Dynamics of two-dimensional turbulence in a pure electron plasma

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Highly magnetised pure electron plasmas confined in Malmberg-Penning traps [1] allow for the experimental study of two-dimensional (2D) fluid turbulence, when the experimental conditions are such that the cold non-relativistic guiding center approximation is valid. In this case, the transverse dynamics of the electron plasma column is well described by the drift-Poisson equations [2, 3], which are isomorphic to the 2D Euler equations for an incompressible, inviscid fluid, whose vorticity corresponds, up to a constant of proportionality, to the electron plasma density. The behaviour of freely decaying 2D turbulence in pure electron plasmas has been extensively studied both by using variational principles (see e.g. Refs. [4, 5, 6]) and by analysing the time scaling of the vortices present in the flow [7]. More recently, the statistics and dynamics of 2D turbulence in an electron plasma has been studied with both Fourier transforms [8] and wavelet analysis [9, 10]. In this work the dynamics of freely decaying 2D turbulence in a pure electron plasma confined in a Malmberg-Penning trap is studied experimentally by performing measurements of the axially averaged electron density. The analysis of these data through the Proper Orthogonal Decomposition (POD) technique [11] allows to extract from the flow the coherent structures that are energetically dominant and to identify the main dynamical processes which drive the time evolution of turbulence.

The experimental data have been obtained in the Malmberg-Penning trap ELTRAP [12]. The time evolution of the system is investigated through an injection-hold-dump cycle and monitored by means of an optical diagnostic system. After being injected into the device, the electrons are trapped for a given time and then dumped onto a phosphor screen. The light emitted by the screen is collected by a charge-coupled device (CCD) camera, so that the light intensity measured at a given position on the CCD sensor is proportional to the axially averaged electron density. A 2D image acquired by the CCD provides thus the density distribution and represents also the vorticity $\zeta(x,y,t)$ of the 2D fluid. The time evolution is studied by repeating the above described machine cycle several times with fixed injection parameters and increasing the trapping time. The shot-to-shot reproducibility of initial conditions is very

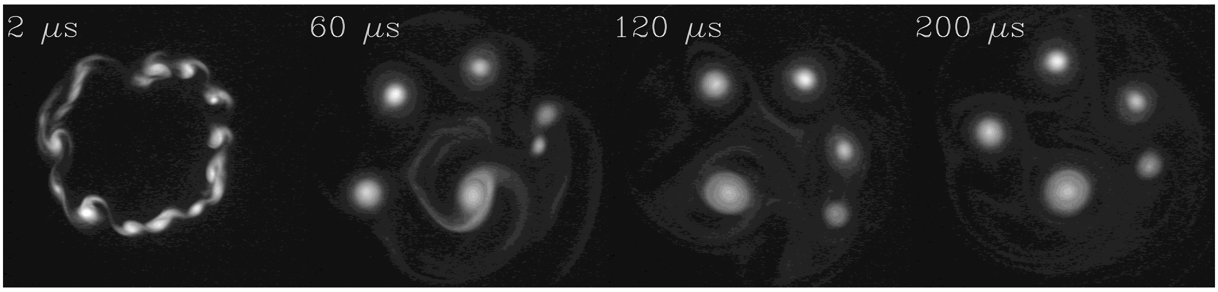

Figure 1: Evolution of the plasma density for the analysed sequence. The trapping time is indicated at the top left corner of each frame.

high, as the typical variation of the measured charge at a given position is less than 0.1 %. The plasma density evolution for the sequence considered in this work is shown in Fig. 1. The first frame (corresponding to a trapping time $\tau = 2$ μs) reflects the shape of the initial annular vorticity distribution distorted by the diocotron (Kelvin-Helmholtz) instability which rapidly leads to a nonlinear evolution of the flow. The interaction between vortices leads to merger events and to the formation of a meta-equilibrium state characterised by the presence of five strong vortices rotating within a diffuse background.

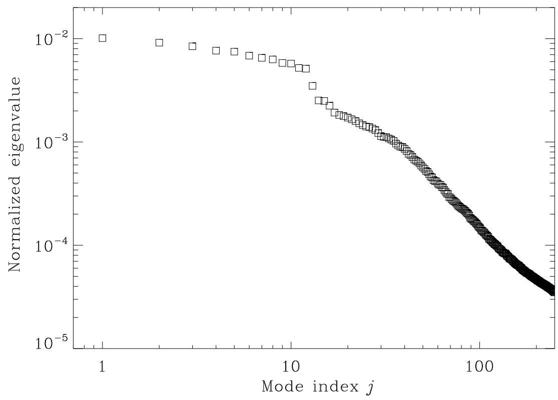

Figure 2: Spectrum of POD mode eigenvalues for the sequence of Fig. 1.

