

Plasma response to a forced oscillation in a 1D modified Vlasov system

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Introduction

Certain tokamaks, including the Mega Amp Spherical Tokamak (MAST)[1], have been fitted with coils in-vessel allowing deliberate excitation of MHD waves. A particular class of wave, the Toroidal Alfvén Eigenmode (TAE), is weakly damped in spherical tokamaks due to vessel geometry. In the presence of supra-thermal particles, with velocities close to the Alfvén speed, TAEs can become unstable leading to explosive behaviour [2], causing potentially unacceptable particle and heat loss from the plasma and possibly damage the vessel. Future reactors will have a significant population of high-energy alpha particles from fusion reactions, thus understanding fast-particle-driven instabilities is a critical issue.

By exciting a stable plasma using the internal coils and measuring the subsequent response, we extract knowledge of the stable modes the plasma can support, including resonant frequency and damping rate. These measurements help to improve theories of fast particle physics. We fit the frequency response from experimental data using the simplest possible model: the damped-driven harmonic oscillator, with natural frequency ω_0 and damping rate γ :

$$\ddot{\xi} + \omega_0^2 \xi - \gamma \dot{\xi} = \Omega^2 A \cos(\Omega t) \quad (1)$$

The term on the right-hand side drives the displacement ξ sinusoidally in time with frequency Ω at an amplitude A . The solutions to this equation provide the system response $R(\Omega) = \xi_{\text{MAX}}(\Omega)$: the envelope of maximum displacement as a function of driving frequency[3]. The response function

$$H(\Omega) = \frac{R(\Omega)}{A} = \frac{1}{1 + \frac{i\gamma\Omega}{\omega_0^2} - \frac{\Omega^2}{\omega_0^2}} \quad (2)$$

has a maximum at resonance $\Omega = \omega_0$ as shown in Fig. (1). Its width gives the damping rate γ .

The fitting of the response function provides the values of resonant frequency and damping rate. When fitting to experimental data, parameters are included in the numerator to improve

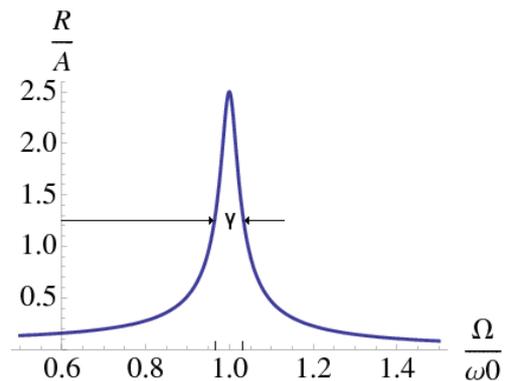


Figure 1: Example frequency response function. Amplitude peaks at the resonant frequency. The width of the curve at half the peak height is the damping of the system.

the fit away from resonance, without affecting the values for resonant frequency or damping [3]. The linear response function Eq. (2) lacks many relevant plasma physics details, such as particle information, spatial distribution and wave-particle dynamics. The model is also unable to provide information on whether the mode is saturated, so it cannot be certain that it is the best to use for experimental data. Whilst it is possible to construct a nonlinear scalar oscillator, we wish to inform this choice by assessing the simplest model that contains the wave-particle effects just described. See Fig. (2).

Kinetic System

We apply a kinetic model that has been widely and successfully used [4]: the one-dimensional modified Vlasov-Maxwell [5] system, which serves as an intermediate step between the complex experimental physics and the heuristic scalar oscillators (see Fig. (2)). The first of our model equations is a modified electrostatic Vlasov equation:

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + E \frac{\partial f}{\partial v} = -\nu_a (f - F_0) \quad (3)$$

The phase-space particle distribution function $f = f(x, v)$ is self-consistently evolved with separate spatial and velocity advectons. A Krook operator is employed on the RHS to model the effects of dissipation and collisional scattering at a rate ν_a , relaxing the particle distribution to the equilibrium distribution F_0 . The electric field E is evolved using a modified displacement current equation:

$$\frac{\partial E}{\partial t} + \int v \delta f dv = -\gamma_D E + \Omega A \cos(\Omega t) \cos(kx) \quad (4)$$

where δf , in the dielectric term, is the difference in f from the spatially averaged distribution. A background damping is applied at a rate γ_D . The other term on the RHS is a forcing term similar to that in Eq. (1), with amplitude A and frequency Ω ; the $\cos(kx)$ term is the applied spatial profile of the forcing; $k = 2\pi/L$ is the wave number of the first harmonic of the electric field in a periodic box of length L .

Treating perturbed quantities as Fourier modes varying in space and time, after substitution into the model equations and linearising, we obtain an expression for the response, $R(\Omega) = E_{\text{MAX}}(\Omega)$, of the electric field in the limit of small forcing amplitude:

$$H(\Omega) = \frac{R(\Omega)}{A} = \frac{\Omega}{\gamma_D - i\Omega - \int_{-\infty}^{+\infty} \frac{v \frac{\partial F_0}{\partial v} dv}{\nu_a - i(\Omega - k_n v)}} \quad (5)$$

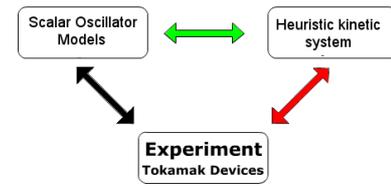


Figure 2: An intermediate kinetic model is introduced. Each level informs the others.

This equation gives the spatial harmonics of the electric field E as a function of the distribution parameters, the collisional and damping parameters and the forcing frequency. The argument of E gives the phase between the forcing term and the plasma response. Full simulations with weak forcing recover the linear result. In this regime Eq. (2) is still applicable and fitting recovers the resonance and damping rate. To ensure the response is in steady state we restrict the scanning rate $\Delta\Omega \ll \gamma$. The overall damping rate γ is the summation of the linear damping rate of the system and the electric background damping, i.e., $\gamma = \gamma_L + \gamma_D$ [6]. Time are normalised with respect to the plasma frequency ω_p .

Nonlinear effects with strong forcing

Confident of the agreement in a weakly driven linear regime, we increase the driving amplitude to see if the response exhibits nonlinear behaviour and if so, how applicable are the current linear tools in such a regime. We see the response (a function of driving frequency) becomes dependent on the direction of scanning, such that the response exhibits hysteresis as shown in Fig. (3). Preliminary analysis suggests that phase-space islands of trapped particles can exist in two modes within the hysteresis region; the larger islands, corresponding to higher electric field amplitude, are sustained, though not induced, by the forcing. Fig. (5). We believe the direction of skewness is related to the phase between the forcing and response, which at frequencies below the resonance or hysteresis region has positive sign, indicating the drive is inhibiting the response oscillation. Conversely, at frequencies above resonance or the hysteresis region, the phase changes sign and the drive leads the response. By leading, the drive supports the momentum of the trapped-particle islands against thermalising collisions attempting to restore the particle distribution to a Maxwellian.

Fitting Eq. (2) to the example shown in Fig. (3) overestimates the magnitude of the system damping rate ($\gamma = -0.0226$) to be $\gamma = -0.034278$ and $\gamma = -0.04311$ for the forward and

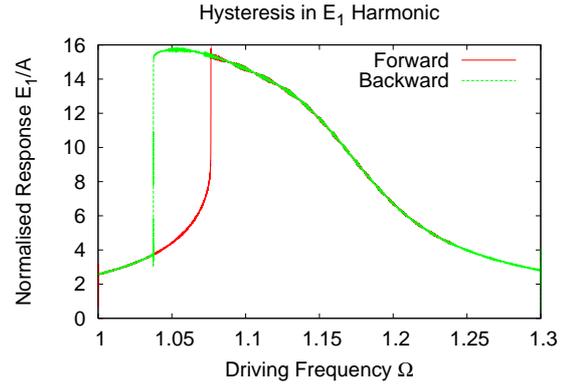


Figure 3: Nonlinear response of the electric field in the kinetic model, exhibiting hysteresis. Shock jumps occur as the system transitions from one stable branch to the other.

