Theory of a spherical emissive probe in a low-density isotropic plasma

Alif Din[1,2], Siegbert Kuhn[1]

[1] Association EURATOM-ÖAW, Institute for Theoretical Physics, Innsbruck University, Technikerstrasse 25, A-6020 Innsbruck, Austria

[2] Theoretical Plasma Physics Division, PINSTECH, P. O. Nilore, Islamabad, Pakistan

The problem of a spherical emissive probe immersed in a time-independent low-density plasma is consider. A fairly general theoretical scenario based on trajectory integration of the Vlasov equation is formulated and specialized to the particular situation considered in [Bernstein and Rabinowitz, Physics of Fluids 2, 112 (1959)], however with the addition of electrons emitted from the probe surface with zero tangential velocity and a waterbag distribution with respect to the radial velocity. Comparison of the potential profiles for the emissive and non-emissive cases shows visible differences, thus demonstrating the effect of electron emission from the probe. Full details can be found in [A. Din, PhD thesis, Universty of Innsbruck, Austria (March 2010)].

Introduction

We consider a spherical probe immersed in a low-density plasma, assuming time-independent conditions. We moreover assume that collisions are negligible in the perturbed region lying between the probe (radius r_p) and the "unperturbed" plasma, the "plasma-probe transition (PPT)" region (outer radius r_{ps}). It is well known that the PPT region exhibits a space-charge dominated "sheath" region, adjacent to the probe and governed by Poisson's equation, and a practically quasi-neutral "presheath plasma" region, leading over to the unperturbed plasma and well approximated by the quasineutrality condition ("plasma equation"). According to [1], the presheath plasma region considered here corresponds to the "geometrical presheath" because the relevant curvature radii are much smaller than the dominant (i.e., smallest) collisional length.

Accordingly, in our PPT region the species-s velocity distribution function (VDF) $f^s(\vec{x},\vec{v})$ satisfies the Vlasov equation

$$\frac{D^s}{Dt} f^s(\vec{x},\vec{v}) = 0, \tag{1}$$

which is formally solved by means of trajectory integration [2]. In a time-independent situation as the one considered here, eq. (1) states that

$$f^s(\vec{x},\vec{v}) = f^s\left(\widehat{\vec{x}},\widehat{\vec{v}}\right) = f^s\left(\widehat{\vec{x}}_{st},\widehat{\vec{v}}_{st}\right), \tag{2}$$

where $(\vec{x}, \vec{v})$ is the point under consideration, $\left(\widehat{\vec{x}}, \widehat{\vec{v}}\right)$ is any arbitrary point along the collisionless trajectory passing through $(\vec{x}, \vec{v})$, and $\left(\widehat{\vec{x}}_{st}, \widehat{\vec{v}}_{st}\right)$ is the starting point of the trajectory.

For the spherically symmetric case considered here, the collisionless trajectories are well known to be characterized by the two constants of motion W (total particle energy) and J (angular momentum). The total energy can be rewritten in the form $W(r, v_r, J) = m^s v_r^2/2 + U^s(r, J)$, where $U^s(r, J) = Z^s eV(r) + J^2/2m^s r^2$ is the "effective potential energy", which governs the radial motion of a particle with angular momentum J. For the particular scenario considered here, the collisionless trajectory passing through a given phase point $(\vec{x}, \vec{v})$ can be of one of the following three types: (i) trajectories entering the PPT region at the "presheath-entrance surface (PSES)" (i.e., at r_{ps}); (ii) trajectories entering the PPT region at the probe surface (i.e., at r_p); and (iii) trajectories confined within the PPT region. In the present case, we consider particles entering at the PSES and at the probe surface, so the trajectories of types (i) and (ii) are relevant while in [3] only type-(i) trajectories were considered. Hence from eq. (2) the ion VDF satisfies $f^i(r, v_r, v_t) = f^i(\hat{r}, \hat{v}_r, \hat{v}_t) = f^i_{ps,in}(\hat{v}_{r,ps}, \hat{v}_{t,ps}) =: \hat{f}^i_{ps,in}$ and, in terms of the constants of motion $f^i(r, v_r, v_t) = g^i_{ps,in}(W, J)$ with $f^i_{ps,in} = g^i_{ps,in}(W, J)$ the incomming-ion VDF. Similarly the emitteds electron VDF reads $f^{e,em}(r, v_r, v_t) = f^{e,em}_p\left(\sqrt{\frac{2}{m^s}\left(W - U^s(r_p, J)\right)}, \frac{J}{m^s r_p}\right) = f^{s,em}_p(W, J)$. Both incomming-particle VDFs are assumed to be known at the boundaries.

