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Dual role of shear flow in turbulent transport of magnetic fields

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Introduction Turbulent transport of magnetic fields is seriously quenched due to the back-reaction of (small-scale) magnetic fields [1, 2] even for a weak large-scale magnetic field far below equipartition. Significant quenching in turbulent transport can also result from shearing by (stable) shear flows, which accelerate the forward cascade to small scales by eddy distortion/disruption, effectively enhancing the overall dissipation in a system [3, 4, 2]. While this shear quenching is vital for plasma confinement in laboratory plasmas (e.g. tokamaks, stellerators, etc [5]), it adds more trouble to explaining observed fast magnetic activities (e.g. solar magnetic cycles, solar flares, corona mass ejection in astrophysical plasmas, saw-teeth and major disruptions in laboratory plasmas). Thus a crucial question is whether or not the transport of magnetic fields is *super slow* under the influence of both magnetic back reaction and shear.

The purpose of this paper is to investigate this problem by comprehensive direct numerical simulations of 2D (single fluid) sheared MHD turbulence to elucidate fundamental physical processes which accelerate or moderate transport. By an extensive exploration of the parameter space, we show that shear flows play an interesting *dual* role: quenching transport by shear distortion whilst simultaneously enhancing it via resonance [6, 7, 8].

Governing equations We consider incompressible MHD equations for the vorticity ($\omega = \nabla \times \mathbf{u}$) and magnetic vector potential A ($\mathbf{B} = \nabla \times A\hat{z}$) given in the dimensionless form:

$$\begin{aligned} [\partial_t + (\mathbf{U}_0 + \mathbf{u}) \cdot \nabla]\, \omega &= \nu \nabla^2 \omega + \frac{1}{M^2} (\mathbf{B} \cdot \nabla) \nabla^2 A + F\,, \\ [\partial_t + (\mathbf{U}_0 + \mathbf{u}) \cdot \nabla] A &= \eta \nabla^2 A\,. \end{aligned} \tag{1}$$

Here, F is an external forcing; $\mathbf{U}_0$ is an imposed shear flow; $\mathbf{u}$ is the turbulent flow evolved consistently under the influence of $\mathbf{U}_0$ and the vorticity driving F; $M = v_c/v_A$ is the Alfvénic Mach number – the ratio of characteristic turbulent velocity v_c to the Alfvén speed v_A associated with a large-scale magnetic field; η and ν are molecular Ohmic diffusivity and viscosity which are assumed to be the same (i.e. magnetic Prandtl number $P_m = \nu/\eta = 1$). The forcing is chosen to have a power spectrum peaked around $|\mathbf{k}| \approx 5$, and temporally random with no characteristic frequency ($\omega_0 = 0$) and correlation time $\tau_C/2\pi = 1/\gamma$, thereby containing frequencies $\omega = \omega_0 \pm 2\pi/\tau_C = [-\gamma, \gamma]$. Here, γ is decorrelation rate. As done in [1], we apply hyperviscosity on scales larger than that of the forcing to keep the characteristic wavenumber of turbulence $k \sim 5$ even in the kinematic limit (without inverse cascade or the formation of zonal flows).

We solve Eq. (1) by using spectral code with the 4th order accuracy (IFRK4) in a $(2\pi)^2$ box with periodic boundary conditions. The shear flow and initial large-scale magnetic field are chosen such that $\mathbf{U}_0 = \Omega \sin(x)\hat{y}$ and $\langle A(t=0)\rangle = A_0(t=0) = \cos x$ ($\mathbf{B}_0 = \nabla \times A_0\hat{z} = \sin(x)\hat{y}$), thus allowing no direct influence of shear on $\mathbf{B}_0$, i.e., $\mathbf{U}_0 \cdot \nabla \mathbf{B}_0 = 0$. Here, $\langle\rangle$ denotes an average over small scale turbulence; Ω represents (maximum) shearing rate and velocity in our units. It is important to note that if $\mathbf{U}_0 \cdot \nabla \mathbf{B}_0 \neq 0$, $\mathbf{B}_0$ would be sheared and stretched directly by $\mathbf{U}_0$, thereby efficiently dissipated. We numerically determine $\eta_T = \langle u_x A\rangle / B_0$ from the decay rate of A_0, which is solely attributed to molecular (η) and turbulent diffusion for $\mathbf{U}_0 \cdot \nabla \mathbf{B}_0 = 0$. In the following, we present the results from numerical simulations obtained by systematically varying M and Ω.

Results Our recent work [8] has demonstrated numerical evidence for a strong suppression of transport by shear flow or magnetic fields when the shearing or Alfvénic timescale is shortest among all the characteristic timescales in the system, respectively. Specifically, without resonance, turbulent magnetic dissipation is quenched by strong magnetic field B_0 ($M > 1$) as $\eta_T \propto B_0^{-4}$ while $\eta_T \propto \Omega^{-2.7}$ by strong shear Ω. The presence of magnetic fields and shear flows can introduce important new dynamics through the excitation of Alfvén waves which interact with shear flows and turbulence. In particular, a strong large-scale magnetic field $\mathbf{B}_0$ transforms turbulence eddies into packets of Alfvén waves of frequency $\omega_B = \mathbf{B}_0 \cdot \mathbf{k}$, with which the shear flow $\mathbf{U}_0$ can resonantly interact when Doppler shifted frequency $\omega_D = \omega - \mathbf{U}_0 \cdot \mathbf{k} = \pm\omega_B$. This leads to an enhancement of the turbulent diffusivity of $\mathbf{B}_0$, which would otherwise be severely quenched, as shown below. Note that for effective transport, irreversibility through this resonance (and its overlap), stochasticity, and/or molecular dissipation is absolutely necessary.

Fig. 1 shows $R_m = \eta_T/\eta$ for $\eta = 0.001$ for different values of Ω and M with $\tau_c = 1$. When the magnetic field is sufficiently weak such that $M^2 > \eta_K/\eta$, η_T is unaffected by magnetic backreaction, recovering the unsheared kinematic result. For $1 < M^2 < \eta_K/\eta$, η_T is suppressed due to magnetic back reaction, similarly to the case without shear flow [8]. However, for $M^2 < 1$, a considerable increase in η_T is noticeable around the resonant point due to the shear flow roughly when $\Omega \sim 1/M$ obtained by taking $\omega \sim 0$ in resonant condition $\omega - \Omega k \sin x \sim \pm k \sin x/M$. These resonant points are denoted by $*$ in Fig. 1, with a good agreement between the location of these points and maximum transport, especially in the limit of strong magnetic field where the turbulence is almost Alfvénic. The resonance at this point is caused by the turbulent eddies being transported by $\mathbf{U_0}$ along the magnetic field lines at the Alfvén speed, which allows the Alfvén waves to be coherently forced, leading to the amplification of the amplitude of the Alfvén waves.

We obtain the scaling of the maximum value of η_T at resonant points as a function of B_0, by choosing the value of Ω satisfying the resonance condition. The results are shown in Fig. 2. In sharp contrast to $\eta_T \propto B_0^{-4}$ scaling obtained with $\Omega = 0$ [8], the scaling of $\eta_T \propto B_0^{-2}$ persists into the very strong magnetic field regime $M < 1$. This is because the shear flow shifts the frequency to match Alfvén frequency, leading to the resonant interaction between Alfvén waves and turbulence, preventing severe reduction of transport $\eta_T \propto B_0^{-4}$. This is an interesting result, highlighting another crucial effect of shear flow.

We demonstrate that these are robust results by performing similar simulations, but by using $\tau_c \to 0$ instead of $\tau_c = 1$. The results are plotted in Fig. 3. Immediately noticeable is the significant reduction in overall value of η_T compared to the case with $\tau_c = 1$ (see Fig. 1). This follows from $|F(\omega)|^2 \propto \gamma/(\omega_0^2 + \gamma^2) \to 1/\gamma$ for $\gamma \gg 1$ ($\tau_c \ll 1$). Furthermore, in the kinematic limit ($M \to \infty$), η_T is quenched $\propto \Omega^{-1}$ for strong shear, with a much weaker dependence on Ω compared to $\Omega^{-2.7}$ in the case of $\tau_c = 1$. This is because for $\tau_c \to 0$, the frequency of forcing can take any value, always satisfying the resonance condition $\omega - \Omega k \sin x = \pm k \sin x / M$ for all values of M and x. Note that this scaling is weaker than the theoretical prediction ($\eta_T \propto \Omega^{-2}$) given in [2] obtained by using an anisotropic forcing. The general tendency of weaker dependency on shear for the forcing with shorter correlation time is however generic, and is also found in previous works (see [2, 6]). The increase of η_T at resonant points, marked by $*$, is also clearly seen in Fig. 3, which again shows the persistence of the scaling of $\eta_T \propto B_0^{-2}$ into $M^2 < 1$ due to resonances.

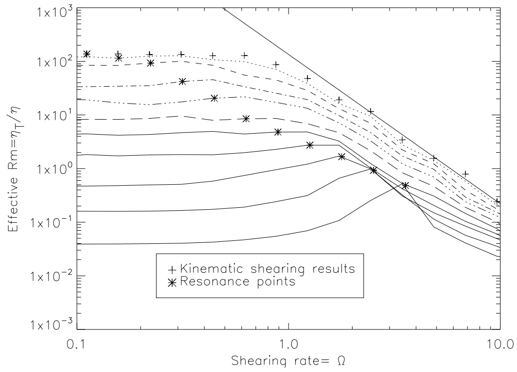

Figure 1: R_m vs Ω for $\tau_c = 1$ and $1/M^2 \in [0, 0.01, .., 10]$, with B_0 increasing from the top to the bottom. The symbol * approximately represents resonance point.

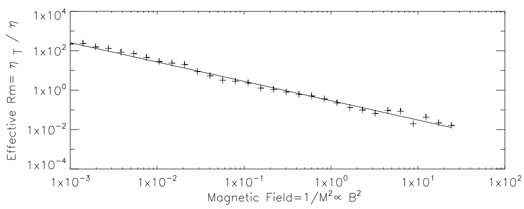

Figure 2: R_m vs $1/M^2$ for $\tau_c = 1$ at resonant points for ($\omega - \Omega k \sim \omega_B$). The line represents the least squares fit with the scaling $\eta_T \propto B_0^{-1.96}$.

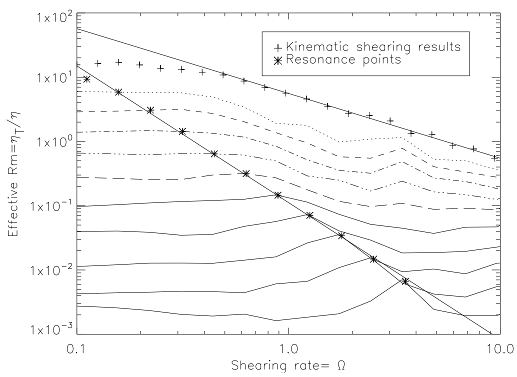

Figure 3: The same as in Fig. 1 but with $\tau_c \to 0$.

Conclusion we have elucidated the key physical processes for transport, especially highlighting the indispensable role of coherent structures (magnetic fields and shear flows) in determining turbulent transport through the excitation of waves, shearing, and resonances [9]. In particular, we have demonstrated (i) that transport quenching by shear flows and resonant interactions are vitally important to understanding turbulence regulation in 2D MHD; (ii) that a shear flow plays a dual role of quenching transport by shearing, whilst enhancing it via resonance and the overlap of resonant layers; (iii) that a strong suppression of transport by shear flow (magnetic fields) occurs when the shearing (Alfvénic) timescale is shortest among all the characteristic timescales in the system (with no resonance between coherent structures and turbulence/waves). Without resonance, $\eta_T \propto B_0^{-4}$ for weak shear and strong B_0; $\eta_T \propto \Omega^{-2.7}$ for strong shear and weak B_0; while with resonance $\eta_T \propto B_0^{-2} \propto \Omega^{-2}$ for both strong shear and B_0.

These results were checked to be robust upon the change in the values of η and ν (across $\eta = \nu \in 0.5, 1, 2, 4, 8 \times 10^{-3}$). We expect that similar results will hold for more general shear flows and equilibrium magnetic fields as long as they are stable.

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