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Propagation and absorption of electron-cyclotron waves in the presence of magnetic islands

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Introduction

We analyse the propagation and absorption of EC waves, using ray tracing techniques, when the Neoclassical Tearing Mode (NTM) is excited inside a tokamak. The general idea for the stabilization of the NTM is to raise the current density in the island's O-point, e.g. by ECCD. So far, the analysis of the EC propagation and absorption was done ignoring the presence of the islands. Our analysis starts with a stable magnetic configuration, which is perturbed and gradually forms a magnetic island. In this unstable geometry, we study the propagation and absorption of EC radiation near the reconnection layer, focusing on the effect of the island topology on the efficiency of the EC absorption.

Standard magnetic field

A simple form of the magnetic field with circular flux surfaces in a torus is the so called Standard Magnetic Field [1]. It consists of two components, the toroidal B_ϕ and the poloidal B_θ

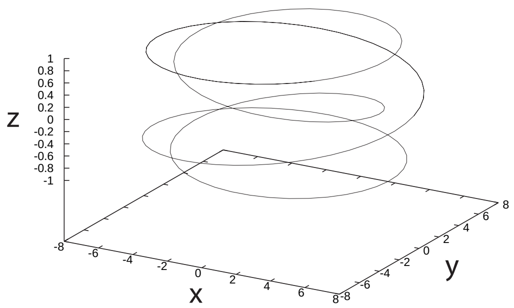
$$\mathbf{B}_0(r,\theta) = B_\phi \hat{e}_\phi + B_\theta \hat{e}_\theta = \frac{B_0}{1+\varepsilon\cos\theta}\hat{e}_\phi + \frac{\varepsilon}{q} B_\phi \hat{e}_\theta \tag{1}$$

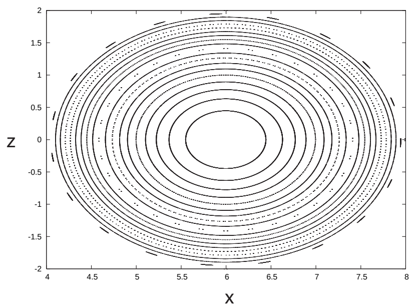
where $\varepsilon = \frac{\alpha}{R_0}$ is the inverse aspect ratio of the tokamak, α the minor radius and R_0 the major radius. The field lines are three-dimensional curves tangent to the vector field $\mathbf{B}$. A field line is the solution of:

$$\frac{d\rho}{ds} = \frac{\mathbf{B}(\rho(s))}{|\mathbf{B}(\rho(s))|} \tag{2}$$

where the variable s parameterizes the field line and ds is the length element of the curve $ds = \sqrt{dr^2 + r^2 d\theta^2 + (R_0 + r\cos\theta)^2 d\phi^2}$. This definition yields $|\frac{\mathrm{d}\rho}{\mathrm{d}s}| = 1$, demonstrating that the parameter s is the arc-length along the field line, measured forward from the point $\rho(0)$. (R, Z, ϕ) are the cylindrical coordinates, ϕ is the toroidal angle. Cylindrical and polar coordinates are related through the following expressions: $R = R_0 + r\cos\theta$, $Z = r\sin\theta$.

To obtain the field lines, Eq. (8) is numerically integrated using a Runge-Kutta method (4th order) For most initial conditions, ergodic field lines are obtained that lie on ergodic flux surfaces, as can be seen in the poloidal cross section below.





Magnetic field perturbation

In the following, we briefly describe how the magnetic field lines can be understood as Hamiltonian trajectories, in what is called "magnetic field line tracing"[2].

In terms of the toroidal and poloidal angle ϕ, θ and the respective fluxes ψ_t, ψ_p, the magnetic field can be represented in the Clebsch form, $\mathbf{B} = \nabla\psi_t \times \nabla\theta - \nabla\psi_p \times \nabla\phi$. The set of variables (ϕ, θ, ψ_t) can be used as coordinates, because the Jacobian of the transformation to the Cartesian coordinates never vanishes. The poloidal flux can be expressed in terms of the new coordinates, $\psi_p = \psi_p(\psi_t, \theta, \phi)$. Along a field line we have $d\psi_t = d\mathbf{r} \cdot \nabla\psi_t = ds\mathbf{B} \cdot \nabla\psi_t$, so Eq. (5) becomes

$$ds = \frac{d\psi_t}{\mathbf{B} \cdot \nabla\psi_t} = \frac{d\theta}{\mathbf{B} \cdot \nabla\theta} = \frac{d\phi}{\mathbf{B} \cdot \nabla\phi}. \tag{3}$$

The method of characteristics leads to Hamiltonian equations in the 2-dimensional space (θ, ψ_t), with ψ_p playing the role of the Hamiltonian and ϕ that of "time"

$$\frac{d\psi_t}{d\phi} = -\frac{\partial\psi_p}{\partial\theta}, \qquad \frac{d\theta}{d\phi} = \frac{\partial\psi_p}{\partial\psi_t}. \tag{4}$$

We illustrate this approach by revisiting the case of the standard magnetic field. The magnetic field in such a configuration is axisymmetric, and due to the axisymmetry it is $\mathbf{B}\nabla\psi_p = 0$, so that the magnetic field lines lie on surfaces of constant ψ_p. In this case, the safety factor does not depend on the poloidal angle, $q = q(\psi_p)$ therefore the canonical equation for the field lines corresponds to straight lines given by $\theta = \phi/q + \theta_0$.

We now consider magnetic perturbations. In this case, the poloidal flux is given by an unperturbed part, related to the standard magnetic field, plus a perturbation, $\psi_p(\psi_t, \theta, \phi) = \psi_{p0}(\psi_t) + \psi_{p1}(\psi_t, \theta, \phi)$. In the previous, the safety factor of the background field is $q(\psi_t) = \partial\psi_t/\partial\psi_{p0}$. The perturbation Hamiltonian is a $2\pi-$ periodic function in θ, ϕ

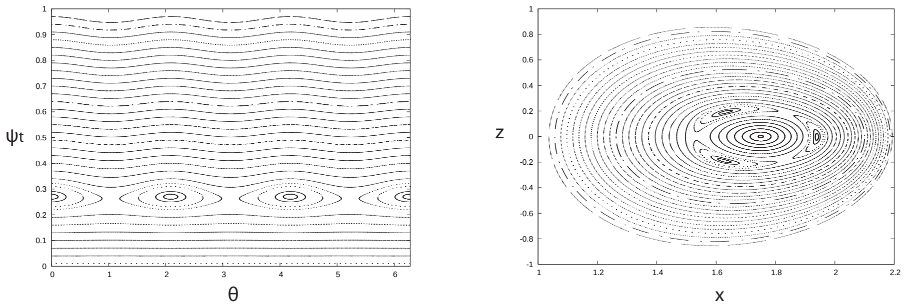
$$\psi_{p1}(\psi_t, \theta, \phi) = \varepsilon\psi_t^{3/2}\cos(m\theta - n\phi), \tag{5}$$

where the integers (m,n) correspond to the rational surface where the perturbation is resonant with the background field, i.e. to the NTM mode responsible for the island formation[3], and

$\varepsilon = 5 \cdot 10^{-3}$ is the perturbation strength. The equations of motion to be integrated become:

$$\frac{d\psi}{d\phi} = m\varepsilon\psi^{\frac{m}{n}} \sin\left(m\theta - n\phi\right), \quad \frac{d\theta}{d\phi} = \frac{\partial \psi_{p0}}{\partial \psi} + \frac{\partial \psi_{p1}}{\partial \psi} = \frac{1}{q\left(\psi\right)} + \frac{m}{n}\varepsilon\psi^{\frac{m}{n}-1}\cos\left(m\theta - n\phi\right) \quad (6)$$

In the following figure we show the poloidal cross section, both in real and $\psi_t - \theta$ space, for the mode $(m,n) = (3,2)$ in AUG equilibrium with Shafranov shift and elongation.



Ray tracing

GO studies wave propagation in the limiting case of small wavelength, in a remarkable analogy with the mechanics of material particles. The Hamiltonian equations for the rays are:

$$\dot{\mathbf{k}} = -\frac{\partial \mathscr{H}}{\partial \mathbf{r}}\ ,\ \dot{\mathbf{r}} = \frac{\partial \mathscr{H}}{\partial \mathbf{k}} \quad (7)$$

where the Hamiltonian is the dispertion function, $\mathscr{H} \equiv det[(\frac{\omega}{c})^2(-k^2\mathbf{I} + \mathbf{kk}) + \varepsilon^h] = 0$. For propagation in cold plasma, adopting the cold plasma tensor yields for the Hamiltonian [4]

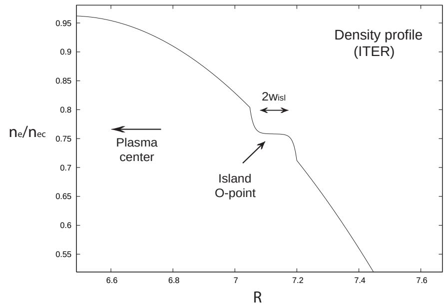
$$\mathscr{H} = SN_\perp^4 + \left[D^2 - PS - S^2 + (P+S)N_\parallel^2\right]N_\perp^2 + PN_\parallel^4 - 2PSN_\parallel^2 + \left(S^2 + D^2\right)P \quad (8)$$

