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PROPERTIES OF CYCLOTRON MASER EMISSION IN INHOMOGENEOUS PLASMA

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Introduction

The electron cyclotron maser instability is a powerful mechanism for producing non-thermal radiation in a plasma. It plays an important role in both devices such as gyrotrons and in atmospheric and space physics applications including phenomena such as auroral kilometric radiation (AKR) [1] and radio emission from certain stars [2] .We suggested that these latter phenomena are produced by a horseshoe shaped distribution in velocity space and have set up an experiment to investigate the effect in the laboratory [3].

One long-standing problem in studies of AKR and similar phenomena is how the radiation generated below the local electron cyclotron frequency gets onto the vacuum propagation branch of the dispersion relation [4,5]. To gain some understanding of the radiation escape possibilities we investigate here the generation and propagation of cyclotron radiation in an inhomogeneous magnetic field, considering a ring distribution for simplicity. For both step function and continuous magnetic field gradients it is shown that there are solutions to the wave equation representing a localised region of instability from which waves radiate outwards. While these results have been obtained for a ring distribution, we show that a horseshoe distribution has a similar dispersion relation and may be expected to show similar behaviour.

Dispersion curves for cyclotron maser instability

We start by studying the topology of dispersive curves in a plasma with electron distribution showing a cyclotron maser instability. For a ring distribution function,

$f(v_{\|}, v_{\perp}) = n_0\, \delta(v_{\|})\, \delta(v_{\perp} - v_0)$, dispersion curves in Fig.1(a) show similar behaviour for parallel and perpendicular propagation with a purely real branch connecting the two unstable branches. For the more realistic horseshoe distribution function $f(v_{\|}, v_{\perp}) = A \cdot e^{-\frac{m}{2Th}\left[\left(\sqrt{v_{\|}^2 + (1 - B/B_0) v_{\perp}^2} - v_0\right)^2 + B/B_0\, v_{\perp}^2\right]}$ and the correspondent dielectric tensor components calculated using formulas we obtained previously [6], we calculated dispersion curves shown in Fig.1(b) which are qualitatively similar to the ring distribution curves.

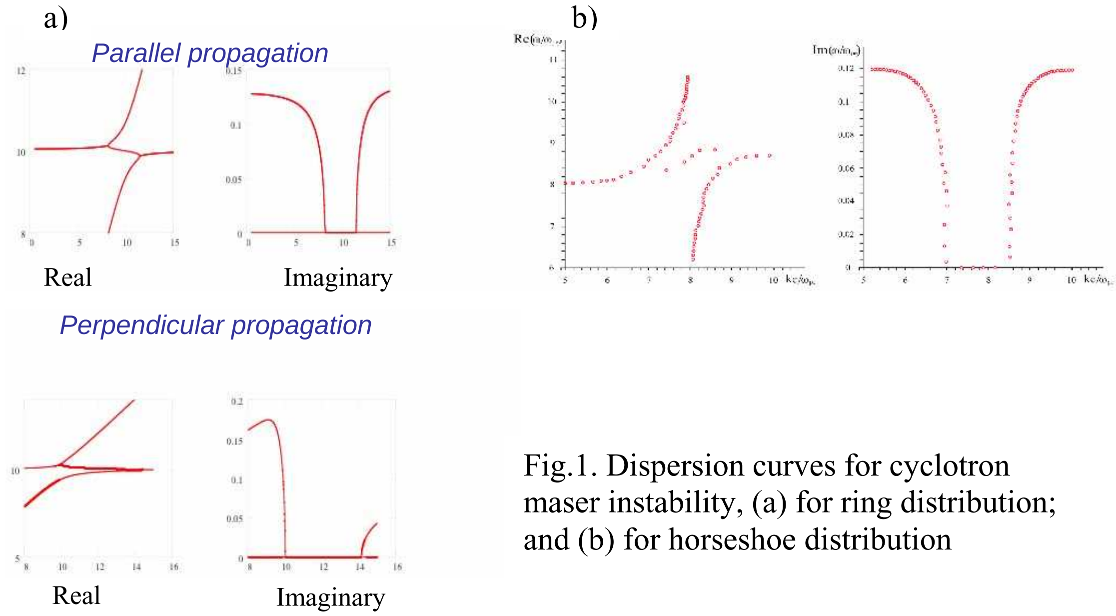

Fig.1. Dispersion curves for cyclotron maser instability, (a) for ring distribution; and (b) for horseshoe distribution

Propagation in inhomogeneous systems

Our further step was to look at inhomogeneous systems, starting with the ring distribution emission with a step function profile of B, with external media dispersion relation $k_0 = \omega / c$. Requiring outgoing waves only gives $i \tan(Kl) = \frac{2K}{k^2 + 1}$ and $F(\omega, K\omega / c) = 0$, where $K = k / k_0$, $l = k_0 L$ and L is the slab's thickness and $F(w, k) = 0$ is the dispersion relation inside the slab. We are interested in solutions with positive $\mathrm{Im}\,\omega$ meaning unstable growing modes. There are multiple solutions shown in Fig.2 where the real part of their frequency follows closely the connecting branch for the infinite plasma dispersion relation in Fig.1(a).

