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# Collisional zonal-flow damping in an impure tokamak plasma

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Zonal flows are widely known to play an important role in improving tokamak plasma confinement by shearing apart turbulent eddies, thus reducing the level of turbulent transport in the plasma. Driven by turbulent Reynolds stress themselves, these flows may be sensitive to other mechanisms which affect either the turbulence itself or the zonal flows. One of these mechanisms is friction due to Coulomb collisions between circulating and trapped particles which slow down the poloidal rotation of the plasma. In this work, we study the effect of heavy, highly charged impurities on the zonal flows. Being much more collisional than the lighter bulk ions, even small amounts of heavy impurities speed up the damping process by increasing the 'effective' collision frequency of the plasma, although their rotation itself hardly contributes to the flow [1]. The main focus of this work is on determining how fast this damping process occurs, and an analytic formula is derived for the damping time $\tau_p = \int_0^\infty (E_r(t)/E_r(\infty) - 1)\, dt$, which describes the time scale on which the zonal-flow potential approaches its residual value. The analytical results are then compared with the numerical results of the NEO code [2].

Following earlier papers by Hinton, Rosenbluth [3] and Xiao, Catto, Molvig [4], the zonal-flow potential is modelled by an initially imposed short-wavelength radial electric field, and the long-time response (i.e. on time scales longer than a few ion bounce times) of the plasma is calculated. The result can then be generalised to arbitrary source terms by convolution with the initial-value solution.

While the ion distribution function is governed by the drift-kinetic equation, the radial electric field has to adjust according to the constraint of conservation of angular momentum. Thus, we first solve

$$\frac{\partial f_a}{\partial t} + (\boldsymbol{v}_\parallel + \boldsymbol{v_d}) \cdot \boldsymbol{\nabla} f_a + \dot{w}\frac{\partial f_a}{\partial w} \quad = \quad C_a(f_a)$$

for both ion and impurity distribution functions, $f_i$ and $f_z$, respectively. Here, $\boldsymbol{v}_d$ is the drift velocity, $w = \frac{1}{2}m_a v^2$ the kinetic energy and the radial electric field is contained in $\dot{w} = -e_a(\boldsymbol{v}_\parallel + \boldsymbol{v_d}) \cdot \boldsymbol{\nabla}\phi$, $\boldsymbol{v}_\parallel$ denoting the velocity along the magnetic field $\boldsymbol{B} = I(\psi)\boldsymbol{\nabla}\varphi + \boldsymbol{\nabla}\varphi \times \boldsymbol{\nabla}\psi$. All particle species (electrons, bulk ions and impurities) are assumed to be in the low-collisional banana regime, and we order $Z \gg Z^2 n_z/n_i \sim Z m_i/m_z \sim 1$, which allows us to neglect impurity-ion collisions as their effect is much smaller than the effect of impurity self-collisions.

Self-collisions are modelled by $C_{aa}(f_{a_1}) = \nu_D^{aa}\left(\mathcal{L}(f_{a_1}) + m_a v_\parallel u_{a_\parallel} f_{a0}/T_a\right)$ and impurity-ion collisions by $C_{iz}(f_{i_1}) = \nu_D^{iz}\mathcal{L}(f_{i_1}) + m_i v_\parallel f_{i0}\nu_D^{iz} V_{z_\parallel}/T_i$, where $\nu_D^{ab}$ denotes the collision frequency of species $a$ and $b$, $\mathcal{L}$ is the Lorentz scattering operator, $V_{z_\parallel}$ the impurity mean flow-velocity and $u_a$ can be calculated to satisfy momentum conservation in self-collisions.

