

THE FUNDAMENTAL CHARACTERISTICS OF RELATIVISTIC RUNAWAY ELECTRON AVALANCHE IN AIR

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Computational technique

Relativistic runaway electron avalanches (RREAs) underlay the high-altitude discharge (HAD) development. For numerical simulations of HADs and their emissions (optical, gamma and neutrons) it is necessary to know dependencies on the field overvoltage $\delta = eE/(F_{\min} \cdot P)$ relative to the minimum of the drag force $F_{\min} = 218 \text{ keV}/(\text{m} \cdot \text{atm})$ of the e -fold enhancement time $t_e(\delta)$ ($1/ct_e$ is the relativistic analog of the Townsend ionization coefficient) and RREA and RREA Bremsstrahlung energy and angular distributions. Below there are presented the most important results of simulations of RREA and Bremsstrahlung in air at 1 atm. carried out by Monte Carlo code ELISA. At the moment $t = 0$ a monoenergetic beam of $N_e(0)$ electrons with kinetic energy ε_0 was assumed to be injected along the electric force $-e\vec{E}$. The calculations were carried out with statistical error 1-2 % for two strongly different ε_0 : 2 and 10 MeV. The electron trajectories were traced down to 1 keV. Electrons with energies below the runaway threshold relax to thermal energies and do not contribute RREA. The temporal dependencies of RE and γ -photon numbers $N_e(t)$ and $N_\gamma(t)$, their energy and angular distributions at different δ were calculated. The $t_e(\delta)$ was determined by the linear segment of $\ln[N_e(t)/N_e(0)]$. The Bremsstrahlung was calculated using two approaches: simulations selfconsistent with RREA simulations and using steady state RE distributions.

Runaway avalanche. Fig. 1 exhibits the $t_e(\delta)$ in the δ range of interest in the HAD

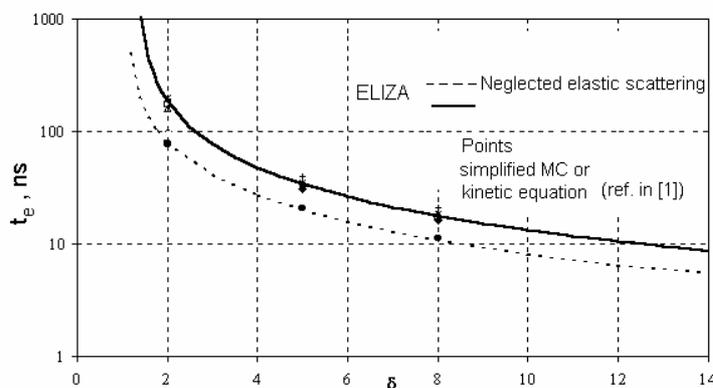
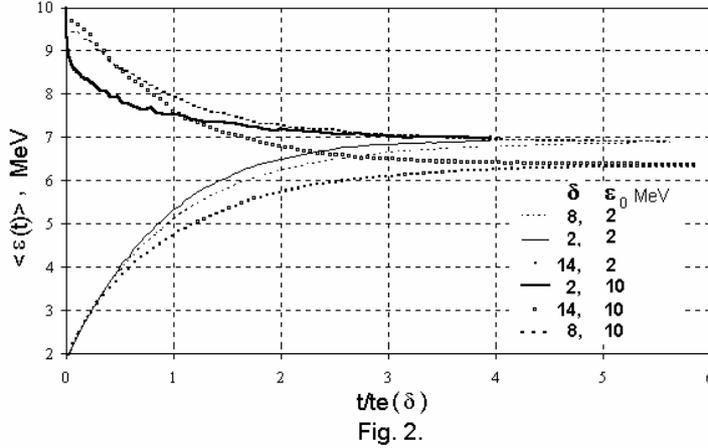


Fig. 1.

problem. Remind, that at $\delta = 14$, the ordinary breakdown occurs. The sharp t_e increase at small δ testifies to approaching to δ value, below which electrons are not involved in the runaway mode. With an accuracy of 5%, the $t_e(\delta)$ is approximated as follows:

$$t_e(\text{ns}) = \exp\left(7.11/\delta^{0.441}\right), \quad \delta \in [2, 10]. \quad (1)$$



The effect of ε_0 on the RREA characteristics was analyzed. For the considered ε_0 differences manifest themselves at $\delta \leq 2$, the time required for RREA to attain the exponential mode being decreased with increasing ε_0 . This time increases with decreasing δ , and only electrons with high ε_0 are

involved in the runaway mode.

