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Dynamic behaviour of type I Edge Localized Modes in the JET

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Abstract

Understanding and control of ELMs are crucial issues for the operation of ITER where the type-I ELMy H-mode has been chosen as the standard operation scenario. Determine whether ELM dynamic is chaotic or random is crucial to correctly describe the ELM cycle. In this paper, the dynamic characteristics of 8 ELM time-series from the JET tokamak are investigated. Characteristic parameters, such as the Hurst exponent and the Maximal Lyapunov Exponent, have been evaluated. The obtained results suggest the presence of deterministic chaos in some of the analysed time series.

I. INTRODUCTION

In large tokamaks the release of a considerable amount of energy during an Edge Localized Modes (ELM) can cause damage to the first wall, especially in the divertor plate. On the other hand, the plasma density in a H-mode without ELMs is generally non-stationary, and can uncontrollably rise until the plasma disrupts. Therefore, the cyclic degradation in confinement due to ELMs has a beneficial effect and allows stationary H-mode operations.

Understanding and control of ELMs are crucial issues for the operation of ITER where the type-I ELMy H-mode has been chosen as the standard operation scenario. A number of different ELM types have been identified, and various models have been proposed [1].Recent work aimed to determine whether ELM dynamic is chaotic or random (noise dominated) [2-3]. It is reasonable to ask whether the ELM time series is predictable before modelling the ELM cycle and trying to forecast its dynamic. Indeed, although chaotically behaving systems appears to be unpredictable, there is always certain determinism present is such system, which makes prediction possible. An additional motivation of interest in this topic is that chaos can be controlled even without any knowledge on the system model.

The aim of this paper is to investigate the dynamic characteristics of ELM non linear time-series data from the JET tokamak. To this purpose, the Hurst exponent is estimated in order to detect determinism in the ELM time series. Moreover, the Maximal Lyapunov Exponent

(MLE) is evaluated to determine whether the observed irregular behaviour is a consequence of deterministic chaotic dynamics.

II. THEORY

II.2 *The Hurst exponent*

Understanding whether a time series shows long term correlations, is fundamental to predictability studies in many engineering fields. The detection of this phenomenon can be obtained by the Adjusted Rescaled Range (*R/S*) analysis [4,5]. Let $\mathbf{x}(t)$ be a scalar measurement $\mathbf{x}(t) = [x(t), x(t+\tau), ..., x(t+N\tau)]$. The analysis considers blocks of *n* successive points in the integrated time series and measures how fast the range of the blocks grows with *n*. In case of long memory process the *R/S* statistic scales as $\left(R/S\right)_m = k \cdot n^H$ where *k* is a constant and *H* is the Hurst exponent. *H* is estimated, on a log-log graph of the *R/S* versus the time lag τ, as the slope of the straight line best fitted by the points. *H*=0.5 indicates lack of correlations, i.e., pure random processes; 0.5<*H*<1 indicates positive correlations (long-term memory) in a data series, whereas 0<*H*<0.5 indicates anti-correlations.

II.1 *The Laypunov Exponents*

LEs quantify the exponential divergence of initially close state-space trajectories and estimate the amount of chaos in a system. The Takens embedding theorem states that the complete dynamics of a system can be reconstructed from a suitable time series **x** derived from the system [6]; therefore LEs can be extracted from such time series.

Let us consider *d*-dimensional vectors $\mathbf{y}(t) = [x(t), x(t+\tau), ..., x(t+(d-1)\tau)]$ whose components provide the coordinate system in which one can identify the attractor structure associated with the observations. There are two critical parameters involved in the construction of the delay coordinate embedding: the embedding dimension *d* and the time delay τ. Criteria to compute the time delay τ are based on the minimum of autocorrelation function [7] and on the minimum of the auto mutual information [8]. The embedding dimension *d* can be estimated through the method of False Nearest Neighbors (FNNs) [9]. A novel method for simultaneously determining both *m* and τ is proposed in [10] and it is based on the minimum of the differential entropy.

By considering the representation of data as a trajectory in the embedding space, let $\mathbf{y}(n)$ be a reference point and $\mathbf{y}(n')$ its neighbour belonging to a neighbourhood U_n of radius ε. If the quantity $Y(\varepsilon, m, t) = \left\langle \ln\left(\frac{1}{U_n} \sum_{\mathbf{y}_{n'} \in U_n} \left| \mathbf{y}(n+t) - \mathbf{y}(n'+t) \right| \right) \right\rangle \Big|_n$ exhibits a linear increase with identical slope for all $m>m_0$ and for a reasonable range of ε, the slope can be taken as an estimate of λ.

Unfortunately, there is not a unique technique to find optimal τ and m. In this paper, a number of tests using the previously cited algorithms has been performed. Moreover, proper parameters have been found by trial and error, comparing the results for invariants such as the MLE, for a whole range of embedding parameters.