The drift-kinetic equation is then expanded in the smallness of the Larmor radius $\delta$ and the resulting 1st order equation is solved. For convenience, we split off the adiabatic part of the 1st order distribution function and write $f_{a_1} = g_a - I v_\parallel e_a \phi' f_{a0}/(\Omega_a T_a)$. There are two different ways of solving the partial differential equation: if one is interested in the complete time evolution of the zonal-flow potential, the equation can be solved via an eigenfunction expansion of the bounce averaged self-collision operator. However, the resulting formula for the damping time is not very intuitive, and in order to have sufficient accuracy it is advisable to calculate the eigenfunctions numerically. Another option is to consider only the long-time behaviour of the system, as in this limit the time derivative of $g_a$ can be shown to be small compared to the other terms and can thus be neglected. Consquently, it is only necessary to solve an ordinary differential equation, which can be done easily without expanding in eigenfunctions. A further advantage is that, as the resulting problem strongly resembles a neoclassical Spitzer problem, it is possible to compare the analytical results with neoclassical simulations.

The solution is then inserted in the equation of angular-momentum conservation, which reads

$$\begin{aligned}\frac{\partial}{\partial t}\Bigg\langle &\frac{(m_i n_i + m_z n_z)|\boldsymbol{\nabla}\psi|^2}{B^2}\phi' \\ &- \int \frac{I v_\parallel}{B}\left(m_i g_{i_0} + m_z g_{z_0} - \frac{I v_\parallel}{B}\phi'\left(\frac{m_i^2}{T_i} f_{i_0} + \frac{m_z^2}{T_z} f_{z_0}\right)\right) d^3v \Bigg\rangle = 0,\end{aligned}$$

and can be rewritten in terms of the (Laplace-transformed) neoclassical polarisation

$$\hat{P} \equiv \sum_{a=i,z}\left\langle \frac{I}{B}\int m_a v_\parallel \hat{g}_{a0} d^3v \right\rangle \Big/ \sum_{a=i,z}\left\langle m_a n_a R^2\right\rangle \hat{\phi}'$$

as an equation for the Laplace-transformed potential response,

$$\hat{\phi}'(p) = \frac{1}{p}\phi'(0)\frac{\left\langle \frac{|\boldsymbol{\nabla}\psi|^2}{B^2}\right\rangle}{\left\langle R^2\right\rangle(1-\hat{P})}.$$

The relation between the neoclassical polarisation and the zonal-flow damping time $\tau_p$ is found to be $\tau_p = \lim_{p\to 0}\int_0^\infty (\phi'(t)/\phi'_\infty - 1)e^{-pt}dt = d\hat{P}/dp(0)$ and can thus be calculated from the

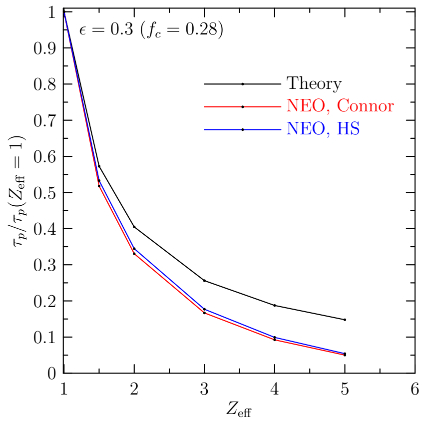


**Figure 1:** Normalised damping time versus effective charge

distribution functions $g_i$ and $g_z$, with the result

$$\tau_p = \frac{I^2}{\langle R^2\rangle B_0^2} f_c \left( \left\{ \frac{1}{\nu_D^{ii} + \nu_D^{iz}} \right\} + \frac{f_c \left\{ \frac{\nu_D^{ii}}{\nu_D^{ii}+\nu_D^{iz}} \right\}^2}{\{\nu_D^{ii}\} - f_c \left\{ \frac{(\nu_D^{ii})^2}{\nu_D^{ii}+\nu_D^{iz}} \right\}} \right)$$

$$\approx \frac{I^2}{\langle R^2\rangle B_0^2} \frac{f_c}{\hat{\nu}_{ie}} \left( \frac{4.51}{Z_{\text{eff}}} + \left( 0.87 + 2.49 \frac{f_c}{1-f_c} \right) \frac{1}{Z_{\text{eff}}^4} \right)$$

