Impurity transport driven by electrostatic turbulence in tokamak plasmas

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Impurity transport is analyzed using a semi-analytical model based on a boundary-layer solution of the gyrokinetic (GK) equation. Analytical expressions for the perturbed density responses are derived and scalings of the mode frequencies and quasilinear fluxes with charge number Z, effective charge Z_{eff}, impurity density scale length and collisionality are determined. An approximate expression of the zero-flux impurity density gradient is derived and used to discuss its parametric dependencies.

Introduction Models of impurity transport driven by microturbulence and neoclassical processes are now well developed, but there are still many open issues regarding the sign and magnitude of the impurity particle flux and its parametric dependencies. To get reliable predictions for the turbulent fluxes, nonlinear electromagnetic GK simulations are needed, but these are costly in computing time. Reduced theoretical models, benchmarked to GK simulations, can ease the interpretation of the results of experiments or numerical simulations and can contribute to the understanding of the underlying processes.

The aim of the present work is to calculate the quasilinear GYrokinetic IMpurity transport driven by ElectroStatic turbulence (GYIMES) using a semi-analytical model based on a boundary layer solution of the GK equation. Following the approach of Ref. [1,2], we use a model electrostatic potential $\phi(\theta) = \phi_0 \left[(1+\cos\theta)/2 + i f_s \sin^2\theta\right] \left[H(\theta+\pi) - H(\theta-\pi)\right]$, where H is the Heaviside function, $f_s = -0.6s + s^2 - 0.3s^3$, s is the magnetic shear and θ is the ballooning angle. This model potential is motivated by variational analysis and GK simulations. By assuming large aspect-ratio, low beta, toroidal symmetry, circular cross section and weak collisionality, analytical expressions can be derived for the ion, impurity and electron perturbed densities. The quasi-neutrality equation is solved numerically to obtain the frequencies and growth rates of the unstable modes, including the effect of impurities on these modes, and the quasilinear impurity particle fluxes. In this paper, we study only ion-temperature-gradient turbulence dominated cases, but the model is suitable for trapped-electron mode turbulence as well. Using the analytically calculated expression for the perturbed impurity density response, we derive an approximate expression for the zero-flux impurity density gradient. Such a zero impurity flux region is relevant to steady state plasmas in the core of tokamaks since the impurity influx occurs through the edge.

Perturbed density responses The perturbed electron, ion and impurity responses are obtained from the linearized GK equation. We assume the following ordering of the electron/ion bounce frequencies and the eigenfrequency of the mode, $\omega_{bi} \ll \omega \ll \omega_{be}$, and consider weakly-collisional plasmas so that $\nu_{\star e} = \nu_e/\epsilon\omega_{be} \ll 1$, where $\epsilon = r/R$ and ν_e is the electron collision frequency. Then, the perturbed electron density response is given by

$$\frac{\hat{n}_e}{n_e}/\frac{e\phi}{T_e} = 1 - \tilde{\phi}\left\{\sqrt{2\epsilon}\left[\hat{\omega}_{\eta*e} - \frac{3}{2}\left(\eta_e\tilde{\omega}_{*e} - \frac{\tilde{\omega}_{Dt}}{2}\hat{\omega}_{\eta*e}\right)\mathcal{F}^1_{5/2}\left(\frac{\tilde{\omega}_{Dt}}{2}\right)\right]\right. \\ \left. - \frac{\Gamma(\frac{3}{4})\sqrt{\epsilon\hat{\nu}_t}}{\sqrt{-i\pi y}}\left[2\hat{\omega}_{\eta*e}\mathcal{F}^{3/2}_{3/4}\left(\frac{\tilde{\omega}_{Dt}}{2}\right) - \frac{3\eta_e\tilde{\omega}_{*e}}{2}\mathcal{F}^{3/2}_{7/4}\left(\frac{\tilde{\omega}_{Dt}}{2}\right)\right]\right\}, \tag{1}$$

