# Impact of the geometry of resonant magnetic perturbations on the control of transport barrier relaxations in fusion plasmas

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Transport barriers in tokamak plasmas are key ingredients of improved confinement regimes. These barriers are thin layers in which turbulent transport of heat and matter is strongly reduced. At the plasma edge, the barrier typically is not stable but exhibits relaxation oscillations associated with high flux peaks [1]. Such barrier relaxations have been studied by means of 3D turbulence simulations [2, 3] and the possible control of these relaxations by externally induced resonant magnetic perturbations has been investigated [4, 5]. In this framework, it has been shown recently that a single harmonic resonant magnetic perturbation localized at the barrier position can also lead to a stabilization of the relaxations [6]. However, in this geometry, the confinement is always degraded.

As shown in these turbulence simulations, a key element for the stabilization of barrier relaxations is the convective energy flux associated with the non-axisymmetric plasma equilibrium in presence of magnetic islands. In fact, when a magnetic island chain is externally imposed inside the plasma, the modified equilibrium pressure and electric potential give rise to a convective flux that plays an important role in the local erosion of the transport barrier and the stabilization of its relaxations [4, 5, 6]. The magnetic island chain can either result from a single harmonic resonant perturbation [6] or from a multiple harmonic resonant perturbation leading to a complex geometry with stochastic regions and residual islands [4, 5]. The aim of the present work is to get further insight into the non axisymmetric plasma equilibrium and the associated convective transport in the presence of a magnetic island. The turbulence model studied here consists of normalized equations for the plasma pressure $p$ and the electric potential $\phi$ [3],

$$\partial_t \nabla_\perp^2 \phi + \left\{\phi, \nabla_\perp^2 \phi\right\} = -\nabla_\parallel^2 \phi - \mathbf{G}\, p + \nu \nabla_\perp^4 \phi + \mu \nabla_\perp^2 \left(\phi_{\mathrm{imp}} - \bar{\phi}\right) , \quad (1)$$

$$\partial_t p + \{\phi, p\} = \delta_c \mathbf{G}\, \phi + \chi_\parallel \nabla_\parallel^2 p + \chi_\perp \nabla_\perp^2 p + S , \quad (2)$$

In toroidal coordinates $(r, \theta, \varphi)$ and in a slab geometry $(x, y, z)$ in the vicinity of a reference surface $r = r_0$, i.e. $x = (r - r_0)/\xi_{\mathrm{bal}}$, $y = r_0\theta/\xi_{\mathrm{bal}}$, $z = R_0\varphi/L_s$, the normalized operators are

$$\nabla_\parallel = \partial_z + \left(\frac{\zeta}{q_0} - x\right)\partial_y - \{\psi_{\mathrm{RMP}}, \cdot\} \quad \text{with} \quad \zeta = \frac{L_s r_0}{R_0 \xi_{\mathrm{bal}}} , \quad \mathbf{G} = \sin\theta\, \partial_x + \cos\theta\, \partial_y .$$