The mean energy $\langle \varepsilon \rangle$ (fig. 2) in the steady-state mode weakly varies in the range $\delta = 2-14$, the RREA steady-state mode being attained during $\sim (4-6) \cdot t_e$. The $\langle \varepsilon \rangle$ weakly decreases with δ , which is caused by involving electrons of progressively lower energies in the runaway mode. The steady-state energy distributions (normalized to 1 MeV) differ weakly in the range $\delta = 2-8$. The maximum difference is in the range 0.01-40 MeV and does not exceed 15%. This universal distribution is approximated as follows with $u = \ln[\varepsilon/(1 \text{ MeV})]$,

$$f_1(\varepsilon) = \exp\left(-0.00108u^6 - 0.004235u^5 + 0.009757u^4\right) \times \exp\left(0.012652u^3 - 0.056372u^2 - 0.43325u - 2.1185\right). \quad (2)$$

The angular electron distributions were obtained integrating over all ε . They approximated as

$$g(\mu, b) = \frac{1-b^2}{2\pi(1-b\mu)^2}, \quad (3)$$

where $b = 0.91$ for $\delta = 2$ and $b = 0.97$ for $\delta = 8$. Here μ is the cosine of the angle between the electron momentum and $-e\vec{E}$. The maximum error of the approximation (3) is at μ close to 1. It is equal to 12% and 34% for $\delta = 2$ and 8. Unlike the energy distributions, the steady-state angular distributions essentially depend on δ becoming prolated with increasing δ .

The normalized steady-state angular-energy RE distributions can be represented in the form

$$f(\varepsilon, \mu) = f_1(\varepsilon) \cdot f_2(\mu|\varepsilon). \quad (4)$$

The normalized steady-state angular distribution at the energy ε is approximated as

$$\ln f_2(\mu|\varepsilon) = \begin{cases} y_1(\varepsilon) - k_1(\varepsilon) \cdot (1 - \mu), & \mu \geq 0 \\ y_2(\varepsilon) + k_2(\varepsilon) \cdot (\mu + 1), & \mu < 0 \end{cases} \quad (5)$$

where

$$y_1(\varepsilon) = 0.5756 \cdot \ln(0.9\varepsilon) - 0.46; \quad k_1(\varepsilon) = 1.24 \cdot (14\varepsilon)^{0.5} - 0.92; \quad (6)$$

$$y_2(\varepsilon) = -2.76 \cdot (2.3\varepsilon)^{0.48} - 2.53; \quad k_2(\varepsilon) = 0.557 \cdot \ln(\varepsilon) + 2.91; \quad \text{for } \delta = 2;$$

and

$$y_1(\varepsilon) = 0.6178 \cdot \ln(\varepsilon) + 0.4145; \quad k_1(\varepsilon) = 3.224 \cdot (5\varepsilon - 0.07)^{0.57}; \quad (7)$$

$$y_2(\varepsilon) = -7.6 \cdot (4\varepsilon)^{0.31} - 0.6; \quad k_2(\varepsilon) = 0.8858 \cdot \ln(\varepsilon) + 5.142; \quad \text{for } \delta = 8.$$

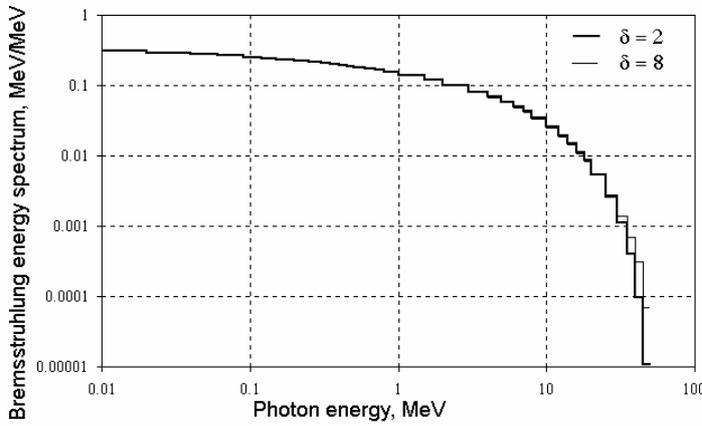


Fig. 3.

RREA Bremsstrahlung.

