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Extended Treatment for the Drift Instability Grow rates in Non-ideal Inhomogeneous Dusty Plasmas

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Abstract

DAW wave propagation in unmagnetized inhomogeneous dusty plasmas has been studied, considering non-ideal state equation, Padé type . In this paper, the drift instability is analyzed in a weakly magnetized dusty plasma including inhomogeneities of the type called gradient instabilities. Fully kinetic approach is achieve using a dispersion relation in a polynomial form through a multipole approximation . The drift instability for the dust acoustic mode is extensively analyzed including the non-ideal effect, using the state equation introduced by Ree and Hoover . These unstable modes are well discriminated and treated as a function of the density gradient and dust grain radius. The threshold where the non-ideal effect is no longer valid is discussed

Introduction

Plasma inhomogeneities across the magnetic field in the presence of finite - size charged grains causes a wide class of instabilities of an inhomogeneous dusty plasma called gradient instabilities. Such instabilities can be studied in the approximations of magnetic fields where we have parallel straight field lines in order to simplify our treatment. We look for instabilities in the very low frequency regime where a new spectrum instabilities and waves appear, induced by the dust collective dynamics: Dust - Acoustic - Waves (DAWS), Dust - Ion - Acoustic - Waves (DIAWS), etc. The frequency of DAWS are around 10 Hz as determined in the laboratory and lower in astrophysical plasmas [1, 2]. In the case grains are in the micron range we expect a non - ideal behavior due to the fact that the particulate are highly charged and intermolecular forces could play certainly an important role. In order to discuss this problem we use the square - well model and the Padè approximant of Ree and Hoover [3] for a hard - spheres gas, as shown in Eq(1), instead of the well known Van der Waals state equation [4]. In this paper we shown an analysis of the electrostatic waves and instabilities growth rates in a weakly non - ideal magnetized dusty plasma with density gradient.Temperature gradient and charge fluctuation are neglected.

$$p_d = n\, k_b\, T \left(1 \,+\, b_0\, n\, \frac{1 \,+\, a_1\, b_0\, n \,+\, a_2\, b_0^2\, n^2}{1 \,-\, b_1\, b_0\, n \,+\, b_2\, b_0^2\, n^2} \right) \;\; ; \;\; b_0 = 2\, \pi\, \sigma^3/3\ . \qquad (1)$$

where σ is the grain radius.

Theoretical Model

In this paper we introduce a new numerical treatment in combination with a more realistic formulation of the state equation to simulate weak non - ideal effects in inhomogeneous Vlasov - Dusty Plasma systems, where a linearized dispersion relation is obtained. Due to the lower frequency range enough energy can be transferred from the particle to the wave and instabilities can be generated. In order to get an adequate linear dispersion relation including magnetic field $\vec{B} = B_0\hat{k}$ with maxwellian multi-species plasmas $(electron, ion\ and\ dust)$, we introduce the well known and very accurate multipolar approximation for the $\mathbf{Z}$ dispersion function. In the presence of a magnetic field we have the distribution function of the specie α, as solution of the kinetic equation

$$\frac{df_\alpha}{dt} = \frac{q_\alpha}{m_\alpha}\nabla\phi \cdot \frac{\partial f_{0\alpha}}{\partial \mathbf{v}} \tag{2}$$

where

$$f(\mathbf{r},\mathbf{v},t) = \frac{q_\alpha}{m_\alpha}\int_{-\infty}^{t} \exp[i\omega(t-t')]\nabla\phi(r(t'))\cdot\frac{\partial f_{0\alpha}}{\partial \mathbf{v}}\, dt' \tag{3}$$

and $\alpha = e, i, d$. Now, the dispersion relation in terms of the dielectric susceptibilities in the low frequency approximation $(\omega,\ k_z v_T \ll \omega_c)$ is

$$1 + \sum_\alpha \chi_\alpha = 0 \tag{4}$$

where,

$$\chi_{0\alpha} = \frac{1}{(\kappa\lambda_{D\alpha})^2}\left[1 + l_\alpha \frac{\omega}{\sqrt{2}k_z}\mathbf{Z}(\xi_\alpha)_{\mathbf{0}\alpha}\mathbf{e}^{-\mathbf{z}_\alpha}\right] \tag{5}$$

with

$$l_\alpha = 1 - \frac{k_y T_\alpha}{m_\alpha \omega_{0\alpha}^2}\left(\frac{d}{dx}\ln n_{0\alpha} + \frac{dT_\alpha}{dx}\frac{\partial}{\partial T_\alpha}\right) \quad ,$$

$$z_\alpha = \frac{k_y^2 T_\alpha}{m_\alpha \omega_{0\alpha}^2} \quad , \quad \xi_\alpha = \frac{\omega}{\sqrt{2}k_z v_{T_\alpha}^2} \quad ,$$

Taking into account the low frequency range, enough energy could be transferred from the particle to the wave inducing instabilities. An adequate linear dispersion relation with a magnetic field defined by $\mathbf{B} = B\,\hat{\mathbf{k}}$ is obtained for Maxwellian multi - species plasmas, which can be approximated by a very accurate multipolar approximation for the $\mathbf{Z}$ dispersion function neglecting temperature gradients $(i.e.\ \frac{\partial T}{\partial x} = 0)$. Further, in order to simplify our expressions, we need:

$$\frac{d}{dT\alpha}\left(\frac{1}{v_\alpha}\right) = -\frac{m_\alpha}{2\,T_\alpha^{3/2}} \;\; ; \;\; \frac{dz_\alpha}{dT_\alpha} = \frac{k_y^2}{m_\alpha \omega_{0\alpha}} \;\; ; \;\; \frac{d\xi_\alpha}{dT_\alpha} = -\frac{\omega}{\sqrt{2}\,k_z v_{T_\alpha^2}\sqrt{m_\alpha T_\alpha}} \tag{6}$$

Now, using the well known identity for the dispersion function

$$\mathbf{Z}' = -\,2\,\left[\,1 + \,\xi_\alpha\, Z(\xi_\alpha)\,\right] \tag{7}$$

In this work we neglect temperature gradients, as was mention before, then we have in this cases the following expression

$$\chi_\alpha = \frac{1}{(\kappa\lambda_{D\alpha})}\left[1+\frac{\omega \mathbf{Z} I_{0\alpha}e^{-z_\alpha}}{\sqrt{2}\,k_z v_{T_\alpha}}\left\{1-\frac{k_y T_\alpha}{m_\alpha}\frac{n'_{0\alpha}}{n_{0\alpha}}\right\}\right] \tag{8}$$

In order to put our dispersion relation in a dimensionless form, we introduce the following suitable definitions:

$$\lambda_{D\alpha} = \sqrt{\frac{T_\alpha}{n_{0\alpha}Z_\alpha e^2}}\ ;\ K=\kappa\lambda_{Di}\ ;\ \xi_\alpha = \frac{\omega}{\sqrt{2}k_z v_{T\alpha}}\ ;\ \mu_\alpha = \frac{n_{0\alpha}}{n_{0i}}\ ;\ \Theta_\alpha = \frac{T_\alpha}{T_i}\ ;$$

$$\omega_{0\alpha} = \frac{z_\alpha eB}{m_\alpha}\ ;\ \kappa\lambda_{D\alpha} = K\sqrt{\frac{\Theta_\alpha}{\mu_\alpha}}\ ;\ \Omega = \frac{\omega}{\omega_{pi}}\ ;\ \Omega_{0\alpha} = \frac{\omega_{0\alpha}}{\omega_{pi}}\ ;\ U_\alpha = \frac{v_{T\alpha}}{c_{si}}$$

Now, using those results and assuming that $\omega \ll \omega_{0i} \ll \omega_{0d}$ we can write down Eq.(9) as

$$1+\chi_{0e}+\chi_{0i}+\chi_{0d} = 0 \tag{9}$$

with

$$\chi_{0\alpha} = \frac{\mu_\alpha}{K^2\ Theta_\alpha}\left[1+\frac{\Omega\ \mathbf{Z} I_{0\alpha}e^{-z_\alpha}}{\sqrt{2}\,K_z U_\alpha}\left\{1-\frac{K_y U_\alpha^2}{\Omega\ \Omega_{0\alpha}}\frac{1}{\Lambda_{n_\alpha}}\right\}\right] \tag{10}$$

Now it is very convenient to define the following relations [4]:

$$\frac{1}{L_p} = \frac{\nabla p_d}{p_d};\ \frac{1}{L_n} = \frac{\nabla n_d}{n_d};\ \frac{1}{L_p} = p_d\frac{1}{L_n} \equiv \frac{\alpha_1 n_d+\alpha_2 n_d^2+\alpha_3 n_d^3+\alpha_4 n_d^4}{1+\beta_1 n_d+\beta_2 n_d^2}\frac{1}{L_n} \tag{11}$$

where

$$\alpha_1 = 1.000000 KT_d\ ;\ \alpha_2 = 0.438507 KT_d b_o\ ;\ \alpha_3 = 0.14482 KT_d b_o^2\ ;$$

$$\alpha_4 = 0.017329 KT_d b_o^3\ ;\ \beta_1 = -0.561493 b_o\ ;\ \beta_2 = 0.081313 b_o^2\ ,$$

Introducing the multipolar approximation for the function **Z**, we can get a rational polynomial expression in the form [5]

$$\sum_j a_i\ \Omega_i \,/ \sum_j b_j\ \Omega_j = 0, \tag{12}$$

where coefficients a_i and b_i are function of the system parameters. In this relation is very easy to find roots of the numerator. Analysis of the solutions spectra permit us to find the gain $\gamma = Im(\Omega)$ in function of $1/K_y$ or simply versus K_y as shown in Fig.(1).

Results

It is shown that for typically laboratory plasmas with $Z_d = 1\times10^3$ and densities of 1×10^{10} the maxima of the instability increase with r_d and decrease and goes wider by smaller radius, toward the ideal region where higher oscillations are expected in dust acoustic electrostatic waves. The range of the main parameters of this low frequency oscillations are established by the approximation that help to simplify the dispersion relation, given by $\Omega, K_z U_d \ll \Omega_{cd}$.

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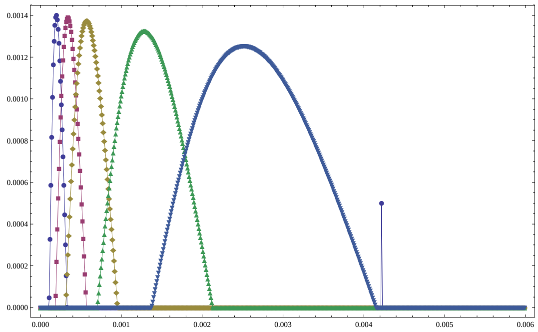

Figure 1: Normalized maximum growth rate as a function of normalized perpendicular wavelength for for a relatively inhomogeneous plasma ($\Lambda = 1.5 \times 10^1$) in function of the grain radius: $r_d = 1$ (circles), 0.7, 0.5, .3 , .1 (microns)