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Neoclassical Electric Field in Presence of Large Gradients and Particle Losses

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Transport barriers with strong density and temperature gradients can appear in regions with large velocity shear where the confinement can reach close to the neoclassical level [1]. The understanding is that large gradient of the radial electric field, associated with the velocity shear, can stabilise the drift mode turbulence when the shearing rate exceed the growth rate of the most unstable drift modes [2]. Radial electric fields several times the neoclassical values have been observed in experiments prior to the formation of transport barriers [3-5]. The radial electric field in the neoclassical theory is determined by the parallel velocity, the density and the temperature gradients of the ions [6,7]. In the banana regime the variation with respect to the ion temperature is weaker and the sign opposite to that of the density. Shaing et al proposed that strong radial electric fields arise due to bifurcation caused by particle losses and reduced collisionality [8]. To be able to study radial electric field and the collisional transport in the banana regime a new orbit averaged Monte Carlo operator was developed [9], which is here improved to enable modelling with stronger gradients.

The neoclassical transport is caused by collisional scattering between trapped and passing particles. Since the averaged flux surface location of a trapped particle in an axisymmetric plasma is approximately given by the location of its turning points, $\psi_T \approx P_\phi /Ze$, the transport of trapped particles can be calculated from the changes of their canonical angular momentum, P_ϕ. The changes of the turning points of a trapped particle are given by $\Delta\psi_T = -mR\Delta v_\phi /Ze$. Since the averaged flux surface location of a passing particle is almost unaffected by the change in its parallel velocity, momentum exchanges between passing and trapped particles by collisions can lead to a significant spatial transport of the trapped particles and give rise to charge separation producing electric fields; collisions between trapped ions of the same species lead only to a redistribution of them without changing the averaged centre of charge. The radial flux of trapped particles of species *i* colliding against particles of species *f* can be described by a stochastic differential equation

$$\Delta P_\phi = \left\langle \dot{\mu}^{(if)}_{P_\phi} \right\rangle_\tau \Delta t + \zeta_1 A^{(if)}_{P_\phi W}\sqrt{\Delta t} + \zeta_2 A^{(if)}_{P_\phi \Lambda}\sqrt{\Delta t} + \zeta_3 A^{(if)}_{P_\phi P_\phi}\sqrt{\Delta t} \ , \qquad (1)$$

where $\langle ... \rangle_\tau = 1/\tau_b \oint ... dt$ with $\tau_b = \oint dt$, ζ_k are random numbers with unit variance and zero expectation value, $\dot{\mu}_{P_\phi}^{(if)} = m_i R \xi \left(\alpha^{(if)} - \gamma^{(if)} / 2v \right)$, and $\xi = v_\parallel / v$. The Monte Carlo operators $A_{IJ}^{(if)}$ are functions of the time derivatives of the co-variances $\dot{\sigma}_{IJ}^{(if)}$ of the invariants of motion (W, Λ, P_ϕ; σ) [10,11], where W denotes the energy of the ions, including an electrostatic potential, which is assumed to be a function of ψ only; $\Lambda = \mu B_0 / W$; σ a label to distinguish different orbits having the same invariants, μ the magnetic moment, B_0 the magnetic field strength on the magnetic axis. The variances and drift terms of test particles of species i colliding against field particles of species f are functions of Chandrasekhar´s Coulomb diffusion coefficients, $\alpha^{(if)}$, $\beta^{(if)}$ and $\gamma^{(if)}$. For an isotropic local Maxwellian distribution function in the thin orbit approximation the contributions from the drift terms cancel, and a diffusion term remains describing the contravariant flux caused by trapped particles colliding with the background

$$\Gamma_{\psi:D}^{(if)} \approx -\frac{1}{2(Z_i e)^2} \int_{trapped(\psi)} \left\langle \dot{\sigma}_{P_\phi P_\phi}^{(if)} \right\rangle_\tau \left(\frac{1}{n_i} \frac{\partial n_i}{\partial \psi} + \left(\frac{W_i}{kT_i} - \frac{3}{2} \right) \frac{1}{T_i} \frac{\partial T_i}{\partial \psi} \right) F_{0i} \sqrt{g} dW d\Lambda , \qquad (2)$$

