Study of the Efficient Photoionization Model 3-Group SP3 for the Modeling of Streamer Propagation

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1. Introduction

Filamentary streamer discharges exhibit a considerable range of spatial scales, from a few millimeters long microdischarges at ground pressure to large-scale electrical discharges present above large thunderstorms, so-called sprites, which can propagate over a few tens of kilometers [1, 2]. The radiation plays an important role in the development of the streamer discharge through two physical processes: photoionization (i.e., ionization of neutral molecules due to the absorption of UV photons emitted from the high-field region of the streamer head) and photoemission (i.e., emission of electrons at surfaces due to absorption of photons). In this work, we focus on the modeling of the photoionization produced by streamers developing in air.

2. Integral and differential models for photoionization in air

In air, the most widely used model for photoionization is the model developed by Zheleznyak *et al.* [3]. In this model, the photoionization source term at position $\vec{r}$ due to the photon emission at $\vec{r}\,'$ is given by

$$S_{\mathrm{ph}}(\vec{r}) = \iiint_{V'} \frac{I(\vec{r}\,')g(R)}{4\pi R^2} dV' \tag{1}$$

where $R = |\vec{r} - \vec{r}\,'|$. In this model, to simplify calculations, the production of photons is assumed to be proportional to the ionization production rate S_i, and then $I(\vec{r})$ is given by

$$I(\vec{r}) = \xi \frac{n_u(\vec{r})}{\tau_u} = \frac{p_q}{p+p_q}\, \xi \frac{\nu_u}{\nu_i} S_i(\vec{r}) \tag{2}$$

where ξ is the photoionization efficiency, $n_u(\vec{r})$ is the density of the radiative excited species u, the ratio $p_q/(p+p_q)$ is a quenching factor, τ_u is the lifetime of the excited state u accounting for the effects of spontaneous emission and quenching, ν_u is the electron impact excitation frequency for level u, and ν_i is the ionization frequency. The function *g(R)* is given by [3]:

$$\frac{g(R)}{p_{O_2}} = \frac{\exp(-\chi_{\min} p_{O_2} R) - \exp(-\chi_{\max} p_{O_2} R)}{p_{O_2} R \, \ln(\chi_{\max}/\chi_{\min})} \tag{3}$$

where $\chi_{min} = 0.035$ Torr^{-1} cm^{-1} and $\chi_{max} = 2$ Torr^{-1} cm^{-1}, and p_{O_2} is the partial pressure of molecular oxygen. The use of equation (1) to calculate the photoionization source term for streamer simulations requires to carry out for each point in the simulation domain and at every time step a quadrature over the entire volume of the discharge.

Recently, two differential approaches to calculate the photoionization term have been proposed to avoid the calculation of the global quadrature over the simulation domain. The first approach is based on the direct numerical solution of an improved Eddington approximation of the radiative transfer equation [4, 5] and is now called the three-group SP_3 ($3G-SP_3$) model. With this model, the photoionization source term is given by

$$S_{\text{ph}}(\vec{r}) = \sum_{j=1}^{3} A_j \xi p_{O_2} c \Psi_{SP_3,0,j}(\vec{r}) \tag{4}$$

where A_j's are constants given in [4] and

$$\Psi_{\text{SP}_3,0,j}(\vec{r}) = \frac{\gamma_2\phi_{1,j} - \gamma_1\phi_{2,j}}{\gamma_2 - \gamma_1} \tag{5}$$

where γ_1 and γ_2 are constants and the functions $\phi_{1,j}$ and $\phi_{2,j}$ are given by

$$\nabla^2\phi_{1,j}(\vec{r}) - \frac{(\lambda_j p_{O2})^2}{\kappa_1^2}\phi_{1,j}(\vec{r}) = -\frac{\lambda_j p_{O2}}{\kappa_1^2}\frac{n_u(\vec{r})}{c\tau_u} \tag{6}$$

$$\nabla^2\phi_{2,j}(\vec{r}) - \frac{(\lambda_j p_{O2})^2}{\kappa_2^2}\phi_{2,j}(\vec{r}) = -\frac{\lambda_j p_{O2}}{\kappa_2^2}\frac{n_u(\vec{r})}{c\tau_u} \tag{7}$$

where λ_j's are absorption coefficients given in [4], and $k_{1,2}$ are constants given in [4, 7]. The variables n_u and τ_u are the same as in equation (2). Based on the radiative transfer theory, it is possible to formulate a consistent set of boundary conditions for the $3G-SP_3$ model [7, 8].

The second approach is called the three-exponential Helmholtz model (3E-Helmholtz) [4, 6]. With this model, the photoionization source term is given by

$$S_{\text{ph}}(\vec{r}) = \sum_{j=1}^{3} S_{\text{ph}}^{j}(\vec{r}) \tag{10}$$

where S_{ph}^{j} is given by

$$\nabla^2 S_{\text{ph}}^{j}(\vec{r}) - (\lambda_j p_{O2})^2 S_{\text{ph}}^{j}(\vec{r}) = -A_j p_{O2}^2 I(\vec{r}) \tag{11}$$

where λ_j's and A_j's are constants given in [4], and $I(\vec{r})$ is given by equation (2). The boundary conditions for equation (11) can be conveniently formulated using the Zheleznyak integral solution [4].

3. Application to streamer propagation in weak field at ground level (P=760 Torr)

The most common and effective model to study the dynamics of streamers is based on the drift-diffusion equations for electrons and ions coupled with Poisson's equation [9]. The transport parameters and reaction rates in air are taken from [10]. The geometry of the simulation domain is identical to the one employed by Liu and Pasko [11] in which a small conducting sphere is placed in a weak uniform electric field E_0. The external homogeneous field E_0 is 10^6 V/m. The radius b and the potential applied to the conducting sphere are 0.1 cm and 6500 V, respectively. In Figure 1, we observe an excellent agreement between the results obtained with the three photoionization models.

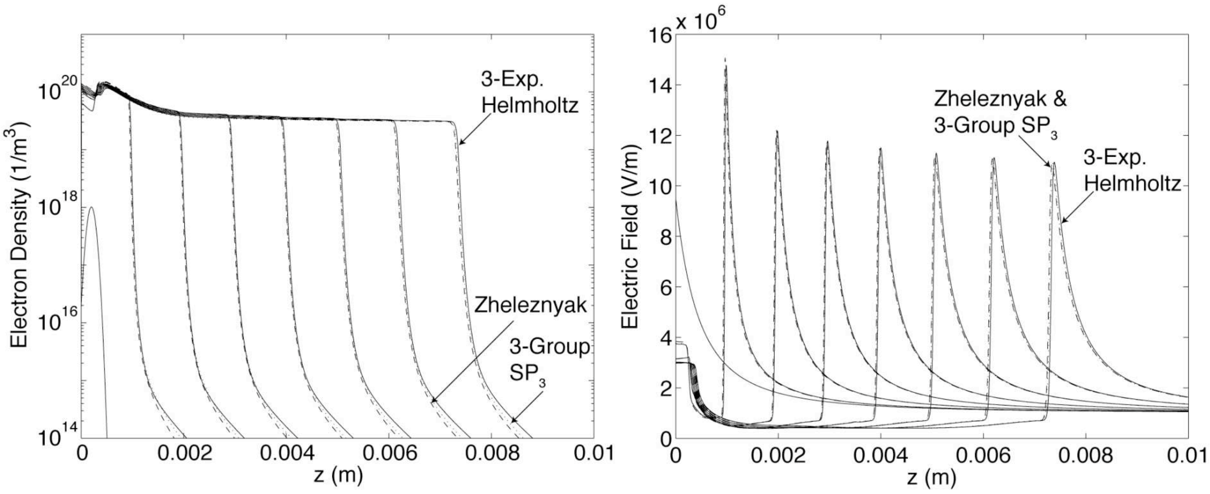

FIGURE 1. Electron density (left) and electric field (right) profiles along the symmetry axis of the domain for the moments of time from t=0 to t=17.5 ns, with a timestep of 2.5 ns calculated using different photoionization models.

4. Application to streamer propagation in weak field at 70 km (P=0.05 Torr)

In this section, following the approach described in [11], we use the similarity relations for streamers to obtain a simulation setup similar to the one used in the previous section, but at a pressure (p=0.05 Torr) corresponding to situation at 70 kilometers altitude in the Earth's atmosphere. The transport and rate parameters are taken to be the same as in [11].

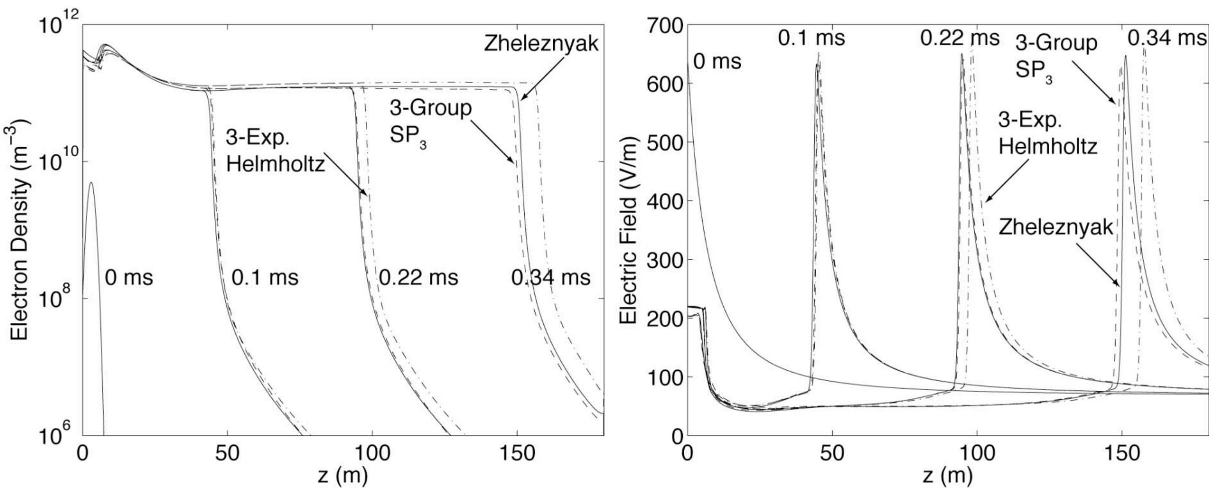

FIGURE 2 Electron density (left) and electric field (right) profiles along the symmetry axis of the computational domain for the moments of time from t=0 to t=0.34 ms calculated using different photoionization models.

In Figure 2, we note that in comparison with the reference Zheleznyak integral model, the 3G-SP_3 and the 3E-Helmholtz models are in good agreement for times up to 0.1 ms. For longer times, we note that the streamer calculated with the 3G-SP_3 remains close to the one calculated using the reference Zheleznyak model whereas a notable discrepancy between the two streamers obtained using the Zheleznyak and the Helmholtz models appears and increases with time.

5. Conclusions

For streamer simulations, the advantage of differential models in comparison with the integral model lies in the simplicity of implementation of this type of models, and in the simplicity of extension of these models to complex two- and three-dimensional simulation geometries. We observe that in the model simulations presented in this paper the three-group SP_3 model exhibits a notably better performance in comparison with the three-exponential Helmholtz model [4, 8].

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