

# **Dust-ion-acoustic Solitons in Magnetized dusty plasma with Trapped Electrons**

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## **abstract**

Dust-Ion-Acoustic Solitons are investigated in Magnetized dusty plasma consisting of positively charged hot ions, trapped electrons and negatively charged dust. The reductive perturbation technique is used to solve a set of nonlinear hydrodynamic equations combined with Poisson's equation. The dust charge fluctuation is considered in the presence of an adiabatic charging process. A multi-dimensional damped Zakharov-Kuznetsov equation is then derived. We found a solitary wave solution which presents a larger amplitude and smaller depth in comparison with the situation corresponding to thermal electrons.

## **Introduction**

The influence of trapped particles in dusty plasmas has been widely investigated in the past decade. In slowly varying electron field and when the particles kinetic energy is less than the potential energy, particles are trapped. As a matter of fact, different physical phenomenon are found to be related to different trapping effects such as solitons motion retard[1],[2]. In the presence of trapped electrons a damped solitons exist due to the monotonically perturbation speed decreasing. Such solitons can be used as a diagnostic tool to characterize the electrons properties[3]. The soliton amplitude has been found dependind on the ratio of free to trapped electrons temperatures while the width was found independent of trapped electrons temperature[4]. In this work we investigate a small but finite amplitude dust ion acoustic waves in magnetized dusty plasma in the presence of adiabatic trapped electrons and charge fluctuation. A comparison is made with the situation in which the electron distribution has been taken isothermal[5].

## **Modelling**

The dusty plasma is collisionnel and magnetized with three species namely trapped electrons, negatively charged dust particles and positive ions. The dynamics of hot ions, in the presence

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of static dust, is governed by fluid equations,

$$\frac{\partial n_i}{\partial t} + \nabla(n_i u_i) = -\nu_{ch} n_i + \nu_i n_e \quad (1)$$

$$\frac{\partial(n_i u_i)}{\partial t} + \nabla(n_i u_i u_i) + \frac{en_i}{m_i} \nabla \phi - \frac{en_i}{m_i} (u_i \times B) = -\tilde{\nu} n_i u_i \quad (2)$$

$m_i$ ,  $n_i$  and  $\nu_i$  are respectively ions mass, density and velocity. The frequencies  $\nu_{ch}$ ,  $\nu_i$  and  $\tilde{\nu}$  correspond respectively to the ion frequency recombination on the dust, the plasma ionization frequency and an ion momentum loss frequency. The pressure effect has been neglected. The electrostatic potential  $\phi$  is governed by Poisson's equation,

$$\nabla^2 \phi = 4\pi e(Z_d n_d^o + n_e - n_i) \quad (3)$$

The dust density  $n_d^o$  is taken constant and the electrons density  $n_e$  follows the Gurivich distribution for trapped and free electrons[1]

$$n_e = n_{eo} \left[ \exp\left(\frac{e\phi}{T_e}\right) \operatorname{erfc}\left(\sqrt{\frac{e\phi}{T_e}}\right) + \frac{2}{\sqrt{\pi}} \sqrt{\frac{e\phi}{T_e}} \right] \quad (4)$$

where  $T_e$  is the electrons temperature. Due to the current following to the dust surface, the dust charge fluctuates and the charge fluctuation equation in the presence of ions (electrons) current  $I_i$  ( $I_e$ ) is

$$\frac{dQ_d}{dt} = I_e + I_i \quad (5)$$

The currents are

$$I_e = -n_e e \pi r^2 \left(\frac{8T_e}{\pi m_e}\right)^{1/2} \exp\left(\frac{e\phi_d}{T_e}\right) \quad (6)$$

$$I_i = \pi r^2 e \left(\frac{8T_i}{\pi m_i}\right)^{1/2} n_i \left[ F_1 - F_2 \left(\frac{e\Phi}{T_e}\right) \right] \quad (7)$$

