

Relativistic plasma expansion in the presence of a magnetic field

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Abstract

Astrophysical relativistic plasma expansion into vacuum is investigated using a one fluid model, where the ideal MHD approximation is considered. The set of nonlinear differential equations is closed by a relativistic equation of state. Based on a quasi-neutral assumption, self-similar solution is found numerically in the presence of a magnetic field that enhances the self similar parameter limits and changes the expanding profiles. The velocity is found to increase with increasing the magnetic field while a cooling effect is noticed, which is more effective for higher magnetic field.

Introduction

Relativistic plasma expansion is commonly observed phenomena in astrophysical medium where the presence of a magnetic field affected the expanding profiles[1]. The gamma-ray bursts (GRB)[2] and powerful astrophysical jets provided by several classes of objects ranging from Active Galactic Nuclei (AGN) to Young Stellar Objects (YSO) are some ultra-relativistic expansion cases[3]. Observations showed that the outflow is formed close to the central object, where thermal energy and magnetic fields were strong enough to allow acceleration. Liang *et al.* [4] studied a new type of plasma expansion, in which a magnetized relativistic plasma freely expands into a vacuum with no external magnetic field. Structures and instabilities of plasmas expanding into a vacuum which contained a uniform ambient magnetic field have also been examined by theoretical and numerical methods. Those studies were relevant to laser plasma experiments [5] and space plasma phenomena [6]. Our attention is paid to the self-similar expansion dynamics of a relativistic plasma of electrons and protons embedded in a nonuniform one component magnetic field.

Basic equations

A single fluid model is used for two-component plasma that behaves as a neutral "gas clouds" in the presence of a magnetic field. In a fully ionized plasma charge neutrality requires $n_e = n_p = n$, where n_e (n_p) is the electron (proton) density.

The electromagnetic field is governed by Maxwell's equations:

$$\nabla \cdot \mathbf{H} = 0, \nabla \cdot \mathbf{E} = 4\pi/cJ^0, \nabla \wedge \mathbf{H} = 1/c\partial\mathbf{E}/\partial t + 4\pi/c\mathbf{J}, \nabla \wedge \mathbf{E} = -1/c\partial\mathbf{H}/\partial t \quad (1)$$

where $J^\nu = (J^0, \mathbf{J})$ is the four-current and H is the magnetic intensity. The plasma motion is governed by relativistic hydrodynamics equations obtained from local conservation laws of the stress-energy tensor,

$$T^{\mu\nu} = T_{Fluid}^{\mu\nu} + T_{EM}^{\mu\nu}, \quad (2)$$

where $T_{Fluid}^{\mu\nu}$ for an ideal fluid is given by:

$$T_{Fluid}^{\mu\nu} = (w/c^2)u^\mu u^\nu - pg^{\mu\nu}. \quad (3)$$

where $g^{\mu\nu} = diag(1, -1, -1, -1)$ is the metric tensor, $w = (p + e)$ is the enthalpy density, sum of pressure p and energy density $e = (mnc^2 + \varepsilon)$, c is the light speed. From the Faraday tensor we have

$$T_{EM}^{\mu\nu} = (u^\mu u^\nu + \frac{1}{2}g^{\mu\nu})H^2 - H^\mu H^\nu \quad (4)$$

where H^μ is the magnetic. In this case, Eq. (2) for total stress-energy tensor becomes:

$$T^{\mu\nu} = (w + H^2)u^\mu u^\nu + (p + \frac{1}{2}H^2)g^{\mu\nu} - H^\mu H^\nu \quad (5)$$

A one dimensional situation corresponds to an energy transported over very long distances into vacuum, such as by means of energetic collimated jets, that leads to $\vec{v} = (v, 0, 0)$ and $\vec{H} = (0, 0, H)$. Thus, the set of differential equations governing the relativistic expansion is

$$\frac{\partial(\gamma n)}{\partial t} + \frac{\partial(\gamma n v)}{\partial x} = 0, \quad (6)$$

$$\gamma^2 \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \right) (mc^2 + 8T - 3TA - 3TB) + 2 \left(\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial t} + \frac{T}{n} \left(\frac{\partial n}{\partial x} + v \frac{\partial n}{\partial t} \right) \right) + \frac{2}{4\pi\gamma^2} \left(\frac{1}{2} \frac{\partial H^2}{\partial x} + H^2 \frac{\partial v}{\partial t} + \frac{v}{2} \frac{\partial H^2}{\partial t} \right) = 0, \quad (7)$$

$$\left(\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} \right) (1 - A^2 - B^2) = -\frac{1}{3} \gamma^2 T \left(v \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \right) \quad (8)$$

where $A = 1/(2 + 3T/mc^2)$ and $B = \eta/(2\eta + 3T/mc^2)$. The equation of state approximation (EOS), for a singly component gas, gives

$$e = \rho c^2 + p \left(\frac{9p + 3\rho c^2}{3p + 2\rho c^2} \right) \quad (9)$$

Self-similar solutions and discussion

The self-similar approach allows great simplification in reducing a system of partial differential equations to ordinary ones and gives often the asymptotic behavior of the flow. To seek self-similar solutions from MHD equations, we introduce an independent similarity dimensionless variable $\xi = x/ct$ and use the following normalization[7]:

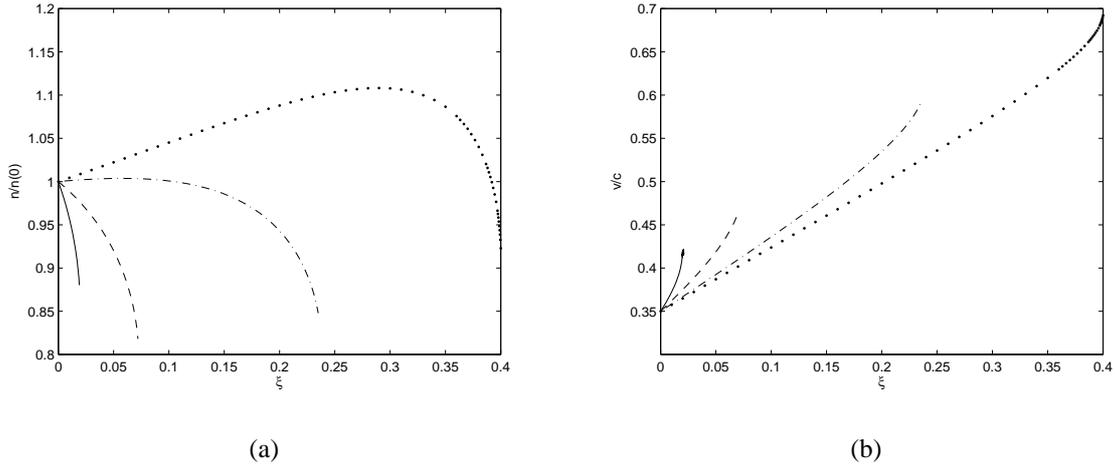


Figure 1: Normalized densities (a) and normalized velocities (b) as function of the similarity variable for different initial values of the magnetic field, $H(0) = 0$ (—), $H(0) = 0.3$ (- - -), $H(0) = 0.5$ (- · -) and $H(0) = 0.6$ (···) for $n(0) = 1$, $v(0) = 0.35$ and $T(0) = 0.1$.

$$\tilde{n} = \frac{n\omega t}{n_0}, \tilde{v} = \frac{v}{c}, \tilde{T} = \frac{T}{mc^2}, \tilde{H}^2 = \frac{H^2}{4\pi mnc^2\gamma^2} \quad (10)$$

$\sigma = \tilde{H}^2$ called the magnetization parameter is the ratio of magnetic field energy density to that of the rest mass. It becomes large for higher magnetic fields and zero for non-magnetized plasma. The MHD models is appropriate when $\sigma \sim 1$. In Figs (1-3), we investigate how the initial magnetic field affects the dynamics of the relativistic flow for the following initial density, velocity and temperature, $n(\tilde{0}) = 1$, $v(\tilde{0}) = 0.35$ and $T(\tilde{0}) = 0.1$. All profiles are dependant on the magnetic field initial value. From Fig. 1.a, The density profiles can be split into two parts. For $H < 0.5$, we have the well known expanding profile, The density decreases as the self-similar parameter increases. However, in our case the second derivative is negative. The second part concerns the situation where $H \geq 0.5$. The density increases then decreases to reach self-similar parameter limited value greater than the previous cases that corresponds to smaller magnetic field initial values. As there no particles creation, density increase can be attributed to an acceleration effect due to the magnetic files. This is easily depicted from the velocity profile (Fig.1.b).

An observational characteristic of many cosmic plasma outflows is that they seem to be accelerated to relatively high speeds which may reach values close to the speed of light in the most powerful AGN jets.

The acceleration mechanisms results from thermal flows or magneto-centrifugal origin. In this last case the acceleration comes from the conversion of Poynting energy flux to kinetic energy flux. The gain in kinetic energy is proportional to the energy that brings the magnetic field lines into rotation, i.e., the energy of the magnetic rotator; this energy is the product of the rotational frequency and the total specific angular momentum [8]. It was shown that relativistic jets in active galactic nuclei undergo extended (parsec-scale) acceleration which cannot be purely hydrodynamic. Vlahakis [9] showed that the parsec-scale acceleration to relativistic speeds can be attributed to magnetic driving. The magnetic energy in the flow can be converted into kinetic energy of the plasma or into fast particles, and from there into heat or radiation. Fig.2 showed a cooling effect that is more effective as the initial value of the magnetic field increases.

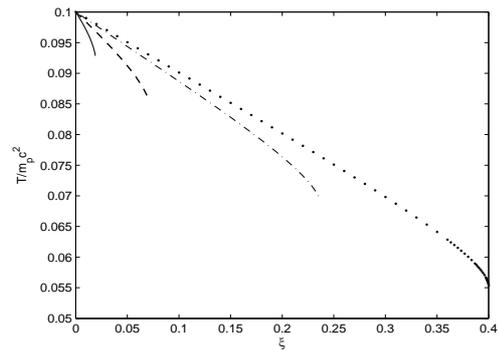


Figure 2: Normalized temperatures vs ξ for different initial values of magnetic field in the same conditions that in Fig.1.

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