Collisional and collisionless beam plasma instabilities

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The Fast Ignition Scenario (FIS) for Inertial Confinement Fusion (ICF) has prompted in recent years many theoretical [7], numerical and experimental works on relativistic beam plasma instabilities [4]. Due to the density gradient of the pre-compressed target (the center is about 10^4 denser than the border), the beam-plasma interaction is collisionless near the REB emitting region, and collisional near the center.

For the collisionless part, it has been established on the one hand that modes propagating perpendicularly (or obliquely) to the beam are the fastest growing ones for typical FIS parameters [2]. The beam is therefore broke-up into finite length filaments, which transverse typical size is the background plasma skin-depth $\lambda_p = c/\omega_p$. On the other hand, the unstable transport in the dense collisional region reveals a qualitatively different picture: the beam is still filamented, but the typical size of the filaments is now the *beam* skin-depth $\lambda_b = c/\omega_b$ [3]. Within the FIS context, this means filaments about 100 times larger than in the collisionless region. The question comes immediately as to know how exactly is operated the transition from one regime to another. Such a bridge is important from the conceptual point of view, and necessary to describe the beam propagation in the intermediate region.

Some recent work investigated this question for the FIS [1]. In addition, the influence of partial electronic plasma degeneracy near the pellet core was discarded, at least with respect to the unstable spectrum. Given the number of effect accounted for, this investigation was restricted to a single set of typical FIS parameters. As a consequence, the transition between the two regimes was not documented in details. The goal of the present work is to fill this gap, accounting for a simpler theoretical model and highlighting the transition threshold in terms of the main variables.

We thus consider a relativistic beam of density n_b, velocity $\mathbf{v}_b$ and Lorentz factor $\gamma_b = (1 - v_b^2/c^2)^{-1/2}$ passing through a plasma of electronic density n_p. The plasma electrons are drifting with velocity $\mathbf{v}_p$ such as $n_b\mathbf{v}_b = n_p\mathbf{v}_p$ and the plasma ionic density n_i is such that $n_i = n_b + n_p$. The return current velocity $v_p = (n_b/n_p)v_p$ can be considered non-relativistic since we are not interested in the fully collisionless region where $n_b \sim n_p$. Collision-wise, the electrons from the beam are supposed collisionless due to their large velocity [3]. The terms collisional/collisionless rather refers to the background electrons. Their collisionality is here characterized by the plasma electron/ion collision frequency ν_{ei}.

The partial degeneracy of the core electrons is neglected, since it has been found that its role on the unstable spectrum is negligible. Finally, the orientation of the perturbation wave vector $\mathbf{k}$ needs to be arbitrary. While this is a source of significant analytical difficulties, such framework is necessary if one wishes to capture the most unstable mode. As will be checked, the fastest growing modes is each regime are usually oblique. An investigation focussing on the filamentation instability with $\mathbf{k} \perp \mathbf{v}_b$, would thus render improperly the beam response by bypassing the most relevant modes in this respect.

After the background plasma ions, which are assumed et rest, electrons from the beam and the plasma share the same continuity equation,

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v}_j) = 0, \tag{1}$$

where the subscript $j = b$ or p for the beam or the plasma. The Euler equation reads for the beam electrons,

$$\frac{\partial \mathbf{p}_b}{\partial t} + (\mathbf{v}_b \cdot \nabla)\mathbf{p}_b = -q\left(\mathbf{E} + \frac{\mathbf{v}_b \times \mathbf{B}}{c}\right) - \frac{\nabla P_b}{n_b}, \tag{2}$$

and for the plasma ones,

$$\frac{\partial \mathbf{v}_p}{\partial t} + (\mathbf{v}_p \cdot \nabla)\mathbf{v}_p = -\frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v}_p \times \mathbf{B}}{c}\right) - \nu \mathbf{v}_p - \frac{\nabla P_p}{n_p}. \tag{3}$$

The beam equation is thus collisionless and relativistic, while the plasma one is non-relativistic and collisional. The pressure terms are expressed in terms of the temperatures through $\nabla P_j = 3k_B T_j \nabla n_j$, where k_B is the Boltzmann constant. Such an adiabatic treatment demands sub-relativistic temperatures, a requirement stronger for the beam than for the plasma [6, 5]. Though lengthy, the derivation of the dispersion equation is quite standard and expressed in terms of the dimensionless variables,

$$\alpha = \frac{n_b}{n_p},\ \ \mathbf{Z} = \frac{\mathbf{k} v_b}{\omega_p},\ \ \beta = \frac{v_b}{c},\ \ \tau = \frac{\nu}{\omega_p},\ \ \rho_j = \sqrt{\frac{3k_B T_j}{m v_b^2}}. \tag{4}$$

Calculations have been conducted aligning the beam velocity $\mathbf{v}_b$ with the z axis, and considering $\mathbf{k} = (k_x, 0, k_z)$. Components k_z and Z_z are therefore the parallel ones, while k_x and Z_x are the perpendicular ones.

Figure 1 shows a typical growth-rate map arising from the numerical resolution of the dispersion equation. Modes localized around $Z_x \sim Z_z \sim 1$ are collisionless ones and produce filaments of transverse size $\sim c/\omega_p$. Note their oblique location, impossible to capture if restricting the exploration to

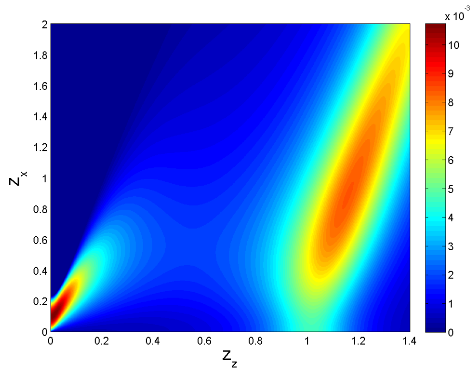

Figure 1: Growth-rate map in terms of $\mathbf{Z}$.

the main axis. Unstable modes at small $\mathbf{Z}$ are collisional ones, as one can check they vanish when setting $\nu = 0$. The full spectrum is here clearly governed by these collisional modes. The fastest growing mode is found for $Z_z = 0.014$ and $Z_x = 0.11$, producing much larger filaments than the collisional modes. The simple relation between the beam and plasma skin-depths, $\lambda_b = \lambda_p/\sqrt{\alpha}$ shows that their size fits here the beam skin-depth, as expected when dealing with resistive filamentation [3].

The unstable spectrum is thus clearly divided into two parts: the "lower" collisional spectrum, and the "upper" collisionless one. Our goal from this junction is two-fold: on the one hand, studying the evolution of the fastest growing mode (and its growth rate) of each part and on the other hand, documenting the transition between the two regimes. In view of the vast numbers of free parameters, we focus on the (τ, α) mapping, choosing for the other variables some FIS relevant values. We thus explore the parameters space $\alpha \in [0, 10^{-1}]$, $\tau \in [0, 0.5]$, $\gamma_b = 4$, $\rho_p = 4.2 \times 10^{-2}$ (T_p=1 keV) and $\rho_b = 0.42$ (T_b=100 keV).

While collisionless modes are mitigated by collisions, collisional ones are numerically found to scale like $\tau^{1/3}$ and $\alpha^{2/3}$. These τ trends make it clear that beyond a given collisionality threshold, collisional modes must surpass collisionless ones. The resulting partition of the (τ, α) domain is pictured on Figure 2, where the beam trajectory from the pellet border to the core is superimposed. Instability wise, the beam clearly starts from the collisionless region to end up in the collisional one. The upper-spectrum is thus relevant at the beginning while the lower one is more important by the end.

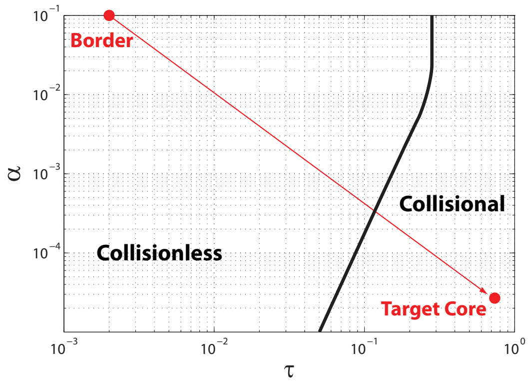

Figure 2: Hierarchy map.

We finally turn to the most unstable wave-vector analysis. Our goal is mainly to check the size of the structures generated. To this extent, Figure 3 pictures the perpendicular Z_{xm} components of the most unstable wave-vector $\mathbf{Z}_m = (Z_{xm}, Z_{zm}$. A detailed analysis (not shown) shows that this quantity Z_x scales like $\alpha^{1/2}$ when $Z_{zm} = 0$, and $\alpha^{\sim 1/3}$ otherwise. Interestingly, the $\alpha^{1/2}$ scaling is exactly what

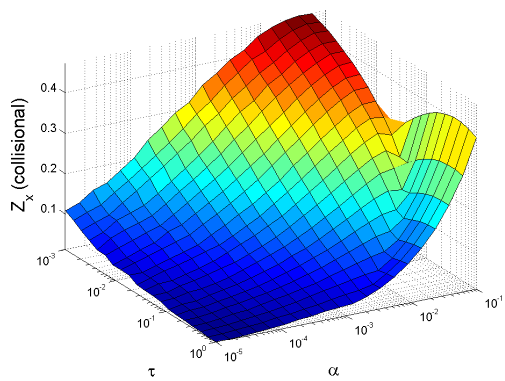

Figure 3: Z_{xm} component of $\mathbf{Z}_m$.

would be expected following the *beam* skin-depth instead of the plasma one. In the oblique collisional regime, we still witness an increase of the filaments size when decreasing α, but the scaling is too slow to keep up with the beam skin-depth.

The present theory presents therefore an unified view of the unstable spectrum in terms of collisionality, and correctly bridges between what was already known about the two regimes.

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