Radial propagation of geodesic acoustic modes

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Introduction

Geodesic acoustic modes (GAMs) are axisymmetric poloidal $E \times B$ flows with finite frequencies of order $\sqrt{2}c_s/R$ (with sound speed c_s) coupled to up-down-antisymmetric pressure perturbations, which provide the restoring force of the oscillation.

In recent works on radially propagating GAMs no general rules for the direction and speed of GAM propagation have been given [1, 2, 3, 4]. The propagation of GAMs may be crucial for the radial windows of GAM activity observed in ASDEX Upgrade [5] or DIII-D [6] and the match between linear theory and experimental GAM frequencies [7].

Since the energy contained in a wave packet is transported with its group velocity, one can calculate the group velocity of a GAM by compairing its total energy to its energy flux (Poynting flux). This provides a powerful tool for determining the direction and speed of GAM propagation, which is more general than the quite complicated direct calculation of the dispersion relation, and even allows predictions for single-null configurations.

The energy approach

We demonstrate the basic concepts of our method using the two-fluid equations for cold ions and infinite safety factor q. For details including the gyrokinetic treatment see Ref. [8]. The units are chosen such that the magnetic drift velocity is unity. Density, n, temperature, T_i and T_e, and electric potential perturbations ϕ are normalized to $\rho^\star n_0$, $\rho^\star T_{0,i/e}$, $\rho^\star T_{0,e}/e$, respectively, where the subscript 0 indicates the corresponding background value and $\rho^\star \equiv \rho_{se}/R$ with the major torus radius R, $c_{se} \equiv (T_e/m_i)^{1/2}$, and $\rho_{se} \equiv (m_i c_{se})/(eB)$. The time scale is $t_0 \equiv R/(2c_{se})$. Flux-surface averaging is indicated by $\langle \ldots \rangle$.

Since the unperturbed equilibrium minimizes the free energy, it is always second order in the fluctuations. In the chosen framework, the free energy functional [9] is given by

$$\langle E \rangle = \langle E_e + E_i \rangle = \left\langle \frac{n^2}{2} + \frac{(\nabla \phi_0)^2}{2} \right\rangle, \tag{1}$$

where E_e and E_i are the electron and the ion free energy density, respectively, $n^2/2$ is the energy of the electron density perturbations, and $(\nabla \phi_0)^2/2$ the ion kinetic energy. The ion density fluctuations obey

$$\dot{n} - \Delta \dot{\phi} - \hat{C}\phi = 0, \tag{2}$$

where $\hat{C} \equiv -\overrightarrow{v}_d \cdot \nabla$ and $\overrightarrow{v}_d \equiv -(1/2)(\hat{\kappa} + R\nabla \ln B) \times \hat{b}$ is the sum of the curvature and ∇B-drifts of the electron density fluctuations, $\Delta\dot{\phi}$ the divergence of the polarization current, $\hat{b} \equiv \overrightarrow{B}/B$, and $\hat{\kappa} \equiv \overrightarrow{\kappa}/\kappa$. The electrons are assumed to be adiabatic

$$n = \phi - \phi_0, \langle n \rangle = 0, \tag{3}$$

because the GAM frequency is much smaller than the electron bounce and transit frequencies. By combining (1), (2), (3), and representing the time derivative of $\langle E \rangle$ as the divergence of a radial Poynting flux one obtains

$$\partial_t \langle E \rangle = -\left\langle \nabla \cdot \overrightarrow{S} \right\rangle = \left\langle -\nabla \cdot \left(\frac{\overrightarrow{v}_d n^2}{2} \right) + \nabla \cdot (n\nabla\dot{n}) \right\rangle. \tag{4}$$

The term $\overrightarrow{v}_d n^2/2$ represents the flow of the energy of electron pressure perturbations in ion magnetic drift direction. The second term is the polarization energy flux.

