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# THEORETICAL STUDY OF ELECTROMAGNETIC WAVE MODES SUSTAINING THE COAXIAL DISCHARGE

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## Introduction

Electromagnetic wave travelling along a dielectric tube can produce plasma inside the tube which is the typical cylindrical plasma column of surface-wave-sustained discharges. The cylindrical plasma column is studied in details and it is well known that the plasma inside the tube is produced by one wave mode only, azimuthally symmetric one ($m$ = 0) in most of the cases. In some particular cases it is possible to produce plasma column by travelling dipolar wave ($m$ = 1) [1]. In that case the azimuthally symmetric wave decays very fast and again only one wave mode is propagating. One of the main characteristics of the cylindrical plasma column sustained by travelling wave is the single mode regime of operation.

Electromagnetic wave can produce plasma also outside the dielectric tube when there is a metal cylinder at the tube axis. Since the plasma is acting as outer conductor, this configuration is named coaxial discharge (figure 1). When the plasma is sustained by traveling electromagnetic wave in a coaxial structure the single mode regime is not proved. The possible exciting of dipolar and higher modes together with the azimuthally symmetric one is experimentally observed in [2].

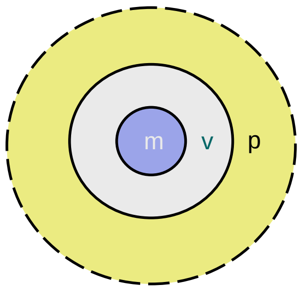


Fig. 1.Coaxial discharge structure

The purpose of this work is to investigate theoretically the wave modes that can produce and sustain plasma in the coaxial structure and their propagation characteristics. For simplicity, we have investigated a coaxial structure which consists of a metal rod in the centre, vacuum and plasma.

## Basic assumptions and relations in the model

In our modelling we consider the stationary state of a plasma at low pressure sustained by electromagnetic (EM) wave ($\omega/2\pi$ =2.45 GHz) travelling along the plasma–vacuum interface. The wave electric field heats the electrons so they absorb the wave energy. As a result the wave energy decreases along the plasma column. The plasma density decreases too and the plasma column is axially inhomogeneous. We assume that the plasma density, the wave number $k_z$ and the wave amplitude are slowly varying functions of the axial coordinate. The investigation is based on one-dimensional axial fluid model and we use radially averaged electron number density. The fluid model is applicable at low pressure when the main process of plasma creation is direct ionization from the ground state and the losses of the charged particles are due to their diffusion. The plasma is considered as a weakly dissipative medium and the collision term in the plasma permittivity can be neglected, i.e. we use the plasma permittivity in the form $\varepsilon_{\rm p} = 1 - \omega_{\rm p}^2/\omega^2$, where $\omega_{\rm p} = \left(4\pi e^2 n/m\right)^{1/2}$ is the plasma frequency, $n$ is the plasma density and $e, m$ – electron charge and mass.

The modelling approach in this paper is similar to the one used for coaxial discharge sustained by azimuthally symmetric ($m$ = 0) TM wave [3] but now the wave electromagnetic field possesses all six components $\mathbf{E} = (E_r, E_\varphi, E_z)$, $\mathbf{B} = (B_r, B_\varphi, B_z)$. Our model is based on Maxwell's equations from which we obtain the wave equation. In cylindrical coordinates $(r, \varphi, z)$ it takes the form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}E_z\right) + \frac{\partial^2}{\partial z^2}E_z + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}E_z + \frac{\omega^2}{c^2}\varepsilon E_z = 0 \qquad (1)$$

Keeping in mind the abovementioned assumptions we consider the solutions in this form:

$$E_z(r,\varphi,z,t) = \mathrm{Re}\left[F_z(r,z)E(z)\exp\left(-i\omega t + i\int_0^z dz' k(z') + il\varphi\right)\right] \qquad (2)$$

The amplitude function $F_z$ is presented as a combination of Bessel or modified Bessel functions in the different media. Similar equations and solutions can be obtained for the other EM field components. The boundary conditions are the conditions for continuity of the electromagnetic field tangential components at the plasma–vacuum interface and the condition for annulment of the $E_z$–component on the metal rod. From boundary conditions we obtain the local dispersion relation symbolically written down as

$$\mathrm{D}\left(\omega,\ k_z,\ l,\ R,\ \varepsilon_{\rm p},\ \eta\right) = 0 \qquad (3)$$

Here $l$ is the azimuthal wave number, $R$ is the plasma radius and $\eta = R_{\rm m}/R$, where $R_{\rm m}$ is the metal rod radius. Since the plasma is axially inhomogeneous the local dispersion relation gives the dependence between the normalized plasma density $N$ ($\omega/\omega_{\rm p} = 1/\sqrt{N}$, $N = n/n_{\text{cut off}}$, $n_{\text{cut off}} = m\omega^2/4\pi e^2$) and the dimensionless wave number $k_zR$, so called phase diagrams. Solving the same dispersion relation at fixed $\omega_{\rm p}$ and varying $\omega$ one obtains the dispersion curves for a given homogenous plasma column. If we assume that our axially inhomogeneous plasma consists of homogenous slices, then each phase diagram could be determined by a series of dispersion curves. We will use this approach to analyze the behavior of the wave propagation characteristics of the EM wave sustaining the coaxial discharge.

