

Dynamics of non-spherical dust grain in plasma

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I. Introduction

In recent years the physics of the interactions of dust grains with plasmas was studied intensively both experimentally and theoretically [1-8]. However, practically all theoretical studies (with few exceptions [9-13]) assume that dust grain is made of homogeneous material and have spherical shape. Meanwhile in many cases (e.g. in fusion plasmas [4, 6]) the material dust grain is made of can be inhomogeneous and the shape of the grain can be far from spherical. These features can significantly alter the very basic properties of grain-plasma interactions and result in new phenomena [14, 15]. In this report we follow Ref. 14, 15 and consider an impact of grain shape on grain dynamics in plasma ignoring the effects of the inhomogeneity of dust material.

II. Equations

Consider a grain immersed into homogeneous plasma flowing with velocity \vec{U} . The dynamics of the grain, which we assume to be a rigid body, is completely described by 6D equations dealing with the motion of the centre of mass of the grain and grain spinning (e.g. see Ref. 16)

$$m \frac{d\vec{V}}{dt} = \vec{F}, \quad (1) \quad \frac{d\vec{M}}{dt} = \vec{K}, \quad (2)$$

where m is the mass of the grain, \vec{V} is the velocity of the grain centre of mass, \vec{M} is the grain angular momentum calculated in the frame of the grain centre of mass, $M_\alpha = I_{\alpha\beta} \Omega_\beta$, $I_{\alpha\beta}$ is the inertia tensor, $\vec{\Omega}$ is the angular velocity of the grain, \vec{F} and \vec{K} are, respectively, the force and torque, calculated in the frame of the grain centre of mass, which are imposed on the grain from plasma. In the absence of magnetic field, and the effects associated with asymmetric properties of grain material, which can lead to both forces and torques discussed in the Introduction (e.g. “rocket” force), the direction of vectors \vec{F} and \vec{K} can only depend on the spatial orientation of the grain with respect to the vectors $\vec{W} = \vec{U} - \vec{V}$ and $\vec{\Omega}$. Moreover in this case both \vec{F} and \vec{K} vanish if $\vec{W} = \vec{\Omega} = 0$.

In many applications dust grain spins rather slowly so that $|\Omega| \tau_{ch} \ll 1$, where τ_{ch} is the characteristic charging time. We notice that for example in fusion plasmas $\tau_{ch} \sim 10^{-9}$ s [6] and inequality $|\Omega| \tau_{ch} \ll 1$ holds in a large range of the magnitude of angular velocity. For this case, both charging processes and the forces imposed on the grain by plasma can be considered in a quasi-stationary approximation. Then for subsonic relative speed W we can keep only linear dependence of both \vec{F} and \vec{K} on \vec{W} and $\vec{\Omega}$. Moreover, if the grain shape does not have intrinsic propeller-like properties, the directions of both \vec{F} and \vec{K} reverse with reversing directions of \vec{W} and $\vec{\Omega}$. As a result, under these assumptions, we can write the expressions for \vec{F} and \vec{K} as follows

$$F_\alpha = \Phi_{\alpha\beta}^{(W)} W_\beta + \Phi_{\alpha\beta}^{(\Omega)} \Omega_\beta, \quad (3) \quad K_\alpha = T_{\alpha\beta}^{(W)} W_\beta + T_{\alpha\beta}^{(\Omega)} \Omega_\beta, \quad (4)$$

where the tensors $\Phi_{\alpha\beta}$ and $T_{\alpha\beta}$ are determined only by the shape of the grain and plasma parameters. The physical meaning of the terms in Eq. () is rather transparent. In Eq. (3) the first term describes the generalized drag force, which for non-spherical grain shape is not necessarily aligned with vector \vec{W} , the second term describes the force caused by the interactions of the spinning of non-spherical grain with plasma. In Eq. (4) the first term describes the torque imposed on non-spherical grain by relative velocity of plasma and grain, and, finally, the second term describes the relaxation of grain spinning due to grain-plasma interactions.

In general case the calculation of the tensors $\Phi_{\alpha\beta}$ and $T_{\alpha\beta}$ is only possible numerically. However, for the grains having rather simple shapes the structure of these tensors can be found just from geometrical arguments. One of such examples will be considered in next section.