where  $F_1 = \sqrt{\pi}/(4u_0)(1 + 2u_0^2)\operatorname{erf}(u_0) + 1/2 \exp(-u_0^2)$  and  $F_2 = \sqrt{\pi}/(2u_0)\operatorname{erf}(u_0)$ . We normalize as follows  $\tilde{n}_j/n_{eo}$ ,  $\tilde{u}_i = u_i/C_s$ ,  $\tilde{\phi} = e\phi/T_e$ . Time and length are normalized respectively by  $\omega_{pi}^{-1} = (m_i/(4\pi e^2 n_{io}))^{1/2}$ , and  $\lambda_D^{-1} = (T_e/(4\pi e^2 n_{io}))^{-1/2}$ . The acoustic speed is given by  $C_s = (T_e/m_i)^{1/2}$ . The requirement quasi-neutrality at equilibrium is  $n_{io} = Z_{do} n_d^o + n_{oe}$ . Small amplitude behavior is investigated using the independent stretched coordinates, i.e.  $X = \varepsilon^{1/4}(x - \lambda t)$ ,  $Y = \varepsilon^{1/4}y$ ,  $Z = \varepsilon^{1/4}z$  and  $T = \varepsilon^{3/4}t$ , where  $\varepsilon$  is a small parameters measuring the weakness of the nonlinearity and  $\lambda$  is the normalized wave speed. The reductive perturbation methods leads to the damped modified Zakharov-Kusnetsov equation,

$$\frac{\partial \phi^{(1)}}{\partial T} + AB\sqrt{\phi^{(1)}} \frac{\partial \phi^{(1)}}{\partial X} + \frac{1}{2}AC\phi^{(1)} + \frac{1}{2}A\alpha \frac{\partial^3 \phi^{(1)}}{\partial X^3} + \frac{1}{2}AD \frac{\partial}{\partial X} \left[ \frac{\partial^2 \phi^{(1)}}{\partial Y^2} + \frac{\partial^2 \phi^{(1)}}{\partial Z^2} \right] = 0 \quad (8)$$

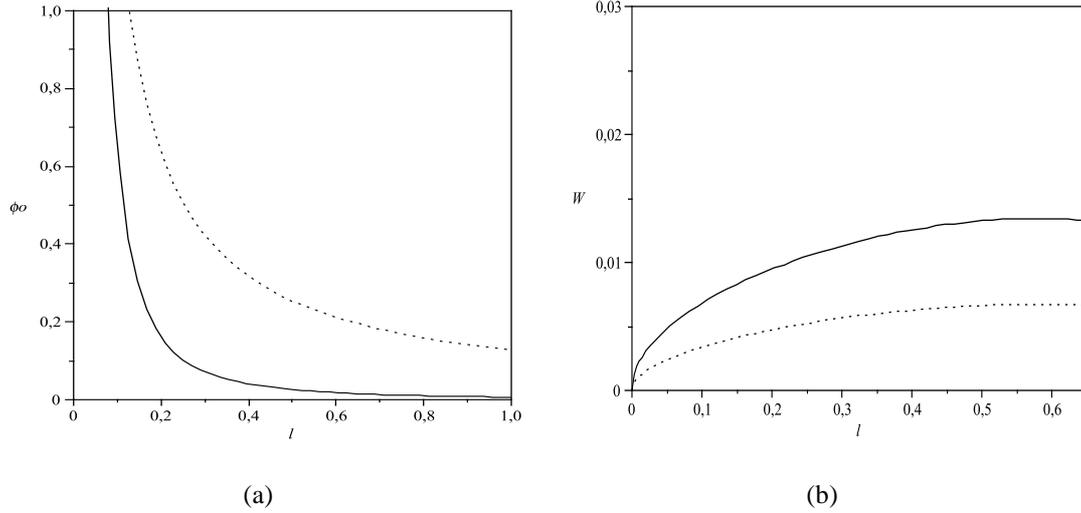


Figure 1: Fig (a):  $\phi_o$  vs  $l$  for  $\alpha = 1.2, \delta = 0.01, n_{eo} = 10^3, Z_d^{(o)} = 20, \lambda = 1.22, r_d = 1.85 \times 10^{-13}, T_i = 0.05, T_{ef} = 0.1, T_{et} = 0.2, u_{ix}^{(o)} = 0.1, v_{cho} = 1.910^{-4}, v_{io} = 2.210^{-4}, v_o = 2.510^{-4}$  and  $T = 3$ . Solid curve for Boltzmann distribution and dashed curve for Gurivich distribution. Fig. (b):  $W$  vs  $l \Omega = 200$ , all others parameters are the same as Fig. (a)