The emission rates and mean photon energy are almost independent of δ :

$$\left(\frac{dN_\gamma}{dt} \right)_{\text{em}} = 10.45 \text{ } \gamma / (\mu\text{s} \cdot \text{electron})$$

with $\langle \varepsilon_\gamma \rangle = 0.477 \text{ MeV}$ at $\delta = 2$

and $10.49 \text{ } \gamma / (\mu\text{s} \cdot \text{electron})$ with 0.472

MeV at $\delta = 8$. Fig. 3 illustrates normalized per 1 MeV steady-state

Bremsstrahlung energy distributions. This universal distribution $f_1(\varepsilon)$ is approximated as

$$\exp(-0.0006 u^6 - 0.0035 u^5 + 0.001 u^4 + 0.0129 u^3 - 0.0964 u^2 - 0.4349 u - 1.8837). \quad (8)$$

Fig. 4 displays angular distributions of Bremsstrahlung energy relative to the electric force

$-e\vec{E}$ for $\delta = 2$ and 8 . They are approximated as follows:

$$g(\mu, b, B) = \frac{(1-b^2)^2}{4\pi(B(1-b^2)^2 + 1)} \cdot (B + (1-b\mu)^{-3}), \quad 2\pi \int_{-1}^1 g(\mu, b, B) d\mu = 1 \text{ MeV} \quad (9)$$

where $b = 0.92$ and $B = 0$ for $\delta = 2$, $b = 0.966$ and $B = 1.4$ for $\delta = 8$.

The steady-state Bremsstrahlung energy distribution over angles and energies (MeV) is represented as (4), where $f_2(\mu|\varepsilon)$ is the normalized steady-state angular distribution of photons with energy ε , which is approximated by the expression

$$f_2(\mu|\varepsilon) = \frac{3(1-b^2)^3}{2\pi(6B(1-b^2)^3 + 2(b^2+3))} \cdot \left(B + \frac{1}{(1-b\mu)^4} \right), \quad (10)$$

where $b = 1 - 1.05/(\varepsilon + 4)$ and $B = 0.055/(\varepsilon + 0.008)$ for $\delta = 2$ and $b = 1 - 0.45/(\varepsilon + 3.3)$ and $B = 0.5/(\varepsilon + 0.02)$ for $\delta = 8$.

In the exponential mode the RE and photon numbers are calculated as follows

$$N_e(t)/N_0 = c_e(\delta) \cdot \exp(t/t_e(\delta)); \quad N_\gamma(t)/N_0 = c_\gamma(\delta) \cdot \exp(t/t_e(\delta)), \quad (11)$$

where $t_e = 189.7$ ns, $c_e(\delta) = 1.084$ for $\delta = 2$; $t_e = 17.8$ ns, $c_e(\delta) = 1.027$ for $\delta = 8$; $c_\gamma(\delta) = 1.028$ for $\delta = 2$ and $c_\gamma(\delta) = 0.1133$ for $\delta = 8$.

Conclusions. The dependence of the characteristic RREA amplification time on the overvoltage $t_e(\delta)$ was calculated at 1 atm. The t_e values at pressure P can be calculated as $t_e(P) = t_e(1 \text{ atm.})/[P(\text{atm.})]$. In the range of $\delta = 2-8$ energy distributions of REs and Bremsstrahlung energy are universal, i.e. almost independent of δ . For $\delta = 2$ and 8 angular distributions of REs of all energies and angular distributions in separate energy groups were calculated. The same was done for the Bremsstrahlung energy. As expected, the RE beam is more directed

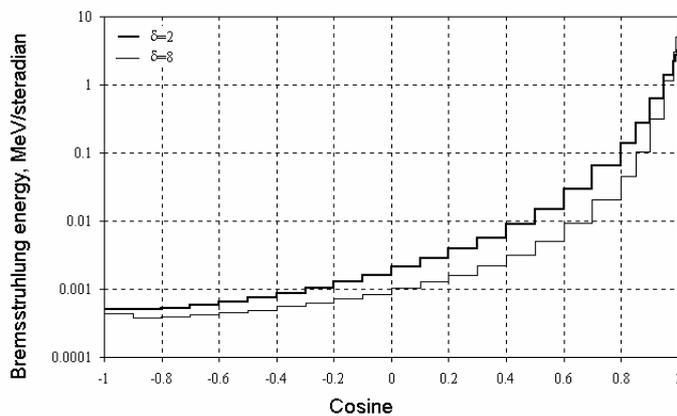


Fig. 4.

along the electric force as δ and electron energy increase. Accordingly behaves the Bremsstrahlung energy flux. The obtained Bremsstrahlung distributions are very close for both approaches of the Bremsstrahlung simulations, and, therefore, the selfconsistent simulations are excessive.

References

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