Non-local transport in the presence of internal transport barriers

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I. Introduction. There is increasing experimental, numerical, and theoretical evidence that transport processes in fusion plasmas can, under certain circumstances, depart from the standard local, diffusive transport description. Some experimental examples include fast pulse propagation phenomena in perturbative experiments, non-diffusive scaling in L-mode plasmas, and non-Gaussian statistics of fluctuations. Non-diffusive transport has also been documented in numerical studies of turbulent transport in fluid and gyrokinetic simulations. From the theoretical perspective, non-diffusive transport descriptions naturally arise from the relaxation of the restrictive assumptions (e.g., locality, scale separation, and Gaussian/Markovian statistics) that lay at the foundation of diffusive models, see for example [1] and references therein.

In previous publications [2,3,4] we proposed an alternative class of models able to capture some of the experimentally and numerically observed non-diffusive transport phenomenology. The models are based on the use of a type of non-local integro-differential operators known as fractional derivatives. These operators provide a unifying framework to describe non- Fickian scale-free transport, non-Markovian (memory) effects, and non-diffusive scaling. In Ref.[4] the model was applied to describe perturbative experiments in JET involving fast cold pulse propagation and ICRH power modulation. Here we present recent result on the role of internal transport barriers (ITB) on non-local transport. In particular we

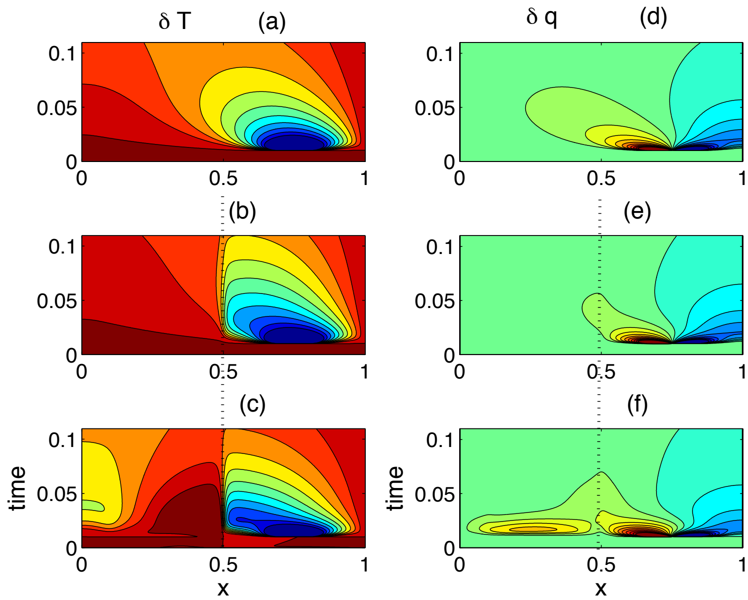

Figure 1: Non-local tunneling across ITB. Left column: spatio-temporal evolution of temperature perturbation, δT. Right column: flux perturbation, δq. Top row: diffusive transport in the absence of ITB. Middle row: diffusive transport in the presence of ITB. Bottom row: non-local transport in the presence of ITB.

explore in detail the interplay between non-locality and transport suppression in perturbative transport. Motivated by experimental results we consider two types of perturbations: cold edge pulses and power modulation. A problem of specific interest is the “tunneling" of perturbations through the ITBs.

II. Model. We consider one-dimensional radial heat transport in a constant density plasma in the slab approximation

$$\partial_t \left[3/2 n T\right] = -\partial_x q + S \, , \tag{1}$$

where n denotes the plasma density, T is the temperature, q is the heat flux, S is the source, and $x \in (0,1)$ is a normalized radial coordinate with $x = 0$ denoting the magnetic axis and $x = 1$ the plasma edge. The flux, $q = q_d + q_{nl}$, is assumed to consists of a diffusive component, q_d, and a non-local component q_{nl}. The diffusive component is modeled using the local Fourier-Fick's prescription $q_d = -\chi_d \, n \, \partial_x T$ where χ_d the heat diffusivity which can depend on x, t, and in the case of nonlinear models on T and its gradients. Following Refs. [3,4] we assume a non-local flux of the form

$$q_{nl} = -\chi_{nl} \, n \, \left[l \, {}_a\mathscr{D}_x^{\alpha} - r \, {}_x\mathscr{D}_b^{\alpha} \right] T \, , \tag{2}$$