The free energy and the Poynting flux, Eqs. (1) and (4), can be evaluated in Fourier space for a circular high aspect ratio magnetic geometry by calculating the second order (in k_r) approximations of the density perturbations with the help of (2). The GAM frequency at $k_r = 0$ is $\omega = 2^{-1/2}$. Inserting the frequency and the density approximations into (1) and (4) one obtains for the radial group velocity

$$v_{g,r} = \frac{\langle S_r \rangle}{\langle E \rangle} \approx -\frac{k_r}{2\sqrt{2}}. \tag{5}$$

implying that group and phase velocities are antiparallel for cold ions. The derivation of this result via the dispersion relation $\omega(k_r)$ would require corrections to the density to higher order in k_r than the energy approach. The validity of (5) is limited to $k_r \ll 1$, because for higher wave numbers the mode loses the character of a GAM due to resonances with other modes.

In case of arbitrary ion to electron temperature ratio τ and finite safety factors q, the Poynting flux of the GAM consists of the magnetic drift and the polarization flux supplemented by FLR correction terms. There exists a critical value of τ ($\tau \approx 0.2$) above which v_g is parallel to the phase velocity. Due to damping by resonant sound waves, GAMs are restricted to $q \gtrsim 3$. The analytical results have been verified using the two-fluid code NLET [10] and the gyrokinetic code GYRO [11].

Geometry effects

Let us now turn to the effects of plasma shaping on the Poynting flux. Assuming, for simplicity, cold ions, infinite safety factor, and neglecting the polarization drift, one can approximate Eq. (2) at $\theta = \pm\pi/2$ (where $\overrightarrow{k}$, $\overrightarrow{v}_d$, and $\overrightarrow{v}_p$ are parallel):

$$n \approx \frac{2v_E}{(\omega - k_r v_{d,r})R} = \frac{2v_E}{Rk_r(v_p - v_{d,r})}, \tag{6}$$

in which $v_E = k_r \phi_0$ is the $E \times B$-drift velocity. For $k_r \ll 1$, equivalent to $v_p \gg v_d$, Eq. (6) implies

$$n^2 \approx \frac{4v_E^2}{\omega^2 R^2} \left(1 + \frac{2v_{d,r}}{v_p} \right), \ \langle n^2 \rangle \approx \frac{4v_E^2}{\omega^2 R^2}, \ \langle v_{d,r} n^2 \rangle \approx \frac{k_r v_d^2}{\omega} \langle n^2 \rangle, \tag{7}$$

$v_{d,r}$ being the radial component of $\vec{v_r}$. Accordingly, the radial Poynting flux (4) can be estimated by

$$\langle S_r \rangle \approx \frac{k_r v_d^2}{\omega} \langle n^2 \rangle - k_r \omega \langle n^2 \rangle. \tag{8}$$

Since $\langle E \rangle \approx \langle n^2 \rangle$ and $v_d = 1$ in the units defined above, the group velocity is of order $O[k_r]$. Thus, GAM propagation is generally much slower than turbulent motions and magnetic drifts, because in physical units $v_{g,r}$ is of order $O(k_r \rho_{se} v_d) \ll O(v_d) \ll O(v_{dia})$.

The Poynting flux has been estimated at $\theta = \pm \pi/2$, but experimentally the radial wavenumber is usually known at the outboard midplane, where k_r is smaller due to, e.g., ellipticity or Shafranov shift. Incorporating correction terms for k_r and the geometry dependence of the GAM frequency at $k_r = 0$ [7], we obtained an estimate of the group velocity at the outboard midplane (in the units defined above) for elliptic Miller geometries [12]

$$v_{g,r}(\kappa) \approx k_0 \frac{0.65 + 2.12\,\delta\kappa + 1.95\,\delta\kappa^2 + 0.64\,\delta\kappa^3}{0.32 + 2.15\,\delta\kappa + 5.31\,\delta\kappa^2 + 5.54\,\delta\kappa^3 + 2\,\delta\kappa^4} \tag{9}$$

with k_0 being the wavenumber at the outboard midplane, $\kappa = 1 + \delta\kappa$ the plasma elongation, $\partial_r \kappa = (\kappa - 1)/r$, Shafranov shift $\partial_r R = -1/3$, $q = 3$, aspect ratio $A = 3.5$ and $\tau = 1$ (for details see. [8]).