II. Dynamics of rotationally symmetrical grain in plasma

Here we consider the main features of the dynamics of the grain having rotationally symmetrical, around some axis, properties (e.g. mass density, shape, etc.). In this case spatial orientation of the grain can be characterized by unit vector \vec{D} , which is directed along symmetry axis (see Fig. 1).

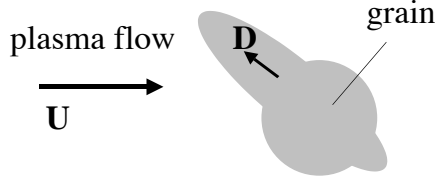


Fig. 1.

We note that \vec{F} , \vec{W} , and \vec{D} are the vectors, while \vec{M} , \vec{K} and $\vec{\Omega}$ are the pseudo-vectors. Therefore, $\Phi_{\alpha\beta}^{(W)}$ and $T_{\alpha\beta}^{(\Omega)}$ should be tensors, while $\Phi_{\alpha\beta}^{(\Omega)}$ and $T_{\alpha\beta}^{(W)}$ should be pseudo-tensors. However, rigid body with rotationally symmetrical properties can only be characterized by the second-order tensors, which in the frame of principal axes of inertia have diagonal form with two equal components associated with the axes perpendicular to the axis of rotational symmetry. As a result, they can be expressed in terms of tensors $\delta_{\alpha\beta}$ and $D_\alpha D_\beta$, where $\delta_{\alpha\beta}$ is the Kronecker delta. On the other hand, rigid body with rotational symmetry can only be characterized by second-order pseudo-tensor $\epsilon_{\alpha\beta\gamma} D_\gamma$, where $\epsilon_{\alpha\beta\gamma}$ is the Levi-Civita symbol. Therefore the tensors $\Phi_{\alpha\beta}^{(W)}$, $T_{\alpha\beta}^{(\Omega)}$, $\Phi_{\alpha\beta}^{(\Omega)}$, and $T_{\alpha\beta}^{(W)}$ can be expressed as follows

$$\Phi_{\alpha\beta}^{(W)} = \Phi_1^{(W)} \delta_{\alpha\beta} + \Phi_2^{(W)} D_\alpha D_\beta, \quad (5)$$

$$T_{\alpha\beta}^{(\Omega)} = T_1^{(\Omega)} \delta_{\alpha\beta} + T_2^{(\Omega)} D_\alpha D_\beta, \quad (6)$$

$$\Phi_{\alpha\beta}^{(\Omega)} = \Phi^{(\Omega)} \epsilon_{\alpha\beta\gamma} D_\gamma, \quad (7)$$

$$T_{\alpha\beta}^{(W)} = T^{(W)} \epsilon_{\alpha\beta\gamma} D_\gamma, \quad (8)$$

where the scalars $\Phi_1^{(W)}$, $\Phi_2^{(W)}$, $T_1^{(\Omega)}$, $T_2^{(\Omega)}$, $\Phi^{(\Omega)}$, and $T^{(W)}$ are determined by particular properties of the grain and plasma parameters.

As a result, the equations (1, 2) can be written as follows [14, 15]

$$m \frac{d\vec{V}}{dt} = \Phi_1^{(W)} \vec{W} + \Phi_2^{(W)} \vec{D} (\vec{D} \cdot \vec{W}) + \Phi^{(\Omega)} (\vec{\Omega} \times \vec{D}), \quad (9)$$

$$\frac{d\vec{M}}{dt} = T^{(W)} (\vec{W} \times \vec{D}) + T_1^{(\Omega)} \vec{\Omega} + T_2^{(\Omega)} \vec{D} (\vec{D} \cdot \vec{\Omega}). \quad (10)$$

