The Influence of Magnetic Islands on ITG Mode Structure and Stability

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1. Introduction

We address the impact of small scale magnetic islands on drift wave stability, focusing on the ion temperature gradient (ITG) mode. The islands modify the density, temperature and flow profiles in their vicinity, which influences both the structure of the ITG mode, and its growth rate. These results suggests that very small magnetic islands may be beneficial for confinement by suppressing turbulent transport near a rational surface, and perhaps triggering the formation of an internal transport barrier there. The following section describes the plasma equilibrium in the island's vicinity, and analyses that equilibrium for stability. Section 3 illustrates some of the 2D mode structure properties. We provide a reduced model that is analytically tractable to interpret the numerical results in terms of a WKB theory. Section 4 provides a summary and conclusions.

2. Equilibrium and ITG stability

Our model is described in detail in Ref [1]; here we reproduce the essential steps. The equilibrium magnetic geometry is taken to be of the form $\boldsymbol{B} = B_0 \nabla z + \nabla \psi \times \nabla z$. We expand about a sheared slab reference state, with $\psi = B_0 x^2 / L_s$ with a Maxwellian distribution function that (spatially) depends only on x. The coordinates (x, y, z) are standard Cartesian coordinates. The magnetic island is introduced as a perturbation to this state, with $\psi = B_0 x^2 / L_s + \tilde{\psi} \cos K_y y$. The island width, w, is then given by $w^2 = 2\tilde{\psi} L_s / B_0$ and we consider the islands to be long and thin, i.e. $K_y w \ll 1$. We evaluate the plasma response to this imposed magnetic perturbation and the electrostatic perturbation, which must be calculated from the quasi-neutrality constraint. We work in the frame where the island is stationary, in which case there is an ExB flow around it associated with a reference radial electric field, E, with potential $\Phi_0 = -Ex$. This is an input to the model, parameterised by $\omega_E = -eL_n E/T_e$. There are two more contributions to the electrostatic potential: $\bar{\phi}$ is independent of time and results from the plasma response to the magnetic island, and a fluctuating piece, $\tilde{\phi} e^{-i\omega t}$, represents the ITG mode fluctuations. We linearise with respect to fluctuating quantities, but retain non-linearities in the time independent quantities.

The electrons are described by an adiabatic response, which is constant on the perturbed flux surfaces of the island. The new profile is parameterised by the profile function $h(\psi)$, which
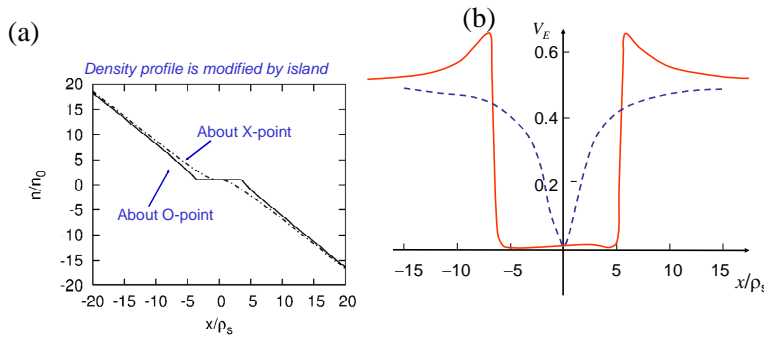
(a)

Density profile is modified by island

About X-point

About O-point

(b)

Figure 1: Density (a) and flow (b) profiles across the island X-point (dashed) and O-point (full).
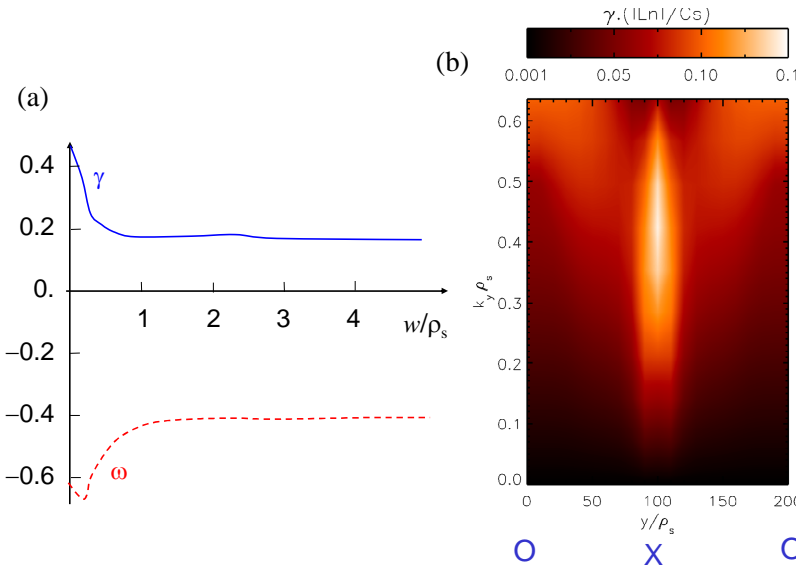
(a)

(b)

$\gamma.(|\text{Ln}|/Cs)$

O   X   O

Figure 2: ITG growth rate as island width increases (a) and contour plot of local growth rate as a function of k_y and x (b).

would be uniquely determined if we imposed a model for cross-field transport. As this is uncertain, we instead adopt a simple model for $h(\psi)$ that is consistent with the necessary boundary conditions (see [1]). The ion response is derived from the non-linear gyrokinetic equation. This has two classes of terms: some are independent of time, while others fluctuate. The linear solution is a particular solution of this nonlinear gyro-kinetic equation, and it too has two types of terms: some

independent of time and some fluctuating. The fluctuating and time-independent electron and ion densities must balance independently. Thus, quasineutrality provides two equations: one determining the equilibrium profiles (i.e. the electrostatic potential) and one determining the fluctuations and hence stability to the ITG mode. Figure 1 shows the resulting equilibrium density and flow profiles.

The eigenmode equation for the fluctuating quantities is simplified by assuming long wavelengths compared to the ion Larmor radius, and treating the sound wave perturbatively:

$$\frac{\partial^2 \widetilde{\varphi}}{\partial x^2} + \left[\frac{\sigma^2 k^2}{(\Omega - \omega_E kS)^2} x^2 - \left[\frac{(\Omega - \omega_E kS) - (1-S)k}{(\Omega - \omega_E kS) + (1-S)k\eta} + k^2 \right] \right] \widetilde{\varphi} = 0 \qquad (1)$$

This is the same as that in a standard sheared slab except that the complex mode frequency, Ω, is Doppler-shifted due to the equilibrium flow shear parameterised by $S(x,y)=1-\partial h/\partial x$. There are also modifications to the diamagnetic frequency due to the influence of the island on the density

gradient, also described by *S*. These equilibrium modifications vary on a short length scale ~*w*. Note, we have Fourier transformed in the y-direction, introducing the Fourier wave-number, *k*, normalised to the sound speed gyroradius, ρ_s. Finally $\eta=(1+\eta_i)/\tau$, where τ is the ratio of electron to ion temperatures and η_i is the standard ITG stability parameter, proportional to the ion temperature gradient. We can convert back to real space through the transformation $k\rightarrow i\rho_s\partial/\partial y$ to deduce the full 2D equation. The growth rate as a function of island width is shown in Fig 2a. Note that the island suppresses the growth rate compared to the sheared slab situation. For long, thin islands the dependence of *h* on *y* is weak. We can then consider a local approximation, and solve Eq (1) for $\Omega(k,y)$. Fig 2b is a contour plot showing a peak in the growth rate at k~0.4 and a value of *y* corresponding to the island X-point (i.e. where the drive is strongest).

