

# The radiative signature of Fermi acceleration at relativistic shocks

Brian Reville, John G. Kirk

*Max-Planck-Institut für Kernphysik, Heidelberg, Germany*

## Abstract

The first-order Fermi process at relativistic shocks requires the generation of strong turbulence in the vicinity of the shock front. Recent particle in cell simulations have demonstrated that this mechanism can be studied self-consistently at weakly magnetised shocks. In this case, strong electromagnetic fields are generated on length-scales comparable to the plasma skin-depth. The resulting diffusion and radiation spectrum in this short-wavelength turbulence can differ significantly from the standard picture. A connection between these two processes can be found which predicts an upper limit to the maximum photon energy that can be produced via synchrotron type radiation at a relativistic shock.

## Introduction

Particle acceleration at relativistic shocks is thought to proceed via the first order Fermi acceleration mechanism (see [2] and references therein). Recent particle in cell (PIC) simulations have demonstrated that the process can indeed occur, under favourable conditions [6, 4, 5]. These simulations are *ab initio* in the sense that the process is reproduced by evolving the particles and fields according to Maxwell's equations and the Lorentz force.

The maximum energy to which particles can be accelerated is determined by several factors. Ultimately, the acceleration ceases when the radiative energy losses that are inevitably associated with the scattering process overwhelm the energy gains obtained upon crossing the shock.

Such effects are inaccessible to current PIC simulations, which do not include radiation reaction. However, for a given model of pitch angle diffusion, it is possible to derive an approximate upper limit for the maximum energy, which depends on the details of the scattering process. Current simulations suggest small angle scattering is a necessary condition for Fermi acceleration to occur. The presence of a large scale field generally acts to suppress the acceleration as particles are effectively transported into the far downstream region.

## Particle transport and acceleration

For the purpose of estimating radiative signatures of accelerated particles, it is convenient to characterise the turbulent fluctuations in terms of their 'strength' parameter  $a$ , defined as the ratio of their length scale to the length defined by their amplitude  $a = \lambda e |\delta B| / mc^2$ . According to this strength parameter, the transport can be divided into two distinct regimes, which we call

‘ballistic’ ( $a < \gamma$ ) and ‘helical’ ( $a > \gamma$ ). In ballistic transport, the scattering mean free path is shorter than the gyroradius in the local field. On the other hand, in the helical transport regime, gyro motion is only slightly perturbed. Particles have sufficient time to gyrate about the field while their pitch angles and guiding-centre positions diffuse.

*Ballistic transport regime:*

At a relativistic shock, a particle remains in the upstream medium until it has been deflected, on average, through an angle of  $1/\bar{\gamma}$  in the upstream rest frame. In the downstream the particle must deflect through a much larger angle  $\sim \pi/2$ . A turbulent fluctuation of strength parameter  $a$ , deflects a particle of Lorentz factor  $\gamma$  through an angle  $a/\gamma$ . Provided this angle is small, the diffusion coefficient is simply  $\mathcal{D}_\theta = a^2 v_{sc}/\gamma^2$ , where  $v_{sc}$  is the mean scattering frequency. The average number of scatterings in the upstream medium between shock encounters is therefore  $N_{scatt,u} \approx (\gamma/a_u \bar{\gamma})^2$ , and likewise in the downstream  $N_{scatt,d} \approx (\gamma/a_d)^2$ .

Assuming similar size strength parameters either side of the shock, radiative losses in the downstream dominate. For kinematic reasons, the average energy gain per cycle is roughly a factor of two [1], so that the acceleration process will saturate when the energy lost in the upstream medium is roughly  $\gamma m c^2$ . This implies that in the absence of inverse Compton cooling, the energy is limited to [3]

$$\gamma < \left( \frac{3mc^2 \lambda_d}{2e^2} \right)^{1/3} \quad (1)$$

*Helical transport regime:*

In the helical transport regime, energy losses are important at all points along a trajectory. In the Bohm limit, the maximum Lorentz factor is [1]:

$$\gamma < \left( \frac{3m^2 c^3}{2e^3 B} \right)^{1/2} \quad (2)$$

At magnetised, relativistic shocks, particle acceleration by the first-order Fermi mechanism is less plausible, since it relies on strong cross-field diffusion. However, if the process does operate, particles can move from the helical to the ballistic regime, as their Lorentz factor increases. There exists a critical strength parameter  $a_{crit}$  such that when  $a = a_{crit}$  the maximum Lorentz factor  $\gamma_{max}$  is achieved just at the point at which the transport changes character from helical to ballistic, i.e.

$$a_{crit} = \left( \frac{3mc^2 \lambda_d}{2e^2} \right)^{1/3} \quad (3)$$

If  $a > a_{crit}$ , all particles remain in the the helical regime. On the other hand, if  $a < a_{crit}$ , particles of the maximum Lorentz factor undergo ballistic transport, but lower energy particles may be in the helical regime.

Combining the constraints from the ballistic regime (1) and the helical regime (2) gives:

$$\gamma_{\max} = \begin{cases} a_{\text{crit}} & \text{for } a < a_{\text{crit}} \\ a_{\text{crit}} \sqrt{a_{\text{crit}}/a} & \text{for } a > a_{\text{crit}} \end{cases} \quad (4)$$

### Radiative signatures

To estimate the maximum photon energy that can be produced by the highest energy particles, we consider the radiation emitted when a particle is scattered by a single fluctuation of strength parameter  $a$ . The character of the emission depends crucially on the “formation” or “coherence” length of the radiated photons. If  $a > 1$ , the formation length is roughly  $\lambda_{\text{coh}} \approx mc^2/eB$ . This is much smaller than the wavelength of the turbulence, so that the individual photons are created in regions in which the field is almost constant and homogeneous. The result is synchrotron radiation, with the emissivity defined by the local value of the field. The emission extends up to the roll-over frequency of the highest energy electrons:

$$\omega_{\max} \approx 0.5a\gamma^2 c/\lambda \quad \text{for } a > 1 \quad (5)$$

If, on the other hand,  $a < 1$ , the particle is deflected through an angle that is small compared to  $1/\gamma$ . In this case, the coherence length is no longer limited by deflection, but is given by the distance moved by the particle in the lab. frame during the time it takes for the photon to move one wavelength ahead of the particle:  $\lambda_{\text{coh}} \approx \gamma^2 c/\omega$ . The maximum frequency is roughly

$$\omega_{\max} \approx 0.5\gamma_{\max}^2 c/\lambda \quad \text{for } a < 1 \quad (6)$$

In each case, most of the power radiated by an individual electron emerges within a decade of the roll-over frequency that corresponds to its Lorentz factor. Therefore, the spectrum radiated by a power-law distribution of electrons, with differential number density  $dn/d\gamma \propto \gamma^{-p}$ , for both the synchrotron and jitter cases, reproduces the standard power-law spectrum at frequencies between the roll-over frequency of the highest and lowest energy electrons:  $dL/d\omega \propto \omega^{-(p-1)/2}$ .

Combining the limit on the Lorentz factor (1) with the expressions for the roll-over frequency (5) and (6), one finds for the maximum frequency that can be radiated by particles accelerated at a relativistic shock front:

$$\frac{\hbar\omega_{\max}}{mc^2} = \begin{cases} (\alpha_f a_{\text{crit}})^{-1} & a < 1 \\ a(\alpha_f a_{\text{crit}})^{-1} & 1 < a < a_{\text{crit}} \\ \alpha_f^{-1} & a > a_{\text{crit}} \end{cases} \quad (7)$$

where  $\alpha_f = e^2/\hbar c$  is the fine structure constant. The results are summarised in Fig. 1.

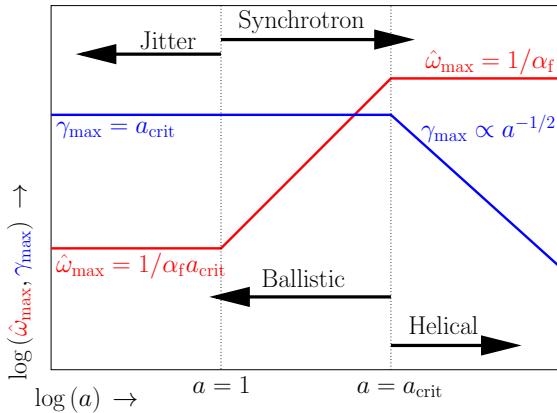


Figure 1: The maximum electron Lorentz factor  $\gamma_{\max}$  and the maximum photon energy  $\hat{\omega}_{\max} = \hbar\omega_{\max}/mc^2$  radiated when scattered by magnetic fluctuations of strength  $a$  at a relativistic shock. The jitter/synchrotron regimes are separated by the vertical  $a = 1$  line; the ballistic/helical transport regimes by the  $a = a_{\text{crit}}$  line.

## Discussion

As mentioned in the introduction, PIC simulations exhibiting Fermi acceleration require the presence of small scale turbulence. For sufficiently weak magnetisation, the field is generated via the Weibel instability. This generates magnetic structures of size on the order of the plasma skin-depth  $\lambda = \ell_w c / \omega_p$  where  $\omega_p$  is the local plasma frequency, and  $\ell_w \sim 10$ .

For magnetised shocks mediated by the synchrotron maser instability, the characteristic length in the downstream plasma  $\lambda_{s,d}$  is dictated by the requirement that the incoming particles be significantly deflected, giving  $\lambda_s = \ell_s \bar{\gamma} mc^2 / eB_d$  with  $\ell_s \sim 1$ .

It follows that for a given shock, the critical strength parameter is

$$a_{\text{crit}} = \begin{cases} 1.4 \times 10^6 \ell_w^{1/3} \bar{\gamma}^{1/6} (n/1 \text{ cm}^3)^{-1/6} & \text{Weibel} \\ 2.1 \times 10^4 \ell_s^{1/3} \bar{\gamma}^{1/3} (B/1 \text{ mG})^{-1/3} & \text{Synch. Maser} \end{cases} \quad (8)$$

Current PIC simulations suggest strength parameters  $\sim \bar{\gamma}$  [5], implying a maximum photon energy of  $\sim \bar{\gamma}/\alpha_f a_{\text{crit}}$ . Even for the extremely relativistic shocks assumed in GRBs this does not reach gamma-ray energies, and an alternative radiation mechanism, such as Compton scattering of the soft jitter or synchrotron photons to produce gamma-rays, must be considered.

## References

- [1] Achterberg, A., Gallant, Y. A., Kirk, J. G., & Guthmann, A. W. 2001, MNRAS, 328, 393
- [2] Kirk, J. G., & Duffy, P. 1999, Journal of Physics G Nuclear Physics, 25, 163
- [3] Kirk, J. G., & Reville, B., 2010, ApJL, 710, L16
- [4] Martins, S. F., Fonseca, R. A., Silva, L. O., & Mori, W. B. 2009, ApJL, 695, L189
- [5] Sironi, L., & Spitkovsky, A. 2009, ApJ, 698, 1523
- [6] Spitkovsky, A. 2008, ApJL, 682, L5