Although to calculate the scalars $\Phi_1^{(W)}$, $\Phi_2^{(W)}$, $T_1^{(\Omega)}$, $T_2^{(\Omega)}$, $\Phi^{(\Omega)}$, and $T^{(W)}$ in Eq. (9, 10) for a non-spherical grains is still possible only numerically, we, nevertheless, can make some evaluation of their magnitudes. First, we take into account that the magnitude of the force on the grain from plasma can be estimated as, $F_{\text{est}} \sim \xi_F \pi R^2 n T W / C_s \equiv \hat{F} W$ (e.g. see Ref. 15), where n and T are the plasma density and temperature, R is the effective size of the grain, C_s is the plasma sound speed, and $\xi_F \sim 10$ is the form-factor depending on the ratio of the Debye length to the grain size. Second, due to grain spinning, there is differential speed of different grain parts and plasma with the magnitude $\delta W \sim R \Omega$, which cause corresponding forces. Therefore, assuming that the grain centre of mass is displaced from the centre of force at the distance $\sim R$ we find: $\Phi_1^{(W)} \sim \Phi_2^{(W)} \sim \hat{F}$, $\Phi^{(\Omega)} \sim \hat{F} R$, $T^{(W)} \sim \hat{F} R$, and $T_1^{(\Omega)} \sim T_2^{(\Omega)} \sim \hat{F} R^2$. Taking these estimates into account we find the effective slowing down frequency, ν_Ω , of grain spinning due to plasma-grain interactions, which is described by the second and third terms on the right-hand side of Eq. (10) can be estimated as

$$v_{\Omega} \sim T_1^{(\Omega)} / I \sim T_2^{(\Omega)} / I \sim \hat{F} R^2 / I \sim \xi_{Fn} \sqrt{TM_i} / R\rho, \quad (11)$$

where ρ is the mass density of grain material, $I \sim \rho R^5$ is the grain moment of inertia, and M_i is the plasma ion mass. For a micron-size grain of mass density $\rho \sim 2 \text{ g/cm}^3$ and plasma parameters typical for the edge of fusion devices $n \sim 3 \times 10^{13} \text{ cm}^{-3}$, $T \sim 10 \text{ eV}$ (e.g. see Ref. 4, 6 and the reference therein) we find $v_{\Omega} \sim 10 \text{ s}^{-1}$. Thus, in fusion plasmas, slowing down of grain spinning occurs on the time-scale much larger than the life-time of the grain $\tau_{lt} \sim 10^{-2} \text{ s}$ (e.g. see Ref. 20) and, therefore, can be neglected. On the other hand, rotation of the grain over angle $\sim \pi$, caused by the torque associated with plasma flow, requires time

$$\tau_{\pi} \sim \left(\frac{\pi I}{\hat{F} R W} \right)^{1/2} \sim \left(\frac{\pi \rho R^2}{\xi_{Fn} \sqrt{TM_i} W} \right)^{1/2}. \quad (12)$$

For the same plasma parameters and grain size as before and rather typical speed $W \sim 0.1 \times C_s$, from Eq. (12) we find $\tau_{\pi} \sim 10^{-5} \text{ s} \ll \tau_{lt}$. So that the impact of torque should be taken into account.

In fusion plasma the grain speed reached few hundred meters per second [6], which usually is much smaller than the speed of plasma flow $U \gtrsim 0.1 \times C_s \sim 3 \times 10^5 \text{ cm/s}$. Therefore in Eq. (9, 10) we can approximate $\vec{W} \approx \vec{U}$. As a result, the equations for grain centre mass velocity and angular momentum in fusion plasma become decoupled

$$m \frac{d\vec{V}}{dt} = \Phi_1^{(W)} \vec{U} + \Phi_2^{(W)} \vec{D} (\vec{D} \cdot \vec{U}) + \Phi^{(\Omega)} (\vec{\Omega} \times \vec{D}), \quad (13)$$

$$\frac{d\vec{M}}{dt} = T^{(W)} (\vec{U} \times \vec{D}). \quad (14)$$

We note that in Eq. (14) we neglected the terms describing slowing down of grain spinning due to grain-plasma interactions. Directing unit vector \vec{e}_Z of the coordinate Z along plasma flow velocity \vec{U} we find that Eq. (14) becomes identical to the equations describing the motion of symmetrical top with fixed lowest point in effective the gravity field such that $m\ell\vec{g} = -T^{(W)}\vec{U}$, where ℓ is the distance from the top's lowest point to the top's centre of mass (e.g. see Ref. 16). However, since Eq. (14) is written in the frame of the grain center of mass and not in the frame of top's lowest point, as in Ref. 16, the moment of inertia $I'_1 = I_1 + m\ell^2$ in the calculations in Ref. 16 should be replaced with I_1 , which is the principal moment of inertia in the frame of the grain center of mass calculated with respect to the axis perpendicular to the axis of rotational symmetry of the grain.

From the solution of the problem of the motion of heavy symmetric top [16] we find that the vector \vec{D} , characterizing the grain orientation, will experience both precession around vector \vec{U} and nutation. For a simple case, corresponding to so-called “fast” top, the precession angular velocity Ω_{pr} is much larger than nutation angular velocity and equals to

$$\vec{\Omega}_{pr} = T^{(W)} \vec{U} \cos \alpha / M, \quad (15)$$

where M is the magnitude of total angular momentum and α is the angle between grain angular momentum and the axis of grain rotational symmetry, which determines the “nutation cone”. For $\alpha \ll 1$, averaging Eq. (13) over fast nutation and using Eq. (15) we find

$$m \frac{d\vec{V}}{dt} = \Phi_1^{(W)} \vec{U} + \Phi_2^{(W)} \vec{D} (\vec{D} \cdot \vec{U}) + \frac{\Phi^{(\Omega)} T^{(W)}}{M} (\vec{U} \times \vec{D}), \quad (16)$$

where the vector \vec{D} precesses around vector \vec{U} with angular velocity Ω_{pr} . As we see from Eq. (16) both second and third terms on the right hand side will cause the oscillation of grain trajectory in the direction perpendicular to the plasma velocity.

To estimate the amplitude of the trajectory oscillation and relative contributions to it of second and third terms in Eq. (16) we note that the “fast” top approximation is valid when Ω_{pr} is much smaller than the angular velocity of grain spinning around the axis of symmetry, Ω_0 . Then estimating M as $I\Omega_0$, using the expression (15), and the estimates for $\Phi_2^{(W)}$, $\Phi^{(\Omega)}$, $T^{(W)}$, and \hat{F} we find that the “fast” top approximation is valid for

$$\Omega_0 \gg \left(T^{(W)} U / I \right)^{1/2} \equiv \Omega_U \sim \left(\frac{\pi \xi_F n M_i}{\rho} \frac{U C_s}{R^2} \right)^{1/2}, \quad (17)$$

and the second term dominates for plasma speed such that $U/C_s \gg \xi_F n M_i / \rho \sim 10^{-10}$, which, in practice, always occurs (we notice that for plasma and grain parameters discussed before $\Omega_U \sim 1 \text{ s}^{-1}$). As a result, the magnitude of the trajectory oscillation, Δ , can be estimated as

$$\Delta \sim \frac{\Phi^{(W)}}{mU} \left(\frac{I\Omega_0}{T^{(W)}} \right)^2 \sim \frac{\rho}{\pi \xi_F n M_i} \frac{\Omega_0^2 R^2}{U C_s}. \quad (18)$$

From Eq. (18) it follows that $\Delta \gtrsim 1 \text{ cm}$, which can be observed in fusion plasmas with fast cameras, can be reached for $\Omega_0 \gtrsim 3 \times 10^4 \text{ s}^{-1}$, which, however, is well within the limits of approximation made while deriving the equations (13, 14).

IV. Conclusions

We perform the analysis of the dynamics of non-spherical dust grain in plasma. Starting with very general principals we derive the equations of motion of non-spherical grain (which, however, does not have propeller-like properties, does not exhibit “rocket” force effects, etc.) immersed in a subsonic plasma flow assuming that the grain spins relatively slow and spinning does not alter grain charging processes. We find that in this case the dynamics of the grain is determined by some tensors, which depend on plasma parameters and grain properties (including shape and mass density distribution). For the grain with rotationally symmetric properties we find the structure and evaluate the magnitude of the components of these tensors. We demonstrate that the dynamics of rotationally symmetric grain spinning is equivalent to the motion of symmetric top in the gravity field. We show that the precession of the grain axis can result in significant oscillations of grain trajectory in the direction normal to plasma velocity.

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