**Results and discussion**

The dispersion relation is solved numerically at various azimuthal wave numbers $l$ from 0 to 7 ($l = 0$ corresponds to azimuthally symmetric wave mode, $l = 1$ – to dipolar one and $l \geq 2$ to higher modes). The obtained phase diagrams are presented in figure 2. One can see that there are two regions in the phase diagrams plot which require special attention. The first one is at small values of $\omega/\omega_p$, which correspond to higher plasma densities. The second one is at $\omega/\omega_p > 0.707$, which is at plasma densities $n < 2n_{cutoff}$ (underdense plasma). In the first region turning back points in the phase diagrams with the azimuthal wave number $l \geq 2$ are observed. After it the wave number decreases and $\omega/\omega_p$ increases to the next turning back point. These two turning back points separate each phase curve ($l \geq 2$) into three parts. The third part passes true the underdense plasma region.

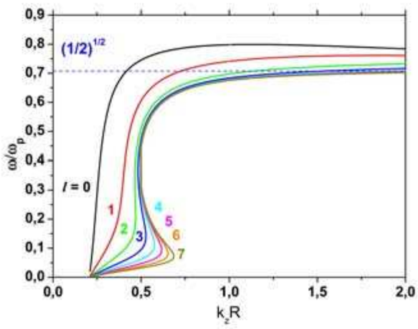


Fig. 2. Phase diagrams at various azimuthal wave numbers

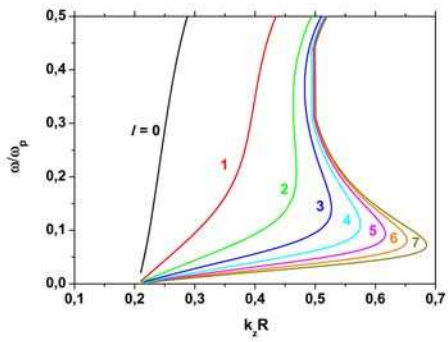


Fig. 3. Phase diagrams at various azimuthal wave numbers

For each phase curve (azimuthal wave number) the first turning back point corresponds to a local maximum of the wave number ($k_{z\ max}$), or a local minimum of the wave length. With the azimuthal wave number increasing, $k_{z\ max}$ increases too. The phase curves of higher modes are below those of lower modes, which means that the plasma density at a given wave number is higher (smaller $\omega/\omega_p$) at the higher modes (figure 3). In addition, one can see from this figure that at $l = 0$ the plasma density decreases rapidly at small variation of wave number while at higher modes the plasma density keeps higher values with a big variation of the wave number before the turning back point. This allow us to conclude that the higher modes can easily produce plasma with higher density in a coaxial structure than the azimuthally symmetric and dipolar waves.

From the course of the phase diagrams it is not possible to conclude if there is a backward wave propagation region (where phase and group velocities would be in opposite directions) between the first and second turning back points. In order to analyze the behavior of the phase diagrams we have calculated a series of dispersion curves for each phase diagram (figures 4 and 5). One can see from these curves that between the first and second turning back points, backward wave propagation region does not exist (neither for lower nor for higher modes).

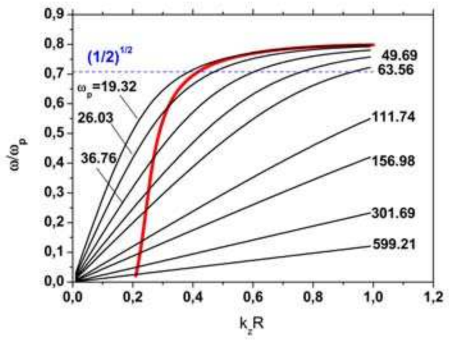


Fig. 4. Dispersion curves and phase diagram for azimuthally symmetric wave ($l$ = 0)

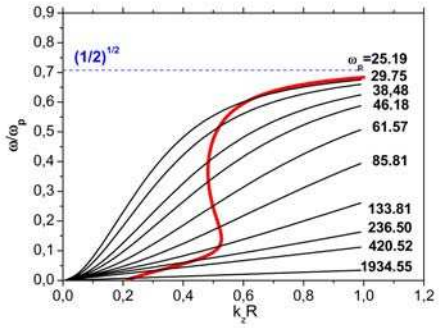


Fig. 5 Dispersion curves and phase diagram for $l$ = 3 wave mode

Region of backward wave propagation is observed at small $\omega/\omega_p$ at underdense plasma after the phase diagrams crosses the $n_{\text{cutoff}}$ limit (dashed blue line in figure 2). The corresponding dispersion curves are presented in figure 6. They also cross the cutoff limit and have maximum after which the backward propagation region occurs. For azimuthally symmetric wave that region is at small wave numbers, while for higher modes it is at bigger values of $k_z R$ and the maximum of the dispersion curves moves to the right with increasing of the azimuthal wave number.

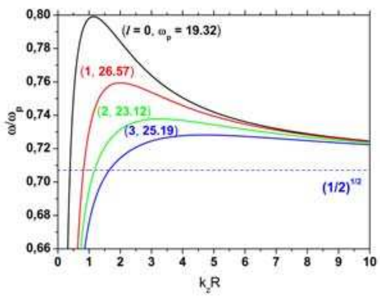


Fig. 6. Dispersion curves in the underdense plasma region

All this investigation confirms experimentally observed possibility of higher azimuthal wave modes to sustain plasma in a coaxial structure. Even more, the higher modes easily produce plasma with higher density.

## Acknowledgments

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