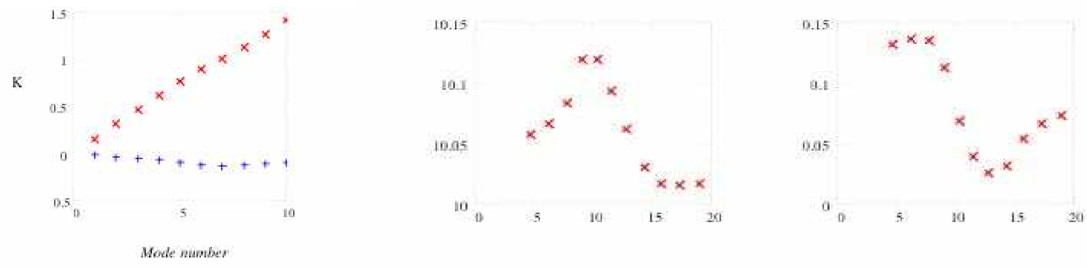

Fig.2. Discrete solutions for step function inhomogeneity dispersion equations for l=20.

The existence of many growing eigenmodes with almost the same frequency is similar to what we found previously [6] for plasma annulus instability for the horseshoe distribution.

The next step is to consider smooth gradient of magnetic field with $\Omega = \frac{eB}{\gamma m} = \Omega_0\left(1+\frac{x}{L}\right)$ where dispersion relations can be generally written in the form $k^2 = F(\omega, \Omega)$ corresponding to the differential equation $\frac{d^2\phi}{dx^2} = -F\left(\omega, \Omega_0\left(1+\frac{x}{L}\right)\right)\cdot\phi$. We firstly consider WKB approximation, finding that if we add a positive imaginary part to ω, then by adjusting its magnitude we can obtain the dispersion curves shown in

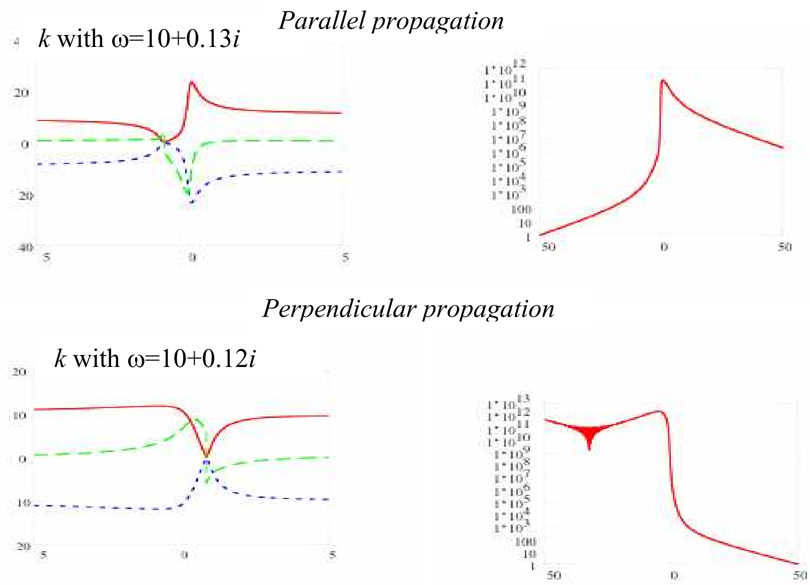

Fig.3. Dispersion curves (a,c) and field amplitude (b,d) for smooth gradient of B; red and blue are real parts of positive and negative square roots and green is imaginary part corresponding to red line.

Fig.3(a,c) for parallel and perpendicular propagation, correspondingly, with the branch going to the left on the low field side connected to the wave going to the right on the high field side for the parallel propagation, and reverse situation for the perpendicular one. The corresponding differential equation solutions for the field amplitude shown in Fig.3(b,d) evidently correspond to outgoing waves.

We investigated a simple analytic approach then, supposing that amplitudes match at $x = x_0$ where the curves on Fig.3(a,c) join together, and then following them away from there. This approach works remarkably well for the ring distribution, and being considerably simpler then solving the initial equations numerically, proved useful in application to the horseshoe distribution. Result look similar for the horseshoe distribution, with the branches of different roots joined at one point and evident propagation away from the instability region.

Conclusion

The results show that evidence that the ring distribution branch connecting cyclotron mode with vacuum mode also exists for the horseshoe distribution suggesting the possibility of radiation escape from the unstable region. For a simple ring distribution of electrons with either a step function variation of magnetic field or a continuous gradient there can exist localised regions of instability from which waves, growing in time, can be radiated outwards. Energy can go in both directions (for step function profile) or to high field or low field region (for gradient magnetic field). Simple analytic method gives good results and will allow investigation of the more complicated horseshoe dispersion relation. Some interesting and surprising properties found here call for further investigation.

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