where $\sqrt{g}$ is the Jacobian, F_{0i} the lowest order approximation of the distribution function of species i and $\dot{\sigma}_{P_\phi P_\phi}^{(if)} = m_i^2 R^2 \left(\xi^2 \beta^{(if)} + 0.5 \left(1 - \xi^2 \right) \gamma^{(if)} \right)$. Eq. (2) implies that collisions scatter trapped particles across flux surfaces; even when $f = i$. The displacements of trapped ions result in space charges that give rise to an electric field normal to the magnetic flux surface. Although the imposed polarisation currents nearly cancel the space charges produced by collisions, the remaining net electric field gives rise to a toroidal precession of the trapped drift orbits with $v_\phi \approx E_r / B_\theta$. The difference in the toroidal mean velocity between passing and trapped ions results in a net friction producing a flux of trapped ions counteracting the diffusive flux described by Eq. (2). To calculate the friction and particle flux we approximate the trapped and passing ions by two shifted Maxwellians, taking as the reference coordinate system the one for which the averaged parallel velocity of the passing ion vanishes, the contravariant flux of trapped ions due to this friction becomes

$$\Gamma_{\psi:d}^{(if)} \approx \frac{1}{Z_i e} \int_{trapped(\psi)} \left\langle \dot{\mu}_{P_\phi}^{(if)} \right\rangle_\tau F_{0i} \sqrt{g} dW d\Lambda . \qquad (3)$$

To compare with neoclassical theory we take the thin orbit limit of Eq. (3)

$$\hat{\Gamma}_{\psi:d}^{(if)} = \int_S \frac{dS}{|\nabla \psi|} \int_{trapped(\psi)} n_{it} \frac{\dot{\mu}_{P_\phi}^{(if)}}{Z_i e} f_{it}(v_\parallel - v_{\parallel i}, v_\perp) d^3 v . \qquad (4)$$

As a model for the shift in parallel velocity of the trapped particles we take $\upsilon_{\|\mathrm{i}} = R\Omega_{\mathrm{if}}(\psi)B_\phi/B$. Using Stix´s approximation of $\alpha^{(if)}$, $\beta^{(if)}$ and $\gamma^{(if)}$ we obtain $\dot{\mu}_{P_\phi}^{(if)} = m_i R C_{if} l_f^2 \left(1+m_i/m_f\right) G\left(l_f \upsilon\right)$, where $C_{if} = 8\pi n_{fp} Z_f^2 Z_i^2 e^4 \ln\Lambda / m_i^2$, $l_\mathrm{i}^2 = m_\mathrm{i}/2kT_\mathrm{i}$, n_fp is the density of the passing field particles, G and Φ (used below) are defined in Ref. [12].

The local contribution to the diffusive flux corresponding to Eq. (2) becomes

$$\hat{\Gamma}_{\psi:D}^{(if)} \approx -\left(\frac{1}{n_i}\frac{\partial n_i}{\partial\psi} - \frac{3}{2}\frac{1}{T_i}\frac{\partial T_i}{\partial\psi}\right)\int_S H_0 \frac{dS}{|\nabla\psi|} - \frac{1}{T_i}\frac{\partial T_i}{\partial\psi}\int_S H_2 \frac{dS}{|\nabla\psi|}, \tag{5}$$

where $H_n = \int\limits_{trapped(\psi)} C(\upsilon)\left(l_i\upsilon\right)^n f_{it}(\upsilon_\| - \upsilon_{\|i}, \upsilon_\perp) d^3\upsilon$ and

$$C(\upsilon) = \frac{m_i^2 R^2 l_f n_{it} C_{if}}{4\left(Z_i e\right)^2 l_f \upsilon_i}\left\langle\left(1-\frac{\upsilon_\|^2}{\upsilon_i^2}\right)\Phi\left(l_f\upsilon_i\right) - \left(1-3\frac{\upsilon_\|^2}{\upsilon_i^2}\right)G\left(l_f\upsilon_i\right)\right\rangle_\tau .$$

The quasi-neutrality condition requires that the fluxes of ions and electrons satisfies

$$Z_i\left(\hat{\Gamma}_{\psi:D}^{(ii)} + \hat{\Gamma}_{\psi:D}^{(ie)} + \hat{\Gamma}_{\psi:loss}^{(ii)} + u_i L^{(ii)} + u_i L^{(ie)}\right) = \hat{\Gamma}_{\psi:D}^{(ei)} + \hat{\Gamma}_{\psi:D}^{(ee)} + \hat{\Gamma}_{\psi:loss}^{(e)} + u_i L^{(ei)} + u_e L^{(ee)}, \tag{6}$$