The sequence under study consists of N = 250 frames with a trapping time step of 2 μs. The POD expansion of the 2D vorticity $\zeta(x,y,t)$ is

$$\zeta(x,y,t) = \sum_{j=0}^{N-1} a_j(t)\varphi_j(x,y) \tag{1}$$

where $\varphi_j(x,y)$ are orthogonal basis functions and $a_j(t)$ are temporal modal coefficients. The functions $\varphi_j(x,y)$ are eigenfunctions of an eigenvalue equation obtained by imposing that the average projection of $\zeta(x,y,t)$ onto $\varphi_j(x,y)$ is maximised, constrained to the unitary norm [11]. The eigenvalues λ_j represent the mean enstrophy of each mode j. As expected, it is found that the first POD mode $j = 0$ nearly corresponds to the enstrophy time average, having a much larger eigenvalue with respect to the other modes and an almost constant modal coefficient. Since the focus here is on investigating the dynamics of the system, in the rest of the paper only the modes $j \geq 1$ are considered.

The spectrum of normalised eigenvalues of POD modes for the vorticity sequence of Fig. 1 is shown in Fig. 2. The normalised eigenvalues represent the fraction of mean enstrophy contained in each POD mode. The kink at $j = 12$ suggests the presence of different dynamical regimes described by the modes with $j \leq 12$ and $j > 12$ respectively. The empirical eigenfunctions of the first 12 modes (two of which are shown in Fig. 3) are characterised by the presence of coherent structures with a size of the order of 5-6 mm, in agreement with the results of Ref. [10] based on wavelet analysis. The modal coefficients of the modes $j \leq 12$ show oscillations dominated by few discrete frequency components between $\simeq$ 14 kHz and $\simeq$ 84 kHz with a separation of $\simeq$ 14 kHz between nearby components. From Fig. 4 it can be seen, for instance, that the mode $j = 1$ shows a main oscillation at $\simeq$ 56 kHz and the $j = 9$ mode at $\simeq$ 84 kHz. The fact that the oscillation frequencies are equally spaced suggests that the dominant POD modes can be identified with diocotron modes [2].

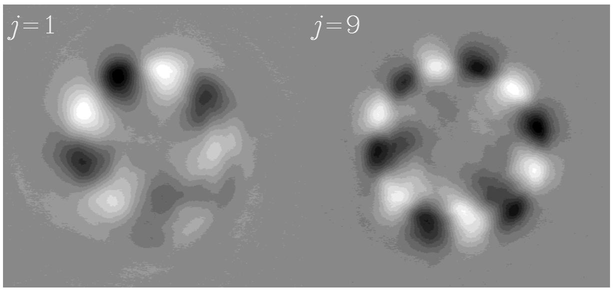

Figure 3: Empirical eigenfunctions of the modes $j = 1$ and $j = 9$ obtained from the POD of the sequence in Fig. 1.

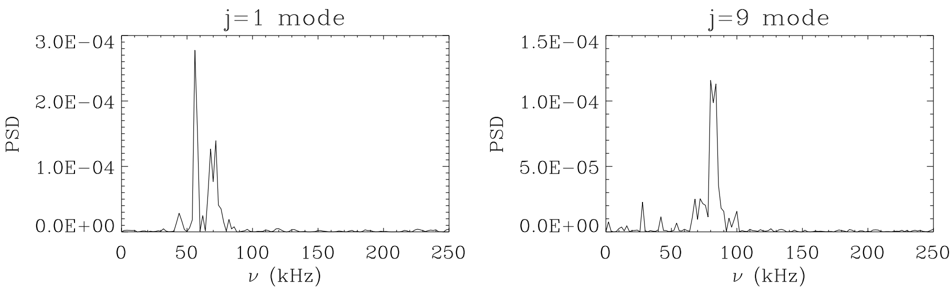

Figure 4: Frequency spectra of the modal coefficients $a_j(t)$ for the modes $j = 1$ and $j = 9$ obtained from the POD of the sequence in Fig. 1.

In the case of an annular electron layer with constant density, the diocotron eigenfrequencies can be computed analytically [2]. For the data of Fig. 1 the linearly unstable diocotron modes are characterised by azimuthal mode numbers $l = 2, \ldots, 7$, with $l = 5$ being the most unstable, in agreement with the spatial structure of the first POD eigenfunctions. The frequency separation between the unstable modes can be estimated to be $\Delta\nu \simeq 13.2$ kHz, also in agreement with the POD analysis results. The time behaviour of the modal coefficients indicates that these modes are active and dominating over the whole evolution of the plasma, that is, both during the initial onset phase of the instability and the subsequent relaxation of turbulence.

In conclusion, in the present work the dynamics of 2D turbulence in a pure electron plasma was studied through the POD technique. A time sequence of 2D fluid vorticity (electron density) measurements, obtained from an annular initial vorticity distribution, was analysed. The structure of the eigenfunctions of the POD modes with the major enstrophy content and the evolution of the corresponding temporal coefficients indicate that these modes are dominated by the contribution of diocotron modes. This method provides a new dynamical characterisation of 2D turbulence in a pure electron plasma and can be applied to the analysis of experiments with different initial conditions.

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