Number densities of plasma electrons and ions

As in [3], the **electrostatic potential** $V(r)$ is assumed to be negative at the probe surface and rises monotonically to V_{ps} at the PSES. The **electron density** in the PPT region is approximated by the Boltzmann distribution $n^e(r) = n^e_{ps} e^{eV(r)/kT^e}$, where n^e_{ps} is the electron density at the PSES and T^e is the (constant) electron temperature.

Integrating the ion VDF over velocity space and transforming the integration variables from (v_r, v_t) to (W, J), the **ion density** is found as described in detail in [4, 5]). If in addition we assume (as in [3]) the VDF of ions incident at the PSES to have a monoenergetic and isotropic VDF (i.e., $g^i_{ps,in}(W, J) \rightarrow G_{ps,in}(W) = m^2 n^i_{ps,in} \delta(W - W_{ps,in}) / 2\pi\sqrt{2mW_{ps,in}}$), we finally arrive at the following expression for the ion number density

$$n^i(r) = n^i_{ps,in}\left\{\sqrt{1 - \frac{ZeV(r)}{W_{ps,in}}} + \mathrm{sgn}(r - r_0)\sqrt{1 - \frac{ZeV(r)}{W_{ps,in}} - \frac{|I^i_{tot}|}{I_r}}\right\}, \qquad (3)$$

where r_0 is the "separating radius" (the regions $r < r_0$ and $r > r_0$ are characterized by the absence and the presence of reflected ions, respectively) [4, 5].

Emitted-electron VDF and velocity moments

We assume emission of electrons from the spherical probe surface directed radially outward, i.e., with zero tangential velocity component:

$$f_p^{e,em}(v_r, v_t) := g_p^{e,em}(v_r)\frac{\delta(v_t)}{2\pi v_t} = B\Theta(v_r - v_{1p})\,\Theta(v_{2p} - v_r)\frac{\delta(v_t)}{2\pi v_t}, \tag{4}$$

where the radial part is assumed to be a "waterbag" distribution and the delta function is normalized to 1. The emitted-electron number density, fluid velocity and radial effective temperature at some arbitrary radius r are found to be $n^{e,em}(r) = n_p^{e,em}\frac{r_p^2}{r^2}\frac{(\Delta v_r)(r)}{(\Delta v)_p}$, $u^{e,em}(r) = \frac{v_{r1}(r)+v_{r2}(r)}{2}$ and $T_r^{e,em}(r) = \frac{m^e(\Delta v_r)^2}{12k}$, respectively.

By appropriately combining these emitted-electron velocity moments and introducing **normalized quantities** as defined in [4], we get the normalized emitted-electron density

$$\tilde{n}^{e,em}(\tilde{r}) = \tilde{n}_p^{e,em}\frac{\tilde{r}_p^2}{\tilde{r}^2}\frac{2}{\sqrt{1-\tilde{V}^*(\tilde{r})}\left[\sqrt{1-\alpha_1\varepsilon+\alpha_2\varepsilon^2}+\sqrt{1+\alpha_1\varepsilon+\alpha_2\varepsilon^2}\right]}, \tag{5}$$

where $\varepsilon := \sqrt{\tilde{T}_p^{e,em}}/\tilde{u}_p^{e,em}$, $\tilde{V}^*(r) := \tilde{V}(\tilde{r}) - \tilde{V}_p/\left(\tilde{u}_p^{e,em}\right)^2$, $\alpha_1 := \sqrt{6}/1-\tilde{V}^*(r)$ and $\alpha_2 := 3/\left\{2\left[1-\tilde{V}^*(r)\right]\right\}$.

With the normalized quantities of [4], the normalized form of Poisson's equation reads

$$\begin{aligned}\frac{1}{\tilde{r}^2}\frac{d}{d\tilde{r}}\left[\tilde{r}^2\frac{d\tilde{V}}{d\tilde{r}}\right] &= \frac{1}{2}\left[\sqrt{1+\frac{Z\tilde{V}}{\tilde{W}_{ps,in}}} + \operatorname{sgn}(\tilde{r}-\tilde{r}_0)\sqrt{1+\frac{Z\tilde{V}}{\tilde{W}_{ps,in}} - \frac{\tilde{I}}{\sqrt{\tilde{W}_{ps,in}}\tilde{r}^2}}\right] \\ &\quad -e^{-\tilde{V}} - \frac{\tilde{n}_p^{e,em}}{\sqrt{1+\left|\tilde{V}^*(\tilde{r})\right|}}\frac{\tilde{r}_p^2}{\tilde{r}^2}\left[1-\frac{3}{4}\frac{\tilde{T}_p^{e,em}}{\left(\tilde{u}_p^{e,em}\right)^2}\frac{2+\left|\tilde{V}^*(\tilde{r})\right|}{\left(1+\left|\tilde{V}^*(\tilde{r})\right|\right)^2}\right],\end{aligned} \tag{6}$$