where $u_i L^{(if)}\left(l_i u_i\right) \equiv \hat{\Gamma}_{\psi:d}^{(if)}\left(l_i u_i\right)$ with $u_\mathrm{i} = R_0\Omega_{\mathrm{if}}(\psi)$ and $\hat{\Gamma}_{\psi:loss}^{(\alpha)}$ is the flux due to particle losses: prompt orbit losses or spatial redistribution of ions. The fluxes in Eq. (6) are non-linear functions of u_i, where u_i is related to the electric field in the lab frame; the non-linearity becomes important as u_i approaches the thermal velocity. To solve Eq. (6) we expand the fluxes in powers of mass ratio, $(m_\mathrm{e}/m_\mathrm{i})^{1/2}$, $u_\mathrm{i} = u_\mathrm{i}^{(0)} + u_\mathrm{i}^{(1)} + \ldots$. Since the ion-ion collision frequency is $\mathrm{O}(m_\mathrm{i}/m_\mathrm{e})^{1/2}$ larger than the ion-electron collision frequency, the ion flux should in the 0:th order vanish to satisfy quasi-neutrality. Neglecting losses by electrons and taking the large aspect ratio limit one obtains

$$u_i^{(0)} = -\gamma_n(u_i^{(0)})\frac{T_i}{Z_i e n_i}\frac{R}{}\frac{\partial n_i}{\partial\psi} - \gamma_\alpha(u_i^{(0)})\frac{R}{Z_i e}\frac{\partial T_i}{\partial\psi} + \gamma_\beta(u_i^{(0)})\frac{\Gamma_{\psi:loss}^{(i)}}{L^{(ii)}(0)} \quad . \tag{7}$$

The coefficients γ_n, γ_α and $\gamma_\beta = L^{ii}(0)/L^{ii}(l_i u_i^{(0)})$ are plotted in Fig. 1 versus $l_\mathrm{i}u_\mathrm{i}^{(0)}$. The coefficient γ_n is found to be independent of u_i and agrees with the standard neoclassical value, $\gamma_\mathrm{n} = 1.0$. For small $\left|u_i^{(0)}\right|$ we obtain $\gamma_\alpha = 0.25$ including also energy diffusion in the collision operator; the standard neoclassical theory gives in the large aspect ratio $\gamma_\alpha = 0.17$ [7]. The deviation from small $\left|u_i^{(0)}\right|$ becomes significant, when u_i is of the order of the thermal velocity of the ions, for which the scale length corresponds to the banana width of thermal ions. The coefficient γ_α changes sign when $l_\mathrm{i}u_i^{(0)} = 0.67$; an increase of the ion

temperature gradient will for large values of $\left|u_i^{(0)}\right|$ result in an increase of $\left|u_i^{(0)}\right|$ instead of a reduction as predicted from standard neo-classical theory.

The non-linearity of γ_α and γ_β, due to larger ion temperature gradients and losses, can give rise to bifurcated solutions with much larger radial electric fields than predicted by the standard neoclassical theory.

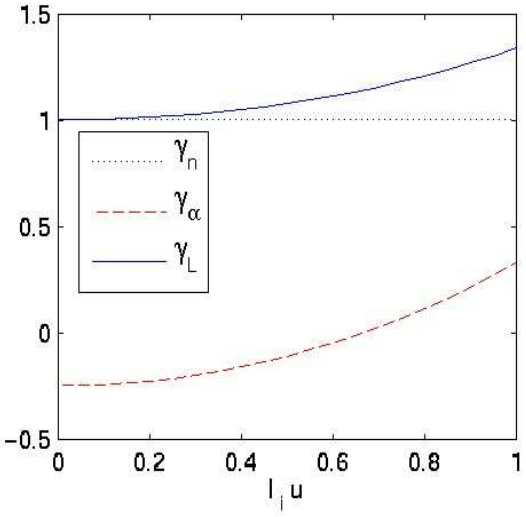

Fig. 1 γ_n, γ_α and γ_β=$L(0)/L(l_i\, u_i^{(0)})$ versus $l_i\, u_i^{(0)}$.

The particle flux is obtained by solving Eq. (7) in the next order and using $u_e^{(0)} \approx u_i^{(0)}$. Since the flux due to the drift term in Eq. (4), that cancels the diffusive flux of the ions, depends on the sign of the charge, the corresponding drift term for the electrons will increase the electron transport and thus resulting in a larger particle flux.

The orbit averaged Monte Carlo operators describing the change in P_ϕ is obtained by shifting the parallel velocity by the trapped particle velocity with $\upsilon_{\|i} = Ru_i^{(0)}B_\phi/R_0B$ with $u_i^{(0}$ given by Eq. (7).

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