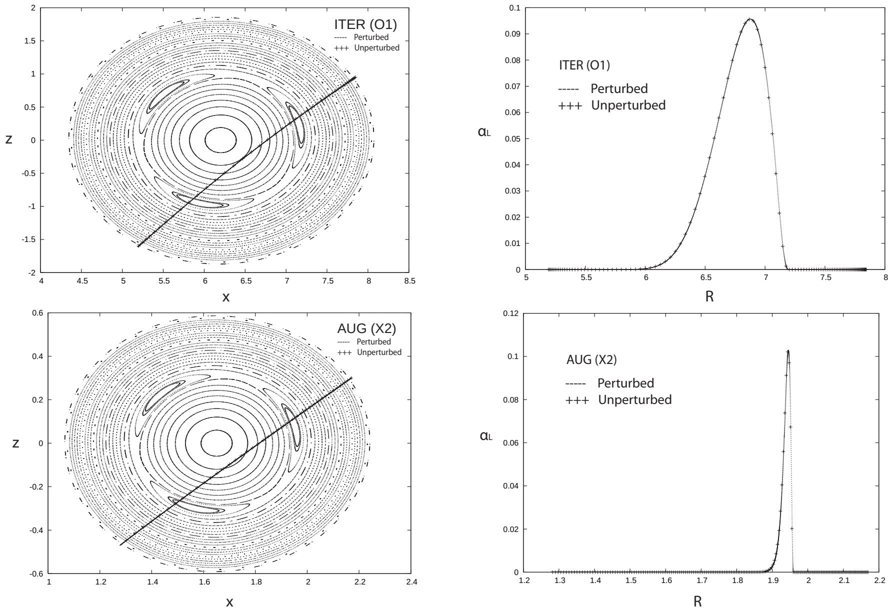
where $S \equiv 1 - \frac{\omega_p^2}{\omega^2}/(1 - \frac{\omega_c^2}{\omega^2})$, $D \equiv -\frac{\omega_c}{\omega} \cdot \frac{\omega_p^2}{\omega^2}/(1 - \frac{\omega_c^2}{\omega^2})$, $P \equiv 1 - \frac{\omega_p^2}{\omega^2}$

We present results for the case of poloidal propagation in ITER and AUG in the presence of magnetic islands due to the mode (3,2). In AUG, the parameters are $r_t = 1.65$m, $r_p = 0.6$m, $B_0(0) = 2.5$T, $n_e \in [1.4, 1.6] \cdot 10^{13}\text{cm}^{-3}$ and $T_e \in [0.2, 2]$KeV. For the wave it is $\omega/2\pi = 140$GHz (X2) and $P_0 = 1$MW. In ITER, these parameters are $r_t = 6.2$m, $r_p = 1.9$m, $B_0(0) = 5.51$T, $n_e \in [1, 10] \cdot 10^{13}\text{cm}^{-3}$,$T_e \in [1, 10]$KeV, $\omega/2\pi = 160$GHz (O1) and $P_0 = 10$MW. The value of the toroidal field on the axis is set such that the resonance resides on the island. The unperturbed equilibrium is circular ($\psi_t(r) = r^2/r_{min}^2$) and the perturbation strength is $\varepsilon = 0.005$.

The wave is injected from the outer flux surface with initial wavenumber relevant to the specific mode. Absorption is calculated according to the linear absorption coefficient emerging from the energy balance. We model the density and temperature flattening in the island region by exponential functions, with proper parameters so that the constant density/temperature is the average of the densities/temperatures in the boundaries of the island (separatrix). The position of the O-point r_{res} is calculated by solving the equation $q(\psi_{res}) = 3/2$ and the half-width from the familiar relation $w_{isl} = r_{min}^2 \frac{m}{n} \sqrt{\frac{\varepsilon \psi^{m/2}}{q'(\psi_{res})}}$.

In both cases, the results show that the magnetic island, for values of the perturbation amplitude relevant to current experiments (up to 0.01), does not play a significant role on the EC absorption. The main reason is that the perturbation is so small that it does not affect the absorption coefficient and the dispersion fuction. The same holds for the density and temperature flattening, because the magnitude of the island is not large enough to change the ray direction or the absorption profile.





Conclusions

The fact that EC propagation and absorption was found to be unaffected by the island topology in the case under study does not in any way imply that the current drive efficiency is not affected by magnetic islands. In this direction, the modeling should include: (a) The case of toroidal injection ($k_{||} \neq 0$), (b) The description of the plasma response in terms of detailed particle dynamics. In addition, the dynamics of the NTM stabilization with ECCD should be studied in terms of self-consistent calculations of the wave and the perturbed magnetic field.

References

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