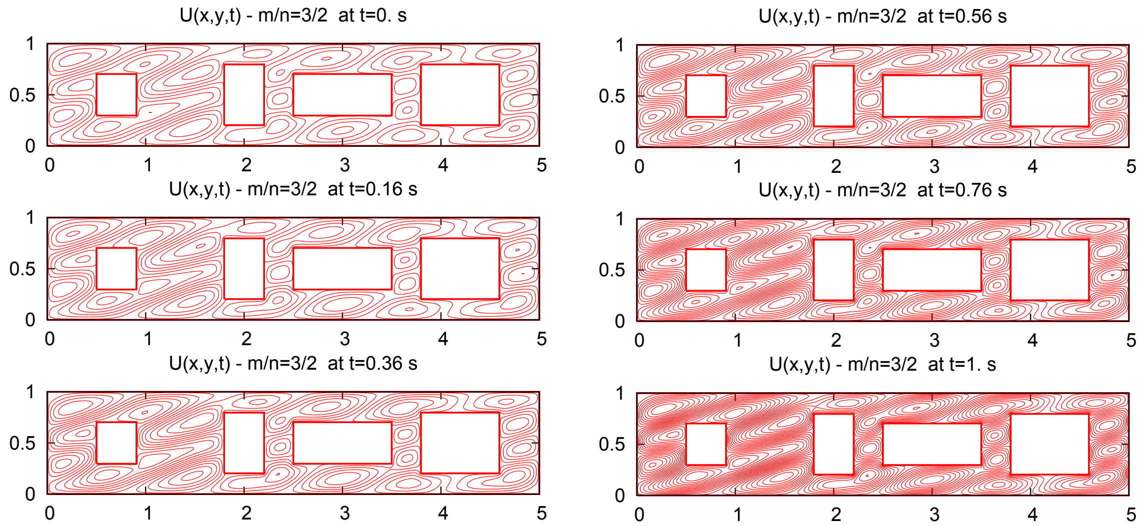

Fig. 3 Constant eddy current stream function $U(x,y,t)$ at different time, excited by a $m/n = 3/2$ external kink mode in a thin wall with holes.

Special attention has been given to the accurate calculation of the influence of the eddy currents on the boundary conditions of the system of equations describing the RWM. We have developed a method for treating singularities which occur in solutions of parabolic partial differential equations (the diffusion equation of the eddy currents) due to sharp corners in the boundary. The method is used in conjunction with the simple explicit finite-difference scheme and subsequently the overall method is explicit. The standard finite-difference in such a neighbourhood was replaced by a truncated series representation of the exact solution at points close to the corner. The coefficients of this truncated series are estimated at each time step in terms of the solution values at points where the influence of the singularity is neglected and which have been derived by an explicit finite-difference scheme from the previous time step. In Fig. 4, the relative error distribution around a singular reentry corner without and with analytical removal of the singularity are given.

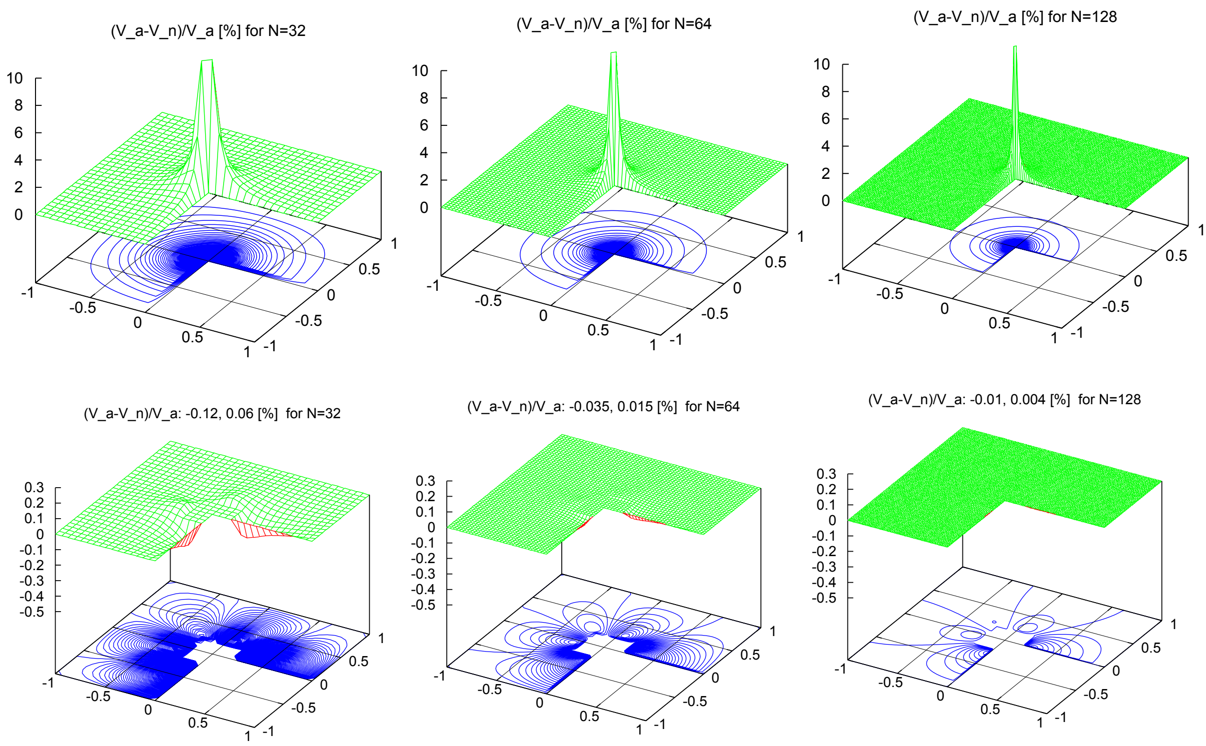

Fig. 4 On the uper row, the relative errors distribution around a reentry corner for different numbers of discrete meshes with respect to the analytical solution without correction. On the lower row, the relative errors distribution around a reentry corner for the same numbers of discrete meshes but with analytically removed singularity.

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