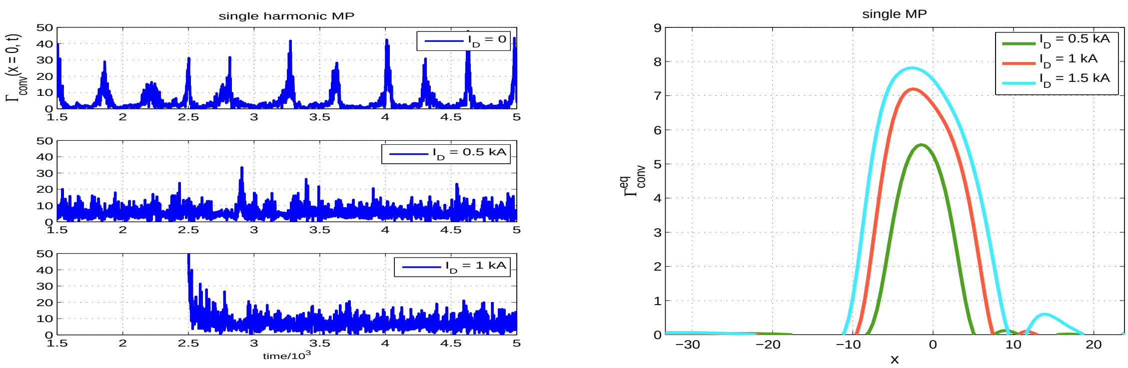


Figure 1: (left): Time evolution of the convective flux $Q_{\text{conv}}$ at the barrier center $x = 0$ for different amplitudes of the magnetic perturbation. Relaxations of the transport barrier are associated with strong flux peaks (top), these peaks are strongly reduced in presence of the magnetic perturbation (bottom). (right): Equilibrium convective flux $Q_{\text{conv}}^{\text{eq}}$ as a function of the radial distance from the resonant surface, for different amplitudes of the magnetic perturbation.

Here, $\xi_{\text{bal}}$ and $L_s$ are the normalization lengths in the directions perpendicular ($\perp$) and parallel ($\|$) to the unperturbed magnetic field, respectively, and $R_0$ is the major radius of the magnetic axis. Time is normalized to the interchange time $\tau_{\text{int}}$. Note that the perpendicular and parallel heat conductivity coefficients in Eq. (2) are normalized using the scalelengths $\xi_{\text{bal}}$ and $L_s$, respectively. Therefore a ratio of $\chi_\| / \chi_\perp \sim 1$ of the normalized coefficients used in the present simulations corresponds to a non-normalized ratio of $L_s^2 / \xi_{\text{bal}}^2 \sim 10^7 - 10^8$.

In the present model, resistive ballooning turbulence is driven by en energy source $S$ located close to the inner boundary of the computational domain that corresponds to the annulus delimited by the the safety factor values $q = 2.5$ and $q = 3.5$ surfaces and including the reference surface $q = q_0 = 3$. When a poloidal rotation $U = \mathrm{d}_x \phi_{\text{imp}}$ with localized maximal shear at $q = 3$ is imposed (via the last term in Eq. (1) where $\bar{\phi}$ denotes the axisymmetric component of $\phi$ and $\mu$ is a friction coefficient), a transport barrier formes at that position. The latter exhibits relaxation oscillations [2, 3]. A typical time trace of the convective flux $Q_{\text{conv}} = \left\langle p \partial_y \phi \right\rangle_{y,z}$ at the center of the barrier is shown in Fig. 1 (left, top). When a resonant magnetic perturbation described by the normalized parallel component of the vector potential

$$\psi_{\text{RMP}} = \psi(x) \cos(m\theta - n\varphi) \quad \text{with} \quad \psi(x) = \psi_0 \exp\left( \frac{m}{\beta_1} \frac{\xi_{\text{bal}}}{r_c} x \right) \tag{3}$$

is imposed at the $q = 3$ surface, (here, $(m, n) = (12, 4)$, $\beta_1 = 0.6$, $\xi_{\text{bal}} / r_c = 1/590$ are parameters typical for the DED device in the TEXTOR tokamak [7]), the barrier relaxations are stabilized [Fig. 1 (left, bottom)]. This stabilization is due to an erosion of the transport barrier that is essentially caused by the convective flux $Q_{\text{conv}}^{\text{eq}} = \left\langle p^{\text{eq}} \partial_y \phi^{\text{eq}} \right\rangle_{y,z}$ associated with the new

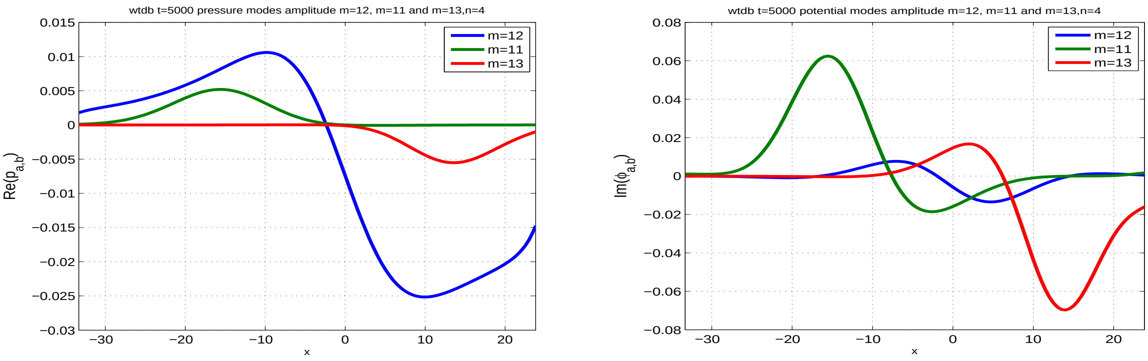


Figure 2: Radial profiles of the real part of the pressure components $p^{\rm eq}_{mn}$ and the imaginary part of the potential components $\phi^{\rm eq}_{mn}$ in an equilibrium in presence of a magnetic island.

plasma equilibrium

$$\begin{pmatrix} p^{\rm eq} \\ \phi^{\rm eq} \end{pmatrix} = \begin{pmatrix} \bar{p}(x) \\ \bar{\phi}(x) \end{pmatrix} + \sum_{m>0} \begin{pmatrix} p^{\rm eq}_{mn}(x) \\ \phi^{\rm eq}_{mn}(x) \end{pmatrix} {\rm e}^{{\rm i}(m\theta - n\varphi)}, \quad (4)$$

in the presence of the magnetic island chain [4, 5, 6]. Typical profiles of $Q^{\rm eq}_{\rm conv}$ are plottet in Fig. 1 (right).

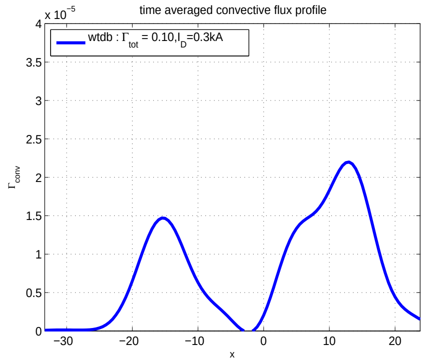


Figure 3: Convective energy flux $Q^{\rm eq}_{\rm conv}$ in an equilibrium in presence of a magnetic island (same parameters as in Fig. 2.)

In order to study this equilibrium, we run the turbulence code for low amplitudes of the source $S$ such that the resulting pressure gradient $\kappa = -1/\chi_\perp \int_{x_{\rm min}}^{x} S\,{\rm d}x' =$ const for $x \geq x_{q=2.5}$ is below the instability threshold. The 3D pressure and electric potential fields then evolve to a stationary state corresponding to the equilibrium. As a first approach, we focus on a case without imposed rotation, i.e. without transport barrier. In this case, the numerical results can be compared to analytical studies concerning the plasma pressure in Ref. [8]. Profiles of $p^{\rm eq}_{mn}$, $p^{\rm eq}_{mn}$ and the associated convective flux $Q^{\rm eq}_{\rm conv}$ are shown in Figs. 2 and 3, respectively.

When inserting the expressions (3), (4) in Eqs. (1), (2) and linearizing for low amplitudes $\psi(x)$, $p^{\rm eq}_{mn}$, $\phi^{\rm eq}_{mn} \ll 1$ and $\bar{p} = -\kappa x$, $\bar{\phi} = 0$, one obtains the following relations for $p^{\rm eq}_{mn}$, $\phi^{\rm eq}_{mn}$:

$$k_y^2 (x - x_m)^2 \phi^{\rm eq}_{mn} - \frac{\rm i}{2}\partial_x \left( p^{\rm eq}_{m-1,n} - p^{\rm eq}_{m+1,n} \right) - \frac{1}{2}\frac{k_y}{m}\left[ (m-1) p^{\rm eq}_{m-1,n} + (m+1) p^{\rm eq}_{m+1,n} \right] = 0 \quad (5)$$

$$-{\rm i}k_y \kappa \phi^{\rm eq}_{mn} + \chi_\parallel k_y^2 (x - x_m)^2 p^{\rm eq}_{mn} + \chi_\parallel \kappa k_y^2 x \frac{1}{2} \psi(x) \delta_{mm_0} = 0 \quad (6)$$

Here, $m_0 = nq_0$, $x_m = (\zeta/q_0)(m - m_0)/m$, $k_y = m\xi_{\rm bal}/r_0$, $L_s = R_0$, the $\delta_c$ term in Eq. (1) has been neglected for simplicity ($\delta_c \ll 1$) and the perpendicular dissipation ($\nu$ and $\chi_\perp$ terms) have not been written. Without the terms related to the magnetic curvature, Eq. (5) would imply $\phi^{\rm eq}_{mn} = 0$. Far from the $q = 3$ resonant surface, the perpendicular heat conduction is negligible

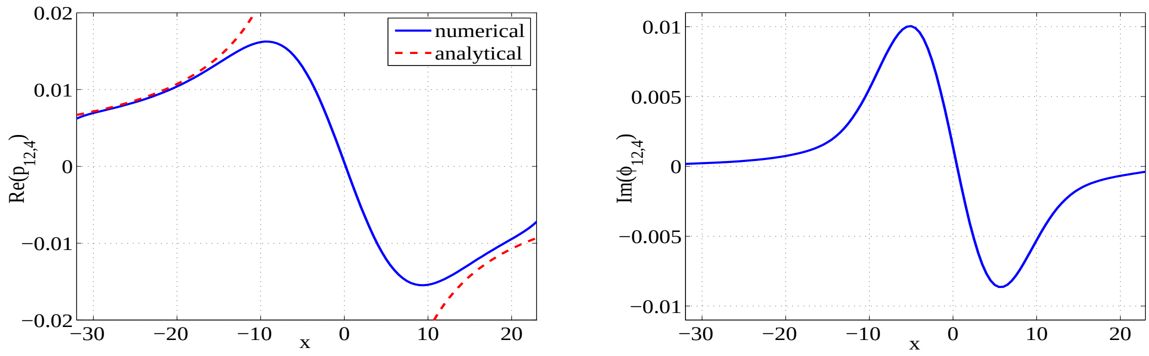


Figure 4: Radial profiles of the real part of $p^{\text{eq}}_{m_0,n}$ and the imaginary part of $\phi^{\text{eq}}_{m_0,n}$ in an equilibrium with magnetic island in a case with cylindrical curvature and constant $\psi$.

with respect to the parallel one and the pressure component $p^{\text{eq}}_{m_0,n}$ is determined – when $\phi^{\text{eq}}_{mn} = 0$ – by the balance of the last two terms in Eq. (6) [8]:

$$p^{\text{eq}}_{m_0,n} = -\kappa \frac{1}{x} \frac{\psi(x)}{2} \,. \tag{7}$$

When accounting for the magnetic curvature, the pressure component $p^{\text{eq}}_{m_0,n}$ is still approximatively given by Eq. (7), and the potential harmonics are then determined by Eq. (5). This can be verified in a simplified cylindrical geometry with $\mathbf{G} = g_0 \partial_y$. Fig. 4 shows the corresponding profiles of $p^{\text{eq}}_{m_0,n}$, $\phi^{\text{eq}}_{m_0,n}$ as well as the asymptotic solution (7) for $g_0 = 0.7$ and $\psi(x) = \psi_0$. Note that close to the resonant surface, according to Ref. [8], no analytical prediction for $p^{\text{eq}}_{m_0,n}$ is available, as the actual island width $W = 4\sqrt{\psi_0} = 7.9$ is comparable to the critical island width $W_c = \sqrt{8}\left(\chi_\parallel/\chi_\perp\right)^{1/4}/\sqrt{k_y} = 18$.

In conclusion, the plasma equilibrium in the presence of a magnetic island chain is characterized by both, a non-axisymmetric pressure structure, arising mainly from the balance between parallel and perpendicular heat conductivity, and a non-axisymmetric potential structure, arising from the coupling between pressure and potential due to magnetic curvature. The phase difference between these structures is such that they give rise to convective transport which is known to play a significant role in the erosion of the transport barrier in turbulence simulations of barrier relaxations.

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