36th EPS Conference on Plasma Phys. Sofia, June 29 - July 3, 2009 ECA Vol.**33E**, O-3.025 (2009)

Parallel propagation of Hall magnetohydrodynamic waves and their stability in flowing solar plasmas

I. Zhelyazkov

Faculty of Physics, Sofia University, BG-1164 Sofia, Bulgaria

Introduction It is well established now that the solar atmosphere, from photosphere to the corona and the solar wind is a highly structured medium. Satellite observations have confirmed the presence of steady flows. Bulk motions are registered along the magnetic field lines which outline the magnetic structures. These structures are in the form of thin flowing plasma layers (or tubes) that are adjacent to each other with differences in their plasma parameters (density, magnetic field, steady flow speed). The magnetohydrodynamics with Hall effect (Hall MHD) gives a fluid description of magnetized plasmas taking into account scales of the order of the ion inertial length, $l_{\rm Hall} = c/\omega_{\rm pi}$, at which the dynamics of ions and electrons separates and the medium becomes dispersive. If in the solar corona plasma β (the ratio of gas to magnetic pressure) is much less than unity, in the solar wind flux tubes it is $\beta \approx 1$. Since we are going to study the wave propagation in flowing solar wind plasma, we can assume that we have a 'high-β' magnetized plasma and treat it as an incompressible fluid. Here, we investigate the influence of flow velocities on the dispersion characteristics and stability of MHD surface waves travelling along an ideal incompressible flowing plasma flux tube surrounded by flowing plasma environment in the framework of the Hall magnetohydrodynamics. For simplicity, we consider a planar jet of width $2x_0$ (embedded together with environments in a constant magnetic field $\mathbf{B}_0$ directed along the z axis), allowing for different plasma densities within and outside the jet, $\rho_{\rm o}$ and $\rho_{\rm e}$, respectively. For investigating the stability of the travelling MHD waves it is convenient to consider the wave propagation in a frame of reference attached to the flowing environment. Thus we can define the relative flow velocity $\mathbf{U}^{\rm rel} \equiv \mathbf{U} = \mathbf{U}_{\rm o} - \mathbf{U}_{\rm e}$ ($U_{\rm o}$ and $U_{\rm e}$ being the steady flow speeds inside and outside the flux tube, respectively) as an entry parameter whose value determines the stability/instability status of the jet. As usual, we normalize that relative flow velocity with respect to the Alfvén speed in the jet and call it Alfvénic Mach number, $M_{\rm A} = U/v_{\rm Ao}$, omitting for simplicity the superscript "rel". Another important entry parameter of the problem is $\eta = \rho_{\rm e}/\rho_{\rm o}$. The third entry parameter is the scale parameter $\varepsilon = l_{\rm Hall}/x_0$ called the *Hall parameter*, in which $l_{\rm Hall}$ is calculated for the plasma density inside the jet.

Basic equations The basic equations for the incompressible Hall-MHD waves are the linearized equations governing the evolution of the perturbed fluid velocity $\mathbf{v}_1$ and wave magnetic field $\mathbf{B}_1$:

$$\rho\frac{\partial \mathbf{v}_1}{\partial t} + \rho(\mathbf{U}\cdot\nabla)\mathbf{v}_1 + \nabla(\frac{1}{\mu_0}\mathbf{B}_0\cdot\mathbf{B}_1) - \frac{1}{\mu_0}(\mathbf{B}_0\cdot\nabla)\mathbf{B}_1 = 0, \tag{1}$$

$$\frac{\partial \mathbf{B}_1}{\partial t} + (\mathbf{U}\cdot\nabla)\mathbf{B}_1 - (\mathbf{B}_0\cdot\nabla)\mathbf{v}_1 + \mathbf{B}_0\nabla\cdot\mathbf{v}_1 + \frac{v_\mathrm{A}^2}{\omega_\mathrm{ci}}\hat{z}\cdot\nabla\nabla\times\mathbf{B}_1 = 0, \tag{2}$$

with the constrains

$$\nabla\cdot\mathbf{v}_1 = 0 \qquad \text{and} \qquad \nabla\cdot\mathbf{B}_1 = 0, \tag{3}$$

where $v_\mathrm{A} = B_0/(\mu_0\rho)^{1/2}$ is the Alfvén speed and μ_0 is the permeability of free space. From these equations, assuming that all perturbed quantities behave like $g(x)\exp(-\mathrm{i}\omega t + \mathrm{i}kz)$, with appropriate boundary conditions at the interface $x = x_0$ one gets the following dispersion relations for sausage and kink surface waves travelling along the jet (for details see Ref. [1]):

$$\begin{aligned}&\left(\frac{\omega - \mathbf{k}\cdot\mathbf{U}}{kv_\mathrm{Ao}}\right)^2 - 1 + \left[\frac{\rho_\mathrm{e}}{\rho_\mathrm{o}}\left(\frac{\omega}{kv_\mathrm{Ao}}\right)^2 - 1\right]\begin{pmatrix}\tanh\\ \coth\end{pmatrix}kx_0\\ &-\varepsilon_\mathrm{o}^2\left[1 + \tilde{\omega}^2\frac{\rho_\mathrm{e}}{\rho_\mathrm{o}}\begin{pmatrix}\tanh\\ \coth\end{pmatrix}kx_0\right]\frac{1 - \tilde{\omega}\rho_\mathrm{e}/\rho_\mathrm{o}}{1 - \tilde{\omega}\left(\rho_\mathrm{e}/\rho_\mathrm{o}\right)^2} = 0,\end{aligned} \tag{4}$$

where

$$\tilde{\omega} = \frac{\omega}{\omega - \mathbf{k}\cdot\mathbf{U}} \quad \text{and} \quad \varepsilon_\mathrm{o} = \frac{\omega - \mathbf{k}\cdot\mathbf{U}}{\omega_\mathrm{ci}}.$$

As can be seen, the wave frequency ω is Doppler-shifted inside the jet. If we are interested in the stability of the surface waves running at the jet interfaces, we have to assume that the wave frequency is complex, i.e., $\omega \to \omega + \mathrm{i}\gamma$, where γ is the expected instability growth rate. When studying dispersion characteristics of MHD waves, one usually plots the dependence of the wave phase velocity v_ph as function of the wave number k. For numerical solving of equations (4) we normalize all quantities by defining the dimensionless wave phase velocity $V = \omega/kv_\mathrm{Ao}$ and wave number $K = kx_0$ to get

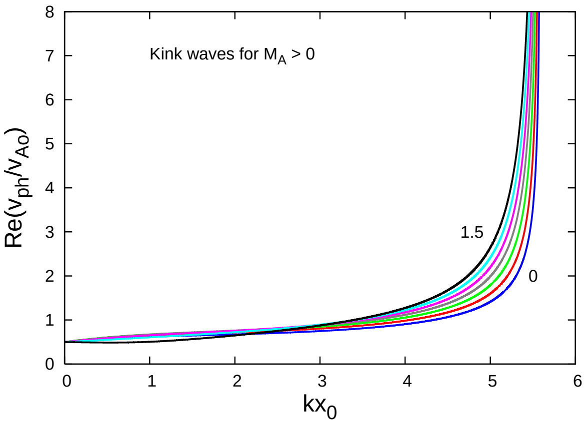
$$\begin{aligned}&(V - M_\mathrm{A})^2 - 1 + \left(\eta V^2 - 1\right)\begin{pmatrix}\tanh\\ \coth\end{pmatrix}K\\ &-K^2\varepsilon^2\left[(V - M_\mathrm{A})^2 + V^2\eta\begin{pmatrix}\tanh\\ \coth\end{pmatrix}K\right]\frac{V(1-\eta) - M_\mathrm{A}}{V(1-\eta^2) - M_\mathrm{A}} = 0.\end{aligned} \tag{5}$$

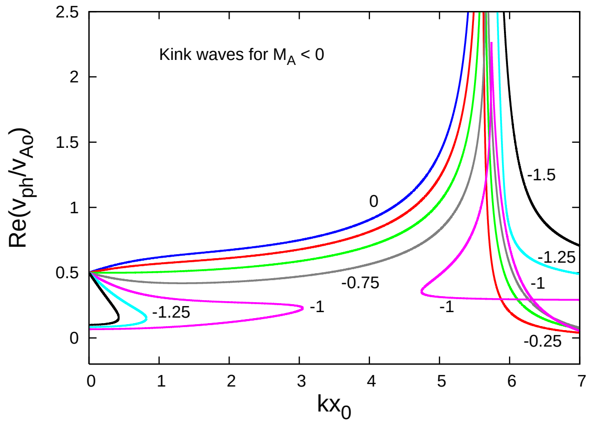
Recall that we consider the normalized wave phase velocity V as a complex number and we shall look for the dependencies of the real and imaginary parts of V as functions of the real dimensionless wave number K at given values of the three entry parameters. It can be easily seen from above equations that they are cubic ones with respect to V. Hence, the dimensionless dispersion relation of, for example, the kink mode can be displayed in the form:

$$AV^3 + BV^2 + CV + D = 0, \tag{6}$$

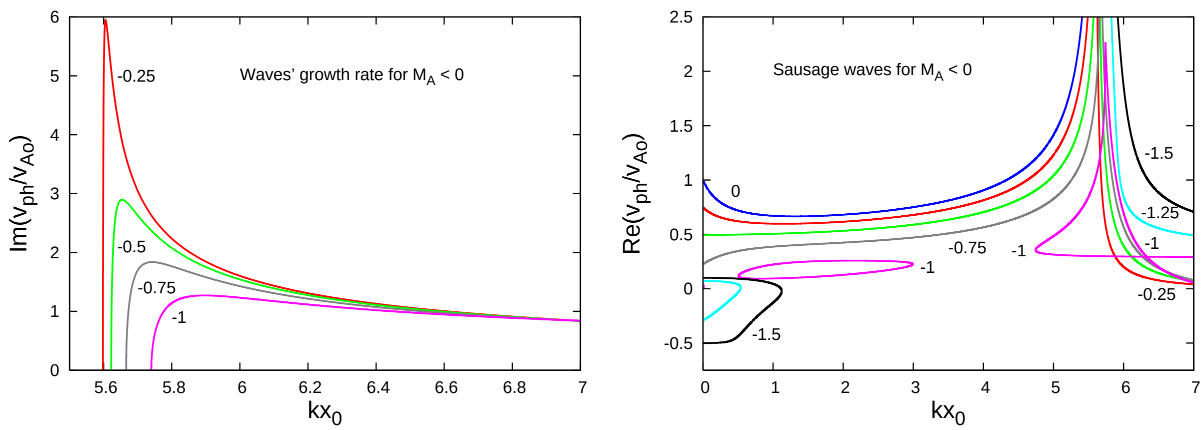
where the coefficients A, B, C, and D are expressed in terms of η, ε, M_A, K and $\coth K$. The dimensionless dispersion relation for sausage mode is similar – one has simply to replace coth by tanh. Each cubic equation by using Cardano's formulas can be solved exactly and in the next section we present the numerical results.

Numerical results and discussion Before starting the numerical solving of dispersion equations (5) we have to specify the entry jet's parameters η, ε, and M_A. Two of them, η and ε are fixed, and the third one is running. Our choice for the fixed parameters is $\eta = 4$ and $\varepsilon = 0.4$. The dispersion relations of both modes represent the dependence of the normalized phase velocity V on the dimensionless wave number K. A specific feature of the surface Hall-MHD waves travelling along an incompressible static plasma layer is that there exists a limiting dimensionless wave number K_limit beyond which the wave propagation is no longer possible. That limiting wave number is given by [2]: $K_\mathrm{limit} = (1+\eta)^{1/2}/\varepsilon$. For our choice of η and ε, $K_\mathrm{limit} = 5.59$. With approaching that wave number the wave phase velocity becomes very large. It is interesting to see whether the steady flow will change that limiting value. We start with $M_\mathrm{A} = 0$ and the





dispersion curves labeled by '0' in the above two figures correspond to a static slab. For positive values of the Alfvénic Mach number (calculated with a step of 0.25) we get a family of dispersion curves plotted in the left figure. All these curves represent real solutions to the complex dispersion relation and correspond to stable kink waves' propagation. As seen, the flow slightly diminishes the value of the K_limit and all curve lie on the left side of the neutral dispersion curve '0'. For $M_\mathrm{A} < 0$ the picture is completely different – the dispersion curves corresponding to various M_As are shown in the right figure. It is clearly seen that for each M_A, in fact, two distinctive dispersion curves merge at $K \approx K_\mathrm{limit}$. The solutions to the dispersion relation for the curves lying on the right side of the merging vertical line at some Ks become complex with positive imaginary part, i.e., there the waves are unstable. However this is true only for kink waves with $M_\mathrm{A} \geqslant -1$. For larger in modulus M_As, say for -1.25 and -1.5, waves are stable as are all kink modes whose dispersion curves lie on the left side of the vertical merging line. The growth rates of unstable kink waves are plotted on the left figure below.



The dispersion characteristics of the sausage surface mode are similar to those of the kink one. For positive values of the relative Alfvénic Mach number, M_A, all dispersion curves describe a stable forward wave propagation. When M_A is negative we get a picture (see the right figure above) which possesses the same features as those of the kink waves at the same circumstances. What is more interesting, the growth rates of unstable sausage waves are exactly identical to those of the kink waves.

One should mention that for both eigenmodes (kink and sausage) for each M_A one can get dispersion curves corresponding to a backward wave propagation [1]. However, such solutions to the waves' dispersion relations are not acceptable, at least for the solar wind, from a physical point of view. That is why they are not considered here.

Conclusion In investigating the wave propagation along a jet moving with respect to the environment with a constant speed U we had to take into account the influence of two factors: (i) the Hall term in the generalized Ohm's law, and (ii) the flow itself. The combining effect of these two factors can be expressed as follows:

The Hall term generally limits the range of propagation of the wave modes not only for static tubes/layers but also for steady flowing ones. The limiting wave number, K_{limit}, is specified by two plasma parameters: the densities ratio of the two plasma media (outside and inside the jet), η, and the Hall parameter, ε. When M_A becomes negative, the real part of the phase velocity of the eigenmode (kink or sausage wave) is forced to go beyond that limiting wave number in a region where the wave becomes unstable. The range of Alfvénic Mach numbers, for which given mode is unstable, however, critically depends of the value of η – for larger ηs it is wider. The instability which occurs should be of Kelvin–Helmholtz type.

References

[1] R. Miteva, I. Zhelyazkov, and R. Erdélyi, New J. Phys. **6**, 14 (2004).

[2] I. Zhelyazkov, A. Debosscher, and M. Goossens, Phys. Plasmas **3**, 4346 (1996).