where  $\lambda_2 = \lambda - 2u_{ix}^{(o)}, A = s^2/(\alpha\lambda_1 E), b = 4/(3\sqrt{\pi}), C = E[\lambda_2 v_{cho}\alpha/s^2 + \lambda v_o\alpha/s^2 - \lambda_2 v_{io}/s], E = [1 + \delta QZ_d^{(0)}/F], D = \alpha[1 + \lambda_1^4 E/(\Omega^2 s^2)]$  and  $B = 3/4b[1 - \delta RZ_d^{(0)}/F]$

### Discussion and conclusion

To study the ion acoustic waves we choose a stationary frame, this enable us to follow one soliton wave in time, we made a translation to a new frame i.e.  $\chi = lX + mY + nZ - UT, U$  being the Mach number  $l, m, n$  are the components of the unit vector ( $l^2 + m^2 + n^2 = 1$ ).

Equation (8) is than transformed to:

$$-U \frac{d\phi^{(1)}}{d\chi} + \frac{2}{3}ABl \frac{d(\phi^{(1)})^{3/2}}{d\chi} + \frac{1}{2}AC\phi^{(1)} + hl \frac{d^3\phi^{(1)}}{d\chi^3} = 0 \quad (9)$$

where  $h = \frac{1}{2}A\alpha l^2 + \frac{1}{2}AD(1 - l^2)$ . To find solution of Eq.8 we put  $C = 0$ , multiplying by  $d\chi$  and using boundary conditions for localized solution ( $\phi^{(1)} \rightarrow 0, d\phi^{(1)}d\chi \rightarrow 0$ . when  $\chi \rightarrow +\infty$ ) gives

$$-U\phi^{(1)} + \frac{2}{3}ABl(\phi^{(1)})^{3/2} + hl \frac{d^2\phi^{(1)}}{d\chi^2} = 0 \quad (10)$$

multiplying the last equation by  $\frac{d\phi^{(1)}}{d\chi}$  and after integration we have the localized solution

$$\left[ \frac{d\phi^{(1)}}{d\chi} \right]^2 = \frac{8}{15} \frac{AB}{h} (\phi^{(1)})^2 \left[ \frac{15U}{8ABl} - (\phi^{(1)})^{1/2} \right] \quad (11)$$

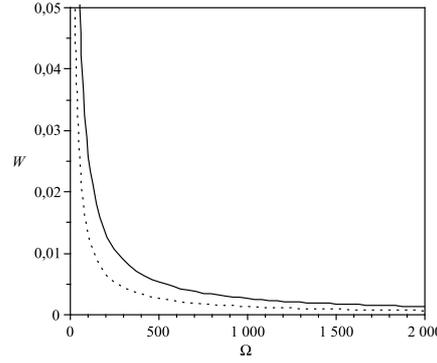


Figure 2:  $W$  vs  $\Omega$  the same as Fig.1 but with  $l = 0.5$ .

which gives

$$\phi^{(1)} = \left( \frac{15U}{8ABl} \right)^2 \operatorname{sech}^4 \left( \frac{\chi}{\sqrt{16lh/U}} \right) \quad (12)$$

The final solution corresponding to ( $C \neq 0$ ) is:

$$\phi^{(1)} = \phi_o \operatorname{sech}^4 \left( \frac{\chi}{W} \right) \quad (13)$$

The amplitude and the width of the soliton are respectively  $\phi_o = (15U/(8ABl))^2 \exp(-1/2ACT)$  and  $W = \sqrt{16hl/U \times [\exp(1/2ACT)]^{1/2}}$ . The variations of the amplitude  $\phi_o$  and the width  $W$  of the ion acoustic wave are investigated using the same parameters as Ref.[6]. We have almost the same results when we compare the actual situation with the one corresponding to the presence of free and trapped electrons of different temperatures. However, The amplitude in the presence of Boltzmann electron is smaller (Fig.1.a). This is not true for the soliton width which is smaller in both cases given here (Fig.1.b). We note also the amplitude is rapidly decreasing when the ion cyclotron frequency  $\Omega$  increases(Fig.2).

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