where

$${}_a\mathscr{D}_x^{\alpha} T = \frac{1}{\Gamma(2-\alpha)} \int_a^x \frac{T'(y) - T'(a)}{(x-y)^{\alpha-1}} \, dy \, , \qquad {}_x\mathscr{D}_b^{\alpha} T = \frac{-1}{\Gamma(2-\alpha)} \int_x^b \frac{T'(y) - T'(b)}{(y-x)^{\alpha-1}} \, dy \, , \tag{3}$$

with $T' = \partial_y T$. The parameters l and r control the asymmetry of the integro-differential operators. Here we will limit attention to the symmetric case $l = r = 1/[2\cos(\alpha\pi/2)]$. For the boundary conditions we assume zero total heat flux at the magnetic axis, and fixed temperature at the edge, $q(x=0,t) = [q_d + q_{nl}]\,(x=0,t) = 0$, and $T(x=1,t) = 0$. Details on the physical motivation and mathematical properties of the non-local operator as well as the numerical method used in the integration can be found in Ref. [1-4]. The local and non-local diffusivities are assumed to be of the form

$$\chi_d = \chi_{d0} - \zeta e^{-(x-x_0)^2/w} \, , \qquad \chi_{nl} = \frac{\chi_{nl0}}{2} \left[\tanh\left(\frac{x - x_c}{L} \right) + \tanh\left(\frac{x_c}{L} \right) \right] - \zeta e^{-(x-x_0)^2/w} \, . \tag{4}$$

The tanh profile in χ_{nl} is introduced to guarantee the vanishing of the non-local flux in the core region where transport is assumed to be dominated by diffusive processes. The ITB is modeled by introducing a dip, $e^{-(x-x_0)^2/w}$, in the diffusivity profiles. In the calculations reported here $\chi_{d0} = 1$, $x_0 = 0.5$, $\zeta = 0.95$, $\chi_{nl0} = 1$, $x_c = 0.1$, and $L = 0.025$. For the cold pulse simulations $w = 0.005$ and for the heat wave modulation simulations $w = 0.0025$. In the non-local simulations, $\alpha = 1.25$. The first step in the simulations is the computation of the steady equilibrium temperature profile, $T_0(x)$, in the presence of an on-axis source of the form

$$S = S_0 \exp\left[-\frac{(x-\mu_s)^2}{2\sigma_s^2}\right], \quad (5)$$

with $\mu_s = 0$, and $\sigma_s = 0.075$. For each run, the source amplitude was selected so that $T_0(0) = 1$. The simulations followed the spatio-temporal evolution of the perturbed temperature, $\delta T(x,t) = T(x,t) - T_0(x)$, and the perturbed flux, $\delta q(x,t) = q(x,t) - q_0(x)$

III. Perturbative transport. For the cold pulse simulations we considered an initial condition of the form

$$\delta T(x,0) = -A \exp\left[-\frac{(x-\mu_p)^2}{2\sigma_p^2}\right], \quad (6)$$

with $A = -0.3$, $\mu_p = 0.75$ and $\sigma_p = 0.03$. Figure 1 shows the spatio-temporal evolution of δT and δq. In the case of pure diffusive transport, $\chi_{nl0} = 0$, in the absence of ITBs the pulse spreads throughout the plasma domain in a diffusive time scale. As expected, in the presence of an ITB the diffusive propagation of the pulse is stopped and the diffusive flux hardly penetrates past the ITB location. However, in the presence of non-local transport the pulse dynamics is fundamentally different. As Fig. 1 shows, in this case the pulse can in fact go through the ITB. This “tunneling" effect is a unique novel property of non-local transport that results from the long range “tongues" in the flux. Another interesting unique signature of non-local transport is observed in the time traces of the normalized temperature perturbation in Fig. 2. Consistent with the results reported in Ref [4], in the absence of ITBS, long and short time responses are observed in the diffusive and non-local cases respectively. However, as the red temperature trace shows, in the presence of ITB, the non-local transport of the *cold* pulse can give rise to *heating* right after the ITB.

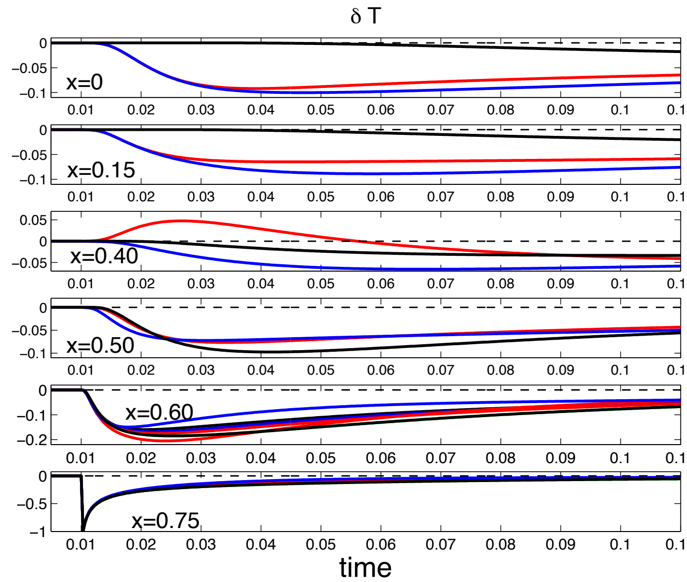

Figure 2: Temperature traces exhibiting heating response from cold pulse due to non-local transport in the presence of ITB. Black curve: diffusive transport with ITB at $x = 0.5$. Blue curve: $\alpha = 1.25$ non-local transport without ITB. Red curve: $\alpha = 1.25$ non-local transport with ITB at $x = 0.5$.

In the power modulation studies we considered, in addition to the on axis source an off-axis source that includes a time-periodic amplitude modulation of the form $S_{oa} = \frac{S_0}{8}\left[3-\cos\left(2\pi\nu t\right)\right]\exp\left[-\frac{(x-\mu_{oa})^2}{2\sigma_{oa}^2}\right]$ where $\mu_{oa} = 0.75$ and $\sigma_{oa}^2 = 0.075$. The propagation properties of the temperature perturbation are determined by the amplitude profiles, $A_n(x)$, and the phase profiles, $\Phi_n(x)$, for the different harmonics $n = 1, 2 \ldots$. Here we focus on the $n = 1$ dominant harmonic. As shown in Fig. 3, in the purely diffusive case it is observed that the heat wave is strongly damped and slowed down by the ITB. However, the behavior is quite different in the presence of nonlocal transport. In this case it is observed that the wave can tunnel through the ITB and reappear on the other with a smaller, but none-glegible amplitude. Also, a region of counter propagating waves is observed on the inner side of the ITB.

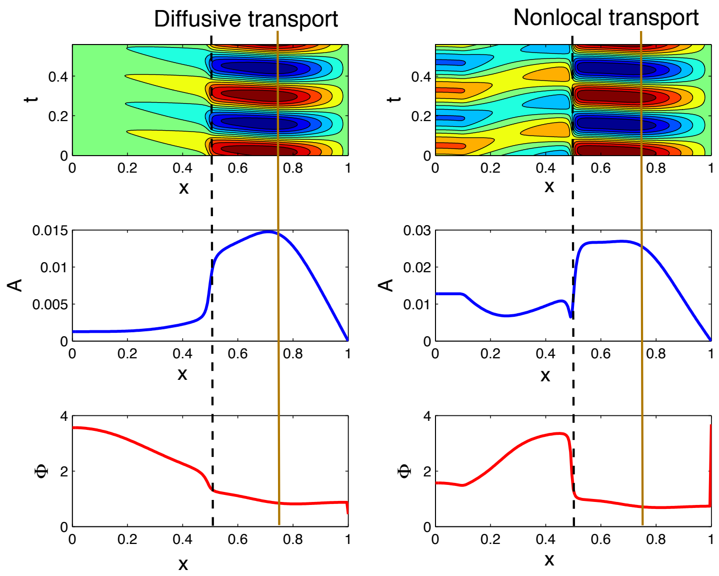

Figure 3: Heat wave propagation in the presence of ITBs. Left column: diffusive transport. Right column: non-local transport. Top row: spatio-temporal evolution of temperature perturbation. Middle row: amplitude of dominant harmonic. Bottom row: phase of dominant harmonic. The dashed vertical line indicates the location of the ITB. The brown solid line indicates the location of the modulating source.

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