Pressure driven resistive modes in the advanced RFP

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1. The advanced RFP

In the conventional Reversed-Field Pinch (RFP) the beta value is often high, but magnetic fluctuations degrade confinement, which leads to modest global energy confinement. The fluctuations are caused by the RFP dynamo, which is a redistribution process of parallel plasma current. The dynamo process is cyclic and during the relaxation phase, tearing and reconnection of magnetic field lines occur, which leads to enhanced energy and particle transport. The dynamo process is thus responsible both for the maintenance of the RFP configuration and the radial energy leakage.

In earlier work [1], a numerical model was examined in which an externally applied auxiliary electric field $\mathbf{E}_a$ was applied to control the current profile. The field was automatically adjusted for continual elimination of the dynamo field $\mathbf{E}_f = -\langle \mathbf{v} \times \mathbf{B} \rangle$ (brackets indicate mean over periodic coordinates) through a feedback routine. The model resulted in strongly increased energy confinement time and poloidal beta β_θ in this *advanced* mode of operation. A poloidal beta scaling law is obtained as function of experimental parameters:

$$\beta_p = 0.246\Theta^{0.23} a^{-0.058} \mu^{0.029} Z_{eff}^{0.058} \left(I / N \right)^{-0.12} I^{-0.12} \; ; \qquad (1)$$

$\Theta = B_\theta(a) / < B_z >$ is the pinch parameter, μ i s the ion to proton mass ratio and a is the pinch radius. The scaling with experimental parameters is weak. For comparison, the scaling law achieved in our earlier conventional RFP study resulted in a $\beta_p \propto \left(I / N \right)^{-0.40} I^{-0.40}$ dependence. The idea to use current profile control (CPC) in the RFP does also find strong support in experiments. Recent improvements made to the original CPC scheme demonstrate that a quasi-steady state with enhanced and nearly constant energy confinement time during a prolonged period of time is possible to realize. Also, in several cases very high poloidal beta values approaching unity are achieved. In Figure 1, the evolution of energy confinement time τ_E, poloidal beta β_θ, radial magnetic field $\langle B_r^2 \rangle$ and the reversal parameter F for different initial conditions of CPC are displayed; for details see [2].

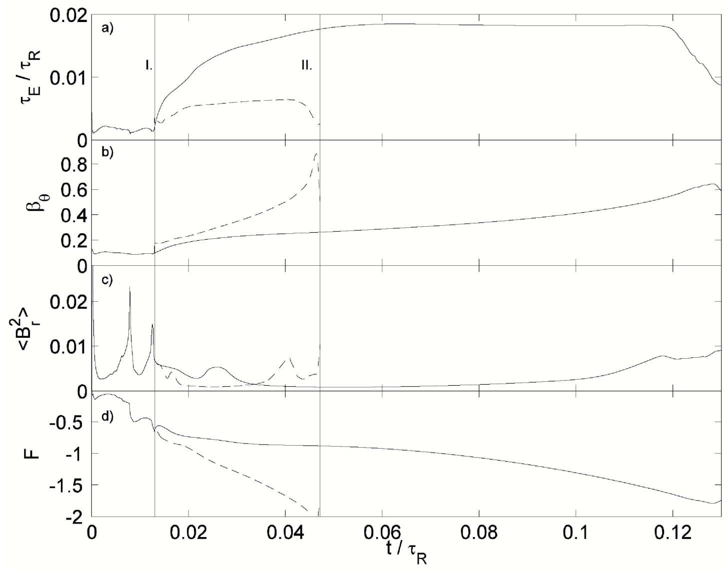

Figure 1.

A new criterion for the beta-limit of $m = 0$ modes is also derived [2], allowing higher poloidal beta values than the earlier criterion $\beta_\theta < 1/2$, derived by Robinson.

One objective of this study is to eliminate the RFP dynamo to relieve the RFP from tearing modes. It thus becomes important to answer to what extent confinement allows pressure driven modes, which will always be present to some extent. We have derived an analytic estimate of $m = 0$ resistive g-mode growth rates, using linearized resistive MHD [2]:

$$\gamma_g = \left[\frac{2k\left(-p' - r^4(\rho u_\theta^2 / r^4)'\right)}{r\left|B_z'\right|} \right]^{2/3} S^{-1/3} \qquad (2)$$

Using code data in the limit of negligible flow, typical values for dominating $(m, n) = (0, 1)$ modes are $\gamma_g = 0.02$ (without CPC) and $\gamma_g = 0.01$ (well into the CPC regime). Nonlinear coupling of unstable $(m, n) = (1, 1$ and $2)$ modes may also contribute to the power spectrum.

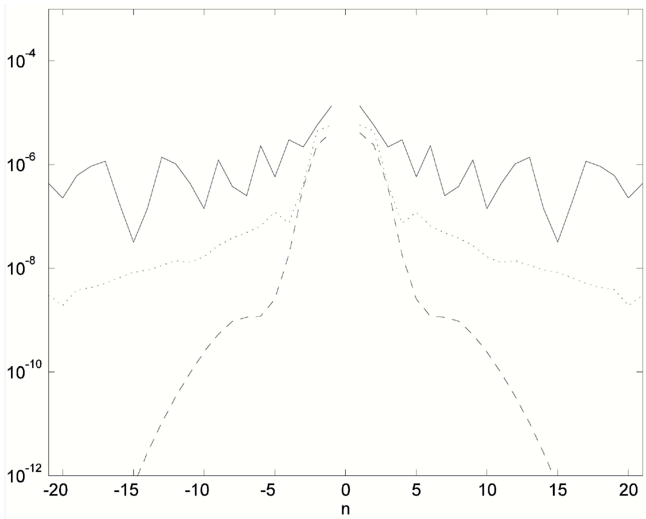

Figure 2.