III RESULTS

Eight time series of Soft-X rays sampled at dt = 1ms have been selected from the JET database. Figure 1 shows an example of the Soft-X rays and Dα data. Data present short term oscillations with varying amplitude, characterized by phases of slow increase and fast decrease. When the signal reaches a critical value it breaks down and the cycle start again. It is important to note that only the part of the pulses with evident ELMs has been analysed.

Results show an H>0.85 for all the time series, indicating a trend reinforcing series. Figure 2 reports the *R/S* plot for pulse #58841, where the slope of the regression line approximates the Hurst exponent.

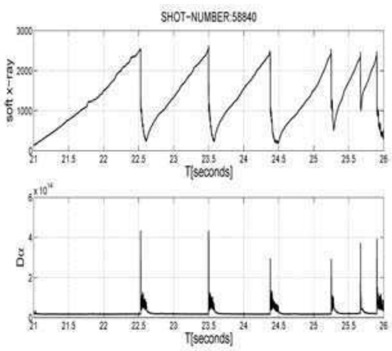


Fig.1: Soft X-ray and Dα for pulse #58840

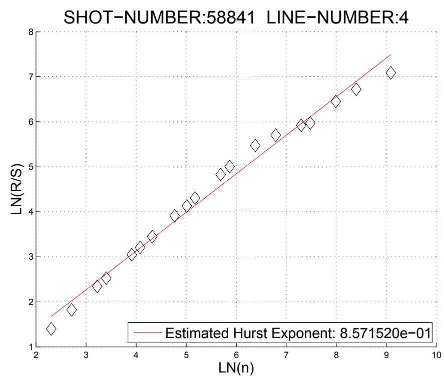
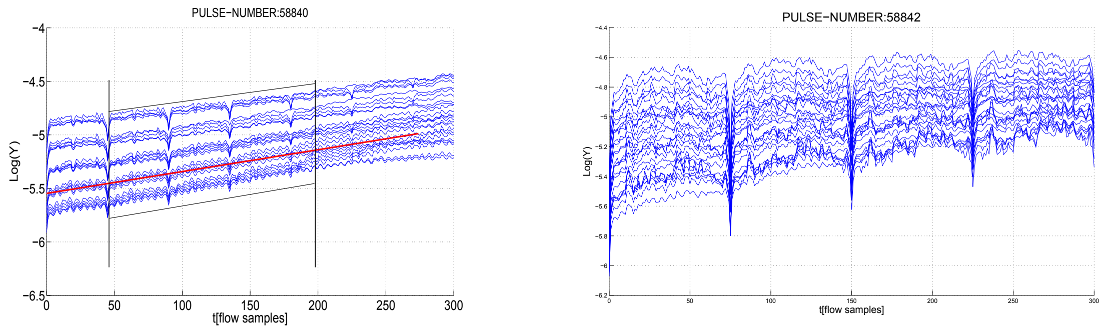


Fig.2 Estimated Hurst exponent of pulse #58841

To test if true structure in the ELM exists, we scramble the series and then calculate H for this series. The scrambled series has the distribution of the original series except that the sequence is random. The Hurst exponents after the scrambling of samples are all very close to 0.5 which is the value of random series. Thus, there must exist some structure in the data.

The MLE has been evaluated using the algorithms of Rosenstein and Kantz implemented in TISEAN [11] in order to verify the presence of a linear increase in Y using both methods. Six time series show a linear increase in the values of Y for different values of m and ε suggesting the presence of chaos. Figure 3 (a-b) reports the values of Y for 2 time series characterized by a different behaviour in terms of MLE. In case of shot #58840 linearly growing values of

$Y(\varepsilon, m, t)$ in the range t = [47-199] can be identified (fig. 3a). Conversely for the pulse #58842 no evidence of linear increase can be identified.



(a) (b)

Fig.3 Estimation of the MLE for pulse: #58840 (a), pulse #58842(b)

IV CONCLUSION

The question whether the apparently random variations in type-I ELMy H-mode time series are due to a deterministic chaotic process has been addressed. Non linear dynamical indicators such as the Hurst exponent and the Maximal Lyapunov Exponent have been evaluated for 8 Soft X-ray time series recorded in the JET database. Hurts exponents are greater then 0.85 for all the time series suggesting a deterministic process, and the Maximal Lyapunov Exponent is positive for some of them. Due to the exiguity of the database, to the shortness of the time series and to the presence of noise, the obtained results do not allow concluding that ELM time series are definitely chaotic or not, but they are found to be consistent with the hypothesis of chaotic dynamics. Further analysis will be carried out employing additional methods to deeply explore the presence of deterministic chaos.

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