where curly brackets denote an average over velocity space, $\{F(v)\} = \int d^3v F m v_\parallel^2 f_0/(nT)$, $f_c = 0.75 \int_0^{\lambda_c} d\lambda\lambda / \left\langle \sqrt{1 - \lambda B/B_0} \right\rangle$ is the effective fraction of circulating particles where $\lambda_c$ denotes the trapped-passing boundary, and the second formula was interpolated in the effective charge $Z_{\text{eff}} = \sum_j (Z_j^2 n_j)/\sum_j (Z_j n_j)$.

In Figs. 1 and 2, the normalised damping time is plotted versus effective charge and inverse aspect ratio $\epsilon$, respectively, together with the results from the NEO code. In regions where the theory is valid, the agreement between theory and simulation is quite good. The simulation in Fig. 1 was done for a carbon species, thus for large $Z_{\text{eff}}$ the impurities cease to be a minority in the plasma and the assumptions in the theory are violated which explains the discrepancy as $Z \to Z_{\text{eff}}$. The discrepancy for small values of $\epsilon$ in Fig. 2 is due to the fact that the plasma is no longer in the banana regime in this limit as the collision frequency was fixed in the simulation. Clearly visible is also the asymptotic character of the theory as $Z \to \infty$.

From both theory and numerical simulations, we find that, when impurity ions are present in the plasma, the damping time is significantly shorter than the damping time in a pure hydrogen plasma. Although the impurities hardly rotate themselves, they are able to slow down the circulating bulk ions considerably. The underlying physical mechanism is that impurities are

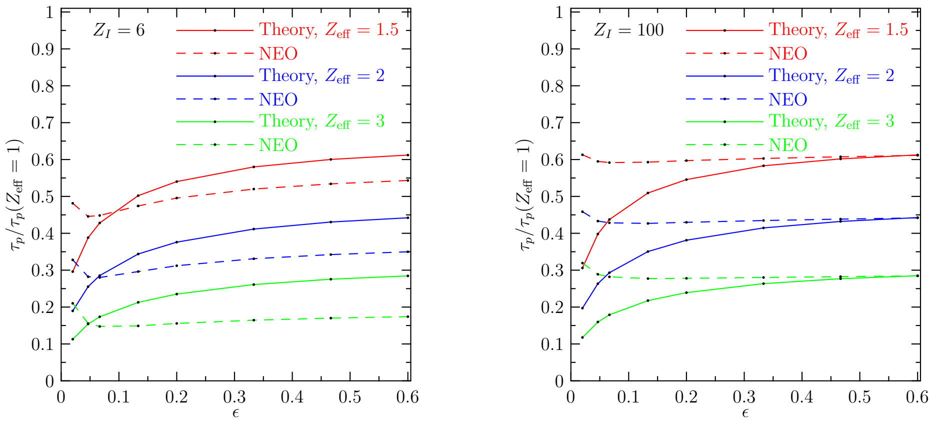


**Figure 2:** Normalised damping time versus $\epsilon$

slowed down on the time scale of impurity collisions, which is much shorter than that of ion collisions. Thus, when the circulating ions collide, their rotation is not only damped by self-collisions with the trapped population as in the case of a pure plasma, but also by collisions with the *entire*, i.e. both trapped and passing, impurity population. Most important is this mechanism in the limit of very large aspect ratio where few trapped particles are present. In this case, the rotation in a pure plasma remains nearly undamped, whereas in an impure plasma damping occurs. For tight aspect ratio, impurities do not matter so much as the number of trapped particles is high enough to damp the rotation nearly immediately.

Insofar as collisional zonal-flow damping plays an important role, this would suggest that impurities inhibit zonal flows and could thus have a deleterious effect on plasma confinement.

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