Turbulent generation and transport of toroidal angular momentum

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The issue of plasma rotation is crucial for current and future fusion devices as it tends to stabilize certain magnetohydrodynamic modes, and appears to play a role in sustaining transport barriers via turbulence saturation by the rotation shear. It has been observed in existing tokamaks that *intrinsic core rotation* is generated even in the absence of external torque. This is particularly relevant for ITER, where injected momentum will be very small. The radial transport of angular momentum is of great importance as it pertains not only to intrinsic rotation but also allows the inwards propagation of rotation initially *external to the core*, either injected or originating from the large scale flows experimentally observed in the scrape-off layer.

We investigate the turbulent generation and transport of toroidal angular momentum using the full-f, global gyrokinetic code GYSELA in the flux-driven regime[1]. We show that the gyrokinetic equations, as formulated by Brizard and Hahm[2], lead to an exact conservation law of toroidal momentum. This equation is conservative, i.e. the force derives from a tensor, which is the sum of a stress tensor and the electric part of a Maxwell stress. We also present a numerical test of this local conservation law.

Local equation for toroidal angular momentum

We consider the gyro-averaged guiding-center distribution function $\bar{F}(\chi,\theta,\varphi,v_{G\|},\mu)$ where χ is the opposite of the poloidal magnetic flux, θ and φ are the poloidal and toroidal angles, $v_{G\|}$ is the parallel velocity and μ is the magnetic moment, which is an adiabatic invariant. The equilibrium magnetic field is axisymmetric: $\mathbf{B} = I\nabla\varphi + \nabla\varphi\times\nabla\chi$. The gyrokinetic equation for each species s can be written in its conservative form[2], valid at order one in the small parameter ρ_*: $\partial_t\bar{F} + \frac{1}{B_{\|}^*}\nabla_{\mathbf{z}}\cdot\left(\dot{\mathbf{z}}B_{\|}^*\bar{F}\right) = \mathscr{C}(\bar{F})$ where $\mathbf{z} = (\chi,\theta,\varphi,v_{G\|},\mu)$ and $\dot{\mathbf{z}} = \mathrm{d}_t\mathbf{z}$. $B_{\|}^* = B + mv_{G\|}/e\mathbf{b}\cdot(\nabla\times\mathbf{b})$ is the Jacobian of the gyrocenter transformation. The collision operator $\mathscr{C}(\bar{F})$ must satisfy some basic properties: relaxation towards a Maxwellian (Boltzmann H-theorem), and conservation of particles, momentum and energy. We restrict the analysis to electrostatic turbulence (the extension to electromagnetic fluctuations does not raise any problem). Self-consistency is then ensured by a quasi-neutrality equation which reads

$$-\sum_s \nabla\cdot\left\{\frac{n_{eq}m_s}{B^2}\nabla_\perp\phi\right\} = \sum_s e_s\int 2\pi B_{\|}^*\mathrm{d}\mu\mathrm{d}v_{G\|}J\cdot\bar{F} \qquad (1)$$

where J is the gyro-averaging operator, and n_{eq} is the equilibrium density of guiding-centers. In the simplest version, electrons are adiabatic.

In the gyrokinetic ordering, the toroidal canonical momentum is $P_\varphi = m_s u_\varphi - e\chi$ where we define $u_\varphi = \frac{I}{B} v_{G\|}$. If the system is axisymmetric, P_φ is an exact motion invariant. Otherwise, for instance when turbulence breaks axisymmetry, $\mathrm{d}_t P_\varphi = -e\partial_\varphi \bar{\phi}$. Considering the expression of P_φ, it is consistent to define the local angular momentum as $\mathscr{L}_\varphi = \sum_s m_s \int \mathrm{d}\tau^* u_\varphi \bar{F}$ where $\int \mathrm{d}\tau^*$ corresponds to the integration over all phase-space variables other than χ. From the conservative Fokker-Planck equation we obtain

$$\partial_t \mathscr{L}_\varphi + \partial_\chi \Pi^\chi_\varphi + \partial_\chi \mathrm{T}^\chi_\varphi = \mathscr{J} \tag{2}$$

where

$$\Pi^\chi_\varphi = \sum_s m_s \int \mathrm{d}\tau^* \bar{F} u_\varphi v^\chi_G \tag{3a}$$

$$\mathrm{T}^\chi_\varphi = \sum_s e_s \int^\chi d\chi \int \mathrm{d}\tau^* \bar{F} \partial_\varphi \bar{\phi} \tag{3b}$$

$$\mathscr{J} = \sum_s e_s \int \mathrm{d}\tau^* v^\chi_G \bar{F} \tag{3c}$$

where $v^\chi_G = \dot{\mathbf{z}} \cdot \nabla\chi$ is the guiding-center toroidal velocity in conventional contravariant notations, which contains both the $E \times B$ drift and the magnetic drifts. Eq.(2) is an *exact* equation of toroidal angular momentum conservation, i.e. it is not dependent on any assumption on ordering once the gyrokinetic and electro-neutrality equations are given. It can be shown that Eq.(2) reduces to the one given by Parra and Catto[3] assuming the ordering considered in their derivation.

The tensor Π^χ_φ is the off-diagonal $(\varphi\chi)$ component of the conventional Reynolds stress. The interpretation of T^χ_φ is less straightforward. An explicit expression can be found by using the quasi-neutrality equation (1) and the low wavenumber expression of the gyroaverage operator $J \simeq 1 + \frac{1}{2}\nabla \cdot \left(\frac{m_s \mu}{e^2 B} \nabla_\perp\right)$. One then finds the following expression

$$\mathrm{T}^\chi_\varphi = -\sum_s \int \frac{\mathrm{d}\theta \mathrm{d}\varphi}{\mathbf{B}\cdot\nabla\theta} \left\{ \frac{1}{2}\frac{m_s}{e_s B^2} \left(E^\chi \mathscr{P}_\varphi + \mathscr{P}^\chi E_\varphi\right) + \frac{n_{eq} m_s}{B^2} E^\chi E_\varphi \right\} \tag{4}$$

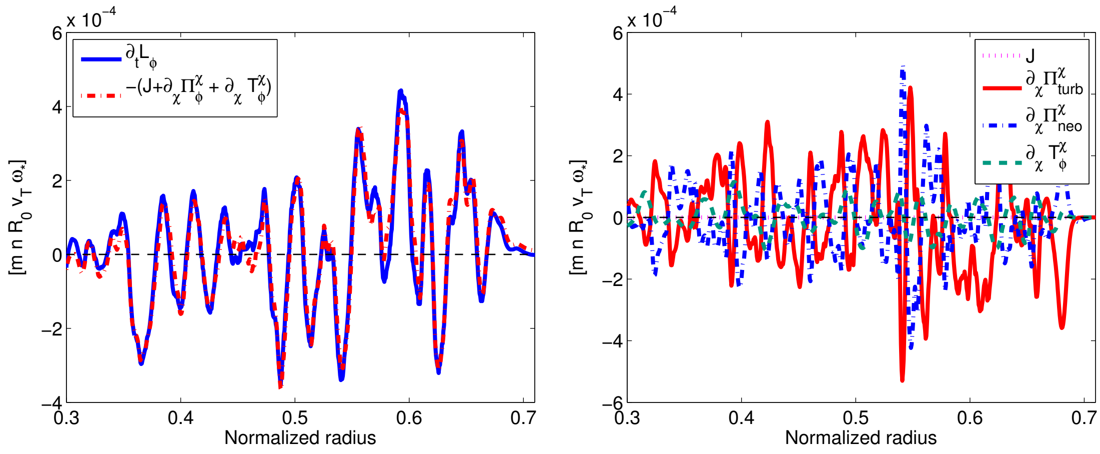
where conventional covariant notations are used, $\mathbf{E} = -\nabla\phi$ is the electric field and $\mathscr{P} = -B\nabla(\bar{P}_\perp / B)$ where $\bar{P}_\perp$ is the gyrocenter perpendicular pressure (the average of μB over the distribution $\bar{F}$). This expression is close to the expression given by McDevitt et al.[4] Considering the limit case $\bar{P}_\perp \approx 0$ (cold plasma limit), using an analogy with a dielectric medium one can identify the term T^χ_φ as the off-diagonal component of the electric part of the Maxwell stress tensor built with the polarized field. In other words it is obtained from the vacuum Maxwell stress by replacing the vacuum permittivity ε_0 by $n_{eq}m/B^2$. The pressure terms account for finite Larmor radius effects, and lead to generalization of the Maxwell stress in a hot plasma.

The right-hand side of Eq.(2) is crucial to the conservation of angular momentum, as any departure from ambipolarity would lead to unphysical sources of momentum. However, all gyrokinetic simulations must verify a charge conservation law that is derived from the gyrokinetic equation and reads $\partial_t \bar{\rho} + \partial_\chi \mathscr{J} = 0$ where $\bar{\rho} = \sum_s e_s \int d\tau^* \bar{F}$. In other words, since boundary conditions impose a vanishing current at the edges, one must have $\mathscr{J} = 0$ in steady-state regime.

Numerical test of this balance

In this section, we present a numerical test of equation (2) for the local balance of toroidal angular momentum using the full-f, gyrokinetic code GYSELA in the flux-driven regime[1]. The model equations implemented correspond to the standard gyrokinetic formulation by Brizard and Hahm[2]. The collision operator is a simplified Lenard-Bernstein[5] operator for ion-ion collisions in the parallel direction. This operator ensures energy and momentum conservation by relaxing the distribution function towards a shifted Maxwellian distribution[6] and is sufficient to recover neoclassical theory[7].

In Figure 1 we present the main results for a simulation with the adimensional parameters $\rho_* = 1/512 \sim \rho_*^{\text{ITER}}$ and $\nu_* = 0.1$. The 5D grid in $(r, \theta, \varphi, v_{G\parallel}, \mu)$ for this case, which simulates a quarter-torus, is $(1024, 1024, 128, 128, 16)$, i.e. approximately 2.7 10^{11} grid points. The simulation required approximately 3 million CPU hours, running on 8 192 cores. The results are time-



(a) (b)

Figure 1: Nonlinear simulation with $\rho_* = 1/512$ and $\nu_* = 0.1$. Figure (a) shows the balance of toroidal angular momentum (equation (2)) while figure (b) identifies the different contributions to this balance

averaged over a significant part of the non-linear saturation phase (approximately $10^4 \omega_c^{-1}$). The quantities are normalized to $m_i n R_0 v_T \omega_*$ where R_0 is the major radius, v_T is the thermal velocity and ω_* is the diamagnetic frequency. Fig. 1(a) compares the time derivative of the toroidal angular momentum to the sum of the other terms in Eq. (2), showing a very good agreement. Fig. 1(b) compares the contributions of the different terms in Eq. (2) to this balance, separating the Reynolds stress (Π_φ^χ) into its turbulent and neoclassical components, i.e. splitting v_G^χ in expression (3a) into $E\times B$ and magnetic drifts. As expected, contributions from the radial currents of guiding-centers ($\mathscr{J}$) are negligible in the steady-state regime (less than 0.1% of the other terms). The balance of toroidal angular momentum results mainly from a competition between turbulent and neoclassical stresses, with a significant contribution from the polarization stress.

Conclusion

The mean toroidal flows in gyrokinetics can be described by a local toroidal momentum conservation equation: $\partial_t \mathscr{L}_\varphi + \partial_\chi \Pi_\varphi^\chi + \partial_\chi \mathrm{T}_\varphi^\chi = \mathscr{J}$. Because of charge conservation and boundary conditions, the radial current of guiding-centers ($\mathscr{J}$) must be vanishing in the steady-state regime. Thus, the force in equation (2) derives from a tensor, which contains the off-diagonal component of the Reynolds stress (Π_φ^χ) and the off-diagonal component of the Maxwell stress built with the polarization field (T_φ^χ), similar to the result of McDevitt et al.[4]. We recover this local conservation equation numerically with excellent agreement using the gyrokinetic full-f code GYSELA and verify that no significant departure from ambipolarity is observed (i.e. $\mathscr{J} = 0$). Thus gyrokinetic codes are capable of correctly predicting the mean toroidal flows. The complete description of mean flows in gyrokinetics also requires the computation of their projections on the radial direction, which is the so-called force balance equation, and on the poloidal direction with a conservation equation similar to Eq. (2).

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