In Figure 2 a mode spectrum for $m = 0$ modes is shown for high beta. The solid curve represents spectrum at dynamo dominated confinement; the dashed line at peak confinement and the dotted line at decreasing confinement. Positive n-numbers refer to modes resonant outside the reversal surface.

2. The time- and parameter generalized weighted residual method (TP-WRM)

A fully spectral method has recently been developed for obtaining semi-analytical solutions to initial-value partial differential equations [3]. By semi-analytical is meant that closed, analytical solutions in *time, space and physical parameters* are obtained as spectral expansions with numerical coefficients. This is of interest for carrying out scaling studies, in which the detailed parametrical dependence preferably is explicit rather than purely numeric. The TP-WRM is based on Chebyshev polynomial expansions, having several desirable qualities. The TP-WRM eliminates time stepping and the associated, limiting Courant or CFL conditions, since the time dependence is calculated by a global minimization, not causally.

Formally, the systems of parabolic or hyperbolic initial-value PDE's

$$\frac{\partial \mathbf{u}}{\partial t} = D\mathbf{u} + f \tag{3}$$

are solved, where $\mathbf{u} = \mathbf{u}(t, \mathbf{x}; \mathbf{p})$ is the solution vector, $\boldsymbol{D}$ is a linear or nonlinear matrix operator and $f = f(t, \mathbf{x}; \mathbf{p})$ is a forcing term. $\boldsymbol{D}$ may depend on *both* physical variables (t, $\mathbf{u}$, $\mathbf{x}$) and physical parameters (denoted $\mathbf{p}$); f is assumed arbitrary but non-dependent on $\mathbf{u}$. Initial $\mathbf{u}(0,\mathbf{x};\mathbf{p})$ as well as (Dirichlet, Neumann or Robin) boundary $\mathbf{u}(t,\mathbf{x}_B;\mathbf{p})$ conditions are assumed.

The RFP plasma has earlier, in the debsp code, been modelled in cylindrical geometry using fully nonlinear 3D resistive MHD equations, including finite pressure, ohmic heating, heat convection, parallel and perpendicular heat conduction. In future TP-WRM work also the Hall term and electron diamagnetism will be included. This will extend, and correct, earlier studies of RFP confinement. In a parallel TP-WRM study, the Vlasov-Fluid equations including the effects of fully kinetic ions and finite electron temperature will be used for determining the effects of finite Larmor radius on the pressure driven resistive g-modes.

The solution (example: 1D, one parameter) is expanded in a Chebyshev series:

$$u = u(t, x; \nu) = \sum_{m=0}^{M}{}' \sum_{n=0}^{N}{}' \sum_{p=0}^{P}{}' a_{mnp} T_m(\tau) T_n(\xi) T_p(V) \tag{4}$$

$$\tau = \frac{t - A_t}{B_t}, \quad \xi = \frac{x - A_x}{B_x}, \quad V = \frac{\nu - A_\nu}{B_\nu} \ , \tag{5}$$

where $A_i = (b_i+a_i)/2$ and $B_i = (b_i-a_i)/2$, with a_i and b_i being domain boundaries. Primes on summation signs denote that the coefficients with zero indices should be halved. We reformulate the PDE as

$$u(t,x;\nu) = u(t_0,x;\nu) + \int_{t_0}^{t} Hu(x,t;\nu)dt \tag{6}$$

The final relation to be solved is a nonlinear algebraic system for the TP-WRM coefficients

$$a_{qrs} = 2\delta_{q0}b_{rs} + A_{qrs} + F_{qrs} \ , \tag{7}$$

where the first right hand term contains initial conditions, the second corresponds to the operator D and the third to the forcing term. In conclusion the TP-WRM is well suited for solving problems with wide separation of time scales, and will be an advantageous method as compared to the explicit Lax-Wendroff (many time steps) and the Crank-Nicholson (expensive matrix inversion for nonlinear equations) schemes.

3. Growth rate analysis of numerical steady states

We examine, using the linearised resistive MHD equations, whether numerically obtained steady states are pressure limited. For theoretical interpretation, it is often convenient to use the variables $p(r)$ and $\mu(r) = \mu_0 \mathbf{j}{\cdot}\mathbf{B} / B^2$. In this way, the effect of pressure alone can be seen by reducing the beta value, retaining equilibrium and the μ profile; the latter being related to current driven tearing instabilities. In normalized variables B_θ, B_z and p can then be obtained by integrating the equations (where prime denotes radial differentiation)

$$B_\theta' - \mu B_z + p' B_\theta / (B_\theta^2 + B_z^2) + B_\theta / r = 0 \tag{8}$$

$$B_z' + \mu B_\theta + p' B_z / (B_\theta^2 + B_z^2) = 0 \tag{9}$$

$$p' + (r\chi / 2)[(\mu / 2)(B_\theta^2 + B_z^2) / B_\theta - B_z / r]^2 = 0 \tag{10}$$

using the on-axis values $B_z(0) = 1$, $B_\theta(0) = 0$ and the boundary value $p(1) = p_c$. The parameter χ represents the Suydam criterion; Suydam instability results for $\chi > 1$. If the pressure profile is given, only the two first equations are used to convert between μ and the magnetic field. A TP-WRM code now exists for this purpose, and is being benchmarked.

4. References

[1] Dahlin J.-E. and Scheffel J., Nucl. Fusion **47** (2007) 9–16.

[2] Dahlin J.-E. and Scheffel J., Nucl. Fusion **47** (2007) 1184–1188

[3] J. Scheffel, TRITA EE-2008-006, Royal Inst. of Technology, Stockholm, Sweden, 2008.