where the last term on RHS is the emitted-electron density in the small emission-temperature approximation (i.e., for $\varepsilon << 1$), which yields the cold-emitted-electron density for $\tilde{T}_p^{e,em} \to 0$. Equation (6) is to be solved for given $\tilde{W}_{ps,in}$, $\tilde{I}$, $\tilde{n}_p^{e,em}$, $\tilde{T}_p^{e,em}$, $\tilde{r}_p^2$. The above definition of $\tilde{r}_0$ implies the two conditions

$$1+\frac{Z\tilde{V}(\tilde{r}_0)}{\tilde{W}_{ps,in}} - \frac{\tilde{I}}{\sqrt{\tilde{W}_{ps,in}}\tilde{r}_0^2} = 0, \quad \left[\frac{d}{d\tilde{r}}\left(1+\frac{Z\tilde{V}(\tilde{r})}{\tilde{W}_{ps,in}} - \frac{\tilde{I}}{\sqrt{\tilde{W}_{ps,in}}\tilde{r}^2}\right)\right]_{\tilde{r}_0} = 0, \tag{7}$$

which for given $\tilde{r}_0$ yield $\tilde{V}(\tilde{r}_0)$ and $\tilde{V}'(\tilde{r}_0)$. The correct value of $\tilde{r}_0$ will be determined by applying as a third condition, the "matching condition" using the analytic-numerical procedure developed in [4].

Calculating the potential profile

Upon setting the RHS of eq. (6) equal to zero and rearranging terms we obtain, independently of the value of $\operatorname{sgn}(r-r_0)$, the quasineutrality condition ("plasma equation")

[5], the solution of which, $\tilde{V}_{qn}^{em}(\tilde{r})$, is shown in fig. 1 for two cases. By using our analytic-numerical procedure [4] we find $\tilde{r}_0$ and $\tilde{r}_{mtch}$ (the radius where the sheath and plasma solutions match in an optimum manner) for the emissive case. Using this value of $\tilde{r}_0$ we calculate the values $\tilde{V}(\tilde{r}_0)$ and $\tilde{V}'(\tilde{r}_0)$ via eqs. (7). Then, using these as initial conditions, we integrate the Poisson's equation (eq. (6)) numerically with $\mathrm{sgn}(\tilde{r}-\tilde{r}_0) = -1$ to obtain the potential curves as shown in fig. 2.

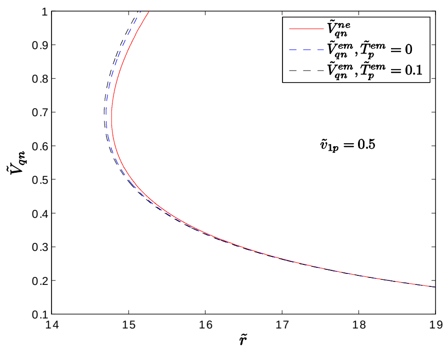

Fig 1: $\tilde{V}_{qn}^{em}$ for $\tilde{n}^{e,em} = 0.2$, $\tilde{I} = 320$, $\tilde{W} = 0.1$ with $\tilde{V}_{qn}^{ne}$ (solid line)

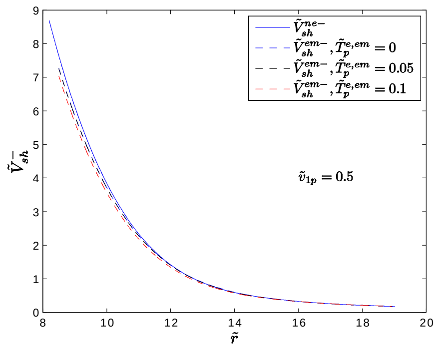

Fig 2: $\tilde{V}_{sh}^{em-}$ for $\tilde{n}^{e,em} = 0.2$, $\tilde{I} = 320$, $\tilde{W} = 0.1$ with $\tilde{V}_{qn}^{-ne}$ (solid line)

Conclusion

We have developed a very general theoretical framework for spherical probes in low-density plasmas, accounting for particles from the probe, from the plasma and trapped. This framework has been specialized to [3], with addition of waterbag-distributed emitted electrons from the probe surface. To our knowledge, this is the first attempt at developing a kinetic theory of the emissive spherical probe. Our first results show that emitted electrons may have a visible effect on the potential profile.

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