3. Mode structure analysis

Naively, one might expect the mode to localise around the y-position of maximum instability. Returning to the 2D calculation again, we plot the full 2D mode structure in Fig 3a. Note that it is not in fact localised at the X-point, but half way between X and the O-points. As a first step to understanding this, let us try to construct a 2D eigenmode structure localised at the position of maximum instability. We write $\phi=F(x,y)\exp(-i\int k\, dy)$ where the exponential describes the short wavelength behaviour, which dominates the *y* variation. The leading order again provides Eq (1) which is solved to yield $\Omega_0(k,y)$ (Fig 2b). The next order provides the solubility condition that $\partial\Omega_0/\partial k=0$. As Ω_0 is complex, it is generally only stationary for a complex value of *k*. This is the key point. The next order provides a solution for *F* that is a Gaussian, centred on the most unstable y-position, $y=y_0$ (corresponding to the X-point in our model). Thus, writing $k=k_R+ik_I$, when $k_I=0$, this provides the expected mode structure, peaked at $y=y_0$. However, in the more general case, one finds that the mode peaks at $y=y_0+k_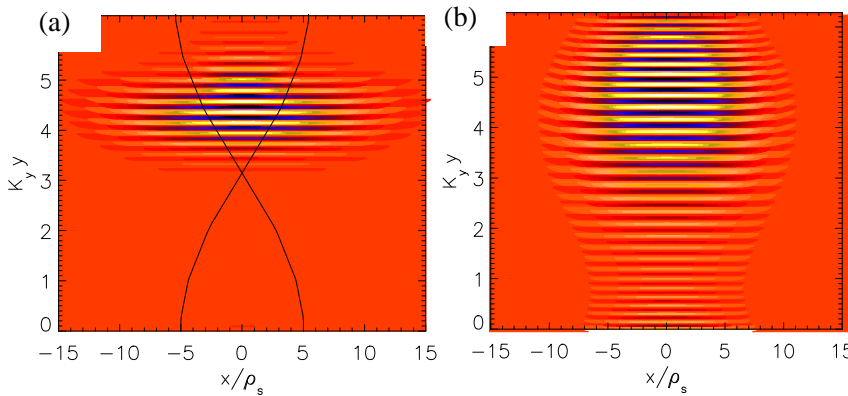I/(2\sigma)$. While this violates our assumption of a mode localised about y_0, it does demonstrate that an eigenmode cannot exist at $y=y_0$.

To illustrate a more consistent WKB theory, we consider a reduced model in which we neglect the flow shear around the

Figure 3: Colour contour plot of potential for full island model (a) and the reduced, analytic model (b).

island and the impact of the profiles on the diamagnetic drift. We retain a modulation of the ion temperature gradient drive, writing $\eta=\eta_0(1-\varepsilon \cos K_y y)/(1+\varepsilon)$. Performing the full 2D calculation for this model, we derive the mode structure shown in Fig 3b. Note that the mode is again not centred at the position of maximum drive, $K_y y=\pi$. The reduced model is analytically tractable. Thus, adopting an eikonal form, $\phi=\exp(-i\int k\, dy)$, results in Eq (1), but with $S=0$, which can be easily solved to yield $k(\Omega,y)$ as an eigenvalue. Now taking the limit of $\eta_0 \gg 1$, this equation has an analytic solution:

$$k = -\frac{(1+i)\Omega}{\sqrt{2\sigma\eta}} - \frac{\Omega}{2\sigma\eta}\left[\frac{\Omega^2}{\sigma}+\sigma+i\right]+O\left(\frac{1}{\eta^{3/2}}\right) \qquad (2)$$

The solution must be periodic in $K_y y$, so k is quantised with mode number $n \gg 1$ and $\Omega=-nK_y\rho_s(\sigma\eta_0/2)^{1/2}(1-i)$. Using this in Eq (2) yields $k= nK_y\rho_s-i(\varepsilon/4)(\sigma-1+\eta_0(nK_y\rho_s)^2)\cos K_y y$. The first term is real, representing the short wave-length nature of the mode. The second term is the small imaginary correction in $1/\eta_0^{1/2}$ and represents the slowly varying envelope in y, that peaks at $K_y y=\pi/2$. This corresponds to the position of the maximum found in the 2D calculations.

4. Conclusions

We have shown that in general, when one has a non-Hermitian system with a complex eigenvalue, it is not possible to localise the mode around the position of maximum instability. Thus, ITG modes in the presence of magnetic islands are not localised around the island X-point where the drive is strongest, but rather half-way between the positions of maximum and minimum drive. The eigenmode structure is rather localised in y, and does not exhibit the plane wave nature of the conventional sheared slab. Thus, only a fraction of the flux surface is influenced by the ITG mode when a magnetic island is present. In addition, the growth rate of the ITG mode is suppressed. The combination of these two effects is expected to dramatically reduce the level of transport when an island is present. This, in turn, raises the intriguing possibility that small scale magnetic islands might actually be beneficial for confinement and may help to establish internal transport barriers in the vicinity of rational surfaces in tokamaks.

References

[1] H.R. Wilson and J.W. Connor, Plasma Phys. Control. Fusion **51** (2009) 115007