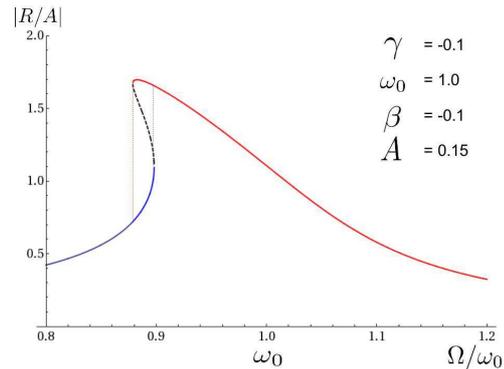


Figure 4: Example plot of the response of a Duffing oscillator with strong forcing. Two stable solutions overlap for certain frequencies. The unstable branch is not seen in practice.

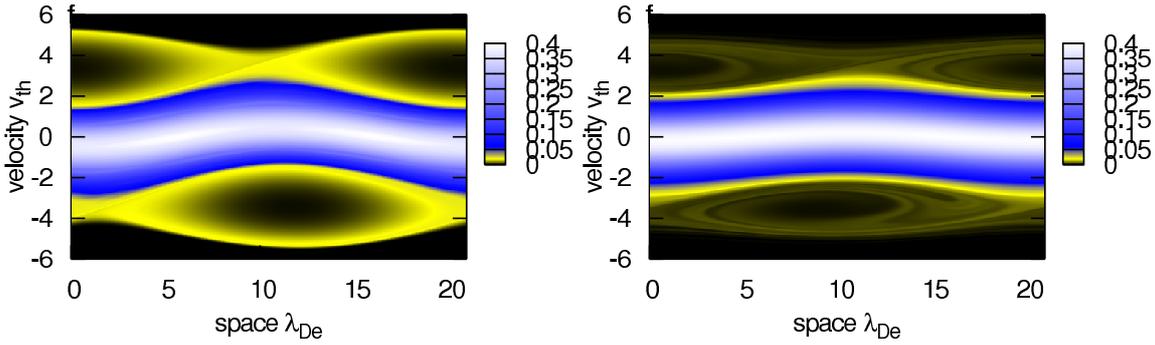


Figure 5: Phase space plots before and after jump transition. Colour represents particle density. The electrostatic island, which is supported by the forcing, is much fainter and the bulk oscillation is subdued.

backward scans, respectively. The linear approximation is therefore inapplicable. We now revisit scalar oscillators and assess whether a simple nonlinear Duffing oscillator model might capture this nonlinear effect.

$$\ddot{\xi} - \gamma \dot{\xi} + \omega_0^2 \xi + \beta \xi^3 = A \Omega^2 \cos(\Omega t) \quad (6)$$

All terms are as before, but a cubic displacement is added to the left-hand side with the parameter β introduced as a control of nonlinearity. We treat Eq. (6) with a perturbative approach similar to that in [7], to yield a response function which is cubic in R^2 :

$$\frac{R^2}{A^2} \{ \gamma^2 \Omega^2 + (\Omega^2 - \omega_0^2 + \frac{3}{4} \beta R^2)^2 \} = \omega_0^4 \quad (7)$$

This gives three real solutions of R , two of which are stable. An example case is shown in Fig. (4); the similarity between Fig. (3) alludes to a connection between the kinetic system and the Duffing Oscillator. Future work will develop a nonlinear transfer function based and fitting algorithm. If the resonance and damping are recovered, which are known in the kinetic model, then the relationship between the driving amplitude and nonlinearity ($\beta(A)$) may be found, and applied to analysing experimental data. We have developed an understanding of fast-particle-driven wave dynamics by showing the behavioural correspondence between the nonlinear Duffing oscillator and the kinetic Vlasov-Maxwell system.

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