Single-null configuration

In single-null configuration near the separatrix, the perturbations vanish at the X-point (which here is assumed to be at the bottom) due to the magnetic null, and the density perturbations are concentrated opposite to the X-point. Consequently, the drift energy flux is $\langle v_{d,r} n^2 \rangle / 2 \sim v_d \langle n^2 \rangle$, and $v_g \sim v_d$ (at the top), while the polarization energy flux is one order smaller and can be neglected. Hence, independent of the ion temperature, the energy of the density perturbations propagates in the ion magnetic drift direction, which is usually directed towards the X-point. Therefore, GAMs propagate radially inward. The group velocity at the outboard midplane in elliptic Miller geometry is therefore given by

$$\frac{k_r}{k_0} v_d = \frac{1 + \partial_r R}{2\kappa - 1} v_d, \tag{10}$$

where k_r is the wavenumber at the top and $\partial_r R$ the differential Shafranov shift. We have analyzed up-down asymmetric magnetic geometries employing Miller type equilibria [12] with the modification $Z(r) = Z_0(r) - \kappa r \sin(\theta)$ to verify our predictions using the NLET two-fluid code.

The up-down asymmetry of v_E^2 resulting from $\partial_r Z_0 \neq 0$ leads to a non-vanishing magnetic drift energy flux $\langle v_{d,r} n^2 \rangle$ at $k_r = 0$, which determines the group velocity. Since the geometry-induced energy transport is always parallel to v_d, the GAM dispersion is slightly shifted compared to the up-down symmetric case as shown in Fig. 1.

Conclusions

We have demonstrated, how estimates for the radial group velocity of GAMs can be obtained exploiting Poynting's theorem. In up-down symmetric magnetic geometries, v_g is smaller than the magnetic inhomogeneity drift, whereas in up-down asymmetric geometries –

Figure 1: NLET computed log-color-coded GAM spectrum for $\tau = 0$, $q \to \infty$, $\partial_r Z_0 = 0.5$; approximate (2nd order in k_r) frequency (dashed) for $\partial_r Z_0 = 0$. For $\partial_r Z_0 > 0$, the flux-surfaces distance is smaller at the bottom than at the top, leading to $v_{g,r} < 0$ at $k_r = 0$.

e.g. single-null configuration – it can be of the same order. In both cases, v_g is much smaller than the diamagnetic velocity. While Refs. [1, 2, 3, 4] observe GAMs to propagate radially outward, our theory predicts inward propagation in case of cold ions and outward propagation for warm ions. In symmetric configurations the propagation direction is determined by *global* cutoff effects induced by e.g. temperature gradients, whereas in up-down asymmetric geometry it depends only on the curvature drift direction, i.e. local effects.

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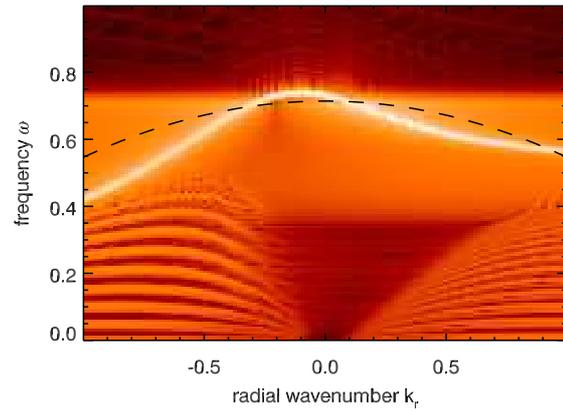[12] R. L. Miller et al., Phys. Plasmas **5**, 973 (1998).