where $\tilde{\phi} = (1+4if_s/5)4\phi_0/(3\pi\phi)$, $\tilde{\omega}_{Dt} = \omega_{D0}/(\omega x_e^2)$, $\omega_{D0} = -k_\theta v^2/\omega_{ce}R$, k_θ is the poloidal wave-number, $\omega_{ca} = e_aB/m_a$ is the cyclotron frequency, $x_a = v/v_{Ta}$, $v_{Ta} = (2T_a/m_a)^{1/2}$, $\hat{\nu} = \nu_e/\omega_0\epsilon$, $\omega_0 = |\Re\{\omega\}|$, $y = \omega/\omega_0$, $\hat{\nu}_t = \hat{\nu}x_e^3$, $\eta_a = L_{na}/L_{Ta}$, with $L_{na} = -[\partial(\ln n_a)/\partial r]^{-1}$ and $L_{Ta} = -[\partial(\ln T_a)/\partial r]^{-1}$, $\omega_{*a} = -k_\theta T_a/e_aBL_{na}$ is the diamagnetic frequency, $\tilde{\omega}_{*a} = \omega_{*a}/\omega$, $\hat{\omega}_{\eta*a} = 1-(1-3\eta_a/2)\tilde{\omega}_{*a}$ and $\mathcal{F}^a_b(z) = {}_2F_0\,(a,b;;z)$, where ${}_2F_0$ is the generalized hypergeometric function. $\mathcal{F}^a_b(z)$ incorporates the full effect of the drift resonances.

For the ions, we neglect the parallel compressibility by assuming $k_\parallel v_{Ti} \ll \omega$, and obtain

$$\frac{\hat{n}_i}{n_i}/\frac{e\phi}{T_i} = -\tilde{\omega}_{*i} + \left(\frac{3\tilde{\omega}_{Dsi}}{2} - b_i\right)\left[\hat{\omega}_{\eta*i} - \frac{5}{2}\left(\eta_i\tilde{\omega}_{*i} - \tilde{\omega}_{Dsi}\hat{\omega}_{\eta*i}\right)\mathcal{F}^1_{7/2}\left(\tilde{\omega}_{Dsi}\right)\right]. \tag{2}$$

Here, $b_a = \langle b_{sa}\rangle_\phi = b_{a0}\left[1+s^2(2\pi^2-12+if_s(2\pi^2-3))/(6(1+if_s))\right]$ is the weighted flux-surface averaged value of the finite Larmor radius (FLR) parameter, $b_{sa} = b_{a0}(1+s^2\theta^2)$, $b_{a0} = (k_\theta\rho_{sa})^2$ and $\rho_{sa} = v_{Ta}/\sqrt{2}\omega_{ca}$. Only the terms linear in b_{i0} were kept. The averaged magnetic drift frequency is $\tilde{\omega}_{Dsa} = [6+(9+16if_s)s\omega_{Da0}]\,/\,[12(1+if_s)\omega]$, where $\omega_{Da0} = -2k_\theta v_{Ta}^2/3\omega_{ca}R$. If $(Z^3m_e/m_i)^{1/2}(n_zZ^2/n_i)\epsilon\nu_{\star e} \ll 1$, collisions can be neglected and we have

$$\frac{\hat{n}_z}{n_z}/\frac{Ze\phi}{T_z} = -\tilde{\omega}_{*z} + \left(\frac{3\tilde{\omega}_{Dsz}}{2} - b_z\right)\left[\hat{\omega}_{\eta*z} - \frac{5}{2}\left(\eta_z\tilde{\omega}_{*z} - \tilde{\omega}_{Dsz}\hat{\omega}_{\eta*z}\right)\mathcal{F}^1_{7/2}\left(\tilde{\omega}_{Dsz}\right)\right]. \tag{3}$$

The dispersion relation follows from the quasi-neutrality condition. We find that for moderate or high charge number ($Z > 10$) the eigenfrequency and stability boundary are only weakly affected by increasing Z for constant $Z_{\rm eff}$, and are approximately equal to the corresponding quantities in a pure plasma [2]:

$$\frac{a}{L_{Tic}} = \frac{\left(1+\tau_i^{-1}\right)(2+3s)a}{3R(1-b_{i0})}, \quad \frac{\omega_{0c}}{\omega_{*e}} = \frac{b_{i0}-1}{\tau_ib_{i0}+1} + \left(1+\frac{1}{\tau_i}\right)\frac{(2+3s)L_{ni}}{(\tau_ib_{i0}+1)2R}, \text{ where } \tau_a = \frac{T_e}{T_a}.$$

Impurity particle fluxes If the mode is far from marginal stability, the effect of increasing charge number and density affects the growth rates and mode frequencies only weakly. Furthermore, the impurity particle flux is only very weakly dependent on the charge number. The normalized impurity flux is reduced with increasing charge number if $Z_{\rm eff}$ is kept constant, as shown in Fig. 1, but that is mainly due to the reducing impurity fraction $n_z/n_e \sim 1/Z^2$. The normalized flux increases for increasing impurity density, and this is more pronounced for impurities with lower Z since the relative increase in $Z_{\rm eff}$ is larger than for high Z. The ion and electron fluxes are expected to be inwards and their absolute values decrease with increasing Z.

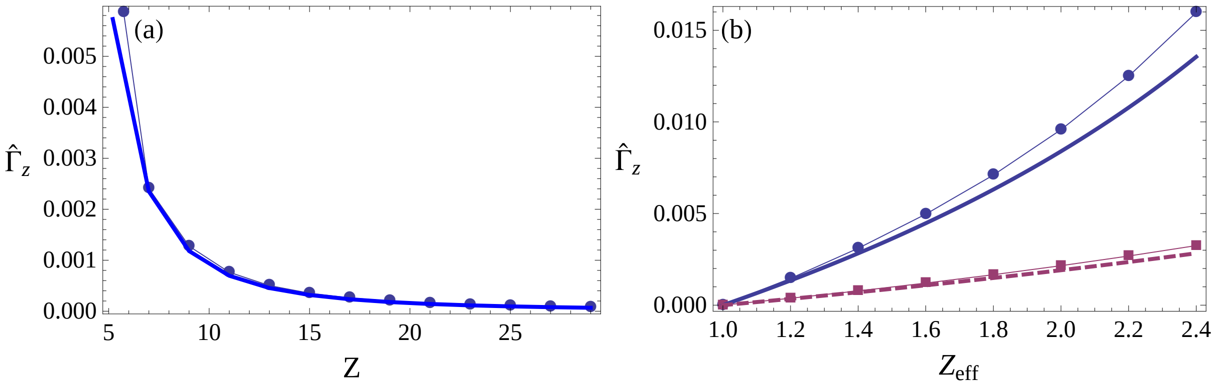

Figure 1: Normalized impurity particle flux $\hat{\Gamma}_z$ (to $k_\theta p_e/eB\,|e\bar{\phi}/T_e|^2$) compared with linear GYRO results (dots). The other parameters are $a/L_{Te} = a/L_{Ti} = 3$, $s = 1$, $q = 2$, $a/R = 1/3$, $r/a = 1/2$, $a/L_{ne} = 1$ and $k_\theta\rho_s = 0.2$ (GA standard case). (a) $\hat{\Gamma}_z$ vs. Z for $Z_{\rm eff} = 1.5$; (b) $\hat{\Gamma}_z$ vs. $Z_{\rm eff}$ for $Z = 6$ (solid), $Z = 10$ (dashed).

Collisions do not affect the mode frequencies, growth rates and impurity fluxes (see Fig. 2a) significantly. We note also that the impurity flux changes sign at approximately the same value of R/L_{nz}, as shown in Fig. 2b, independently of Z, $Z_{\rm eff}$ and many other plasma parameters.

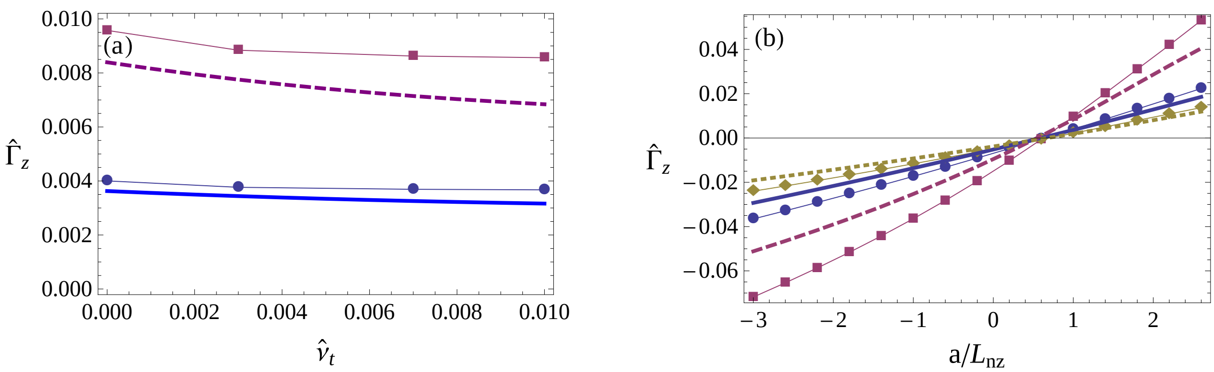

Figure 2: Normalized impurity flux $\hat{\Gamma}_z$ compared with linear GYRO results (dots) for the GA standard case. (a): $\hat{\Gamma}_z$ vs. normalized collisionality (in c_s/a) for $Z = 6$ and (solid) $Z_{\rm eff} = 1.5$, (dashed) $Z_{\rm eff} = 2$; (b): $\hat{\Gamma}_z$ vs. inverse radial impurity density gradient for the parameters: solid $Z_{\rm eff} = 1.5$, $Z = 6$, dashed: $Z_{\rm eff} = 2$, $Z = 6$, dotted: $Z_{\rm eff} = 2$, $Z = 10$.

Zero-flux impurity density gradient For moderate and high Z, an approximate expression for the zero-flux impurity density gradient, R/L_{nzc}, can be derived using the analytical expression for the perturbed impurity density, and is given by:

$$\frac{R}{L_{nzc}} = \frac{(2+3s)}{2} \frac{1 - \frac{2}{1+\hat{\gamma}^2} \frac{k_\theta \rho_s}{Z\tau_z \omega_0^n} \left(\frac{a}{L_{Tz}} - \frac{a}{R} \frac{(2+3s)5}{6}\right)}{1 + \frac{a}{R} \frac{2+3s}{1+\hat{\gamma}^2} \frac{k_\theta \rho_s}{Z\tau_z \omega_0^n}}, \quad (4)$$

where, ω_0^n is ω_0 normalized to c_s/a and $\hat{\gamma} = \gamma/\omega_0$ is the normalized growth rate. Equation (4) shows that for higher impurity temperature gradient or higher $k_\theta \rho_s$, R/L_{nzc} is lower, a trend which is in good agreement with our numerical results. In deriving Eq. (4) we assumed the unstable mode frequencies and growth rates to be constant, as, for the same set of parameters, they do not show a strong dependence on Z or Z_{eff}. However, if, for instance, the inverse electron density scale length a/L_{ne} or the temperature ratio τ_i are changed, the unstable mode frequencies and growth rates will also change and R/L_{nzc} will be affected by that, especially for low Z, when the effect of thermodiffusion cannot be neglected. This means that in scenarios with more peaked electron density profiles or strongly differing electron-to-ion temperature ratios the zero-flux impurity density gradient is expected to be different from that in scenarios with flat density profiles or if $\tau_i = 1$.

Conclusions In this paper we presented a semi-analytical model for impurity transport driven by electrostatic turbulence. The model does not rely on expansions in the smallness of the magnetic drift frequencies, and includes electron-ion collisions modeled by a Lorentz operator. The results agree well with linear gyrokinetic simulations with GYRO. The semi-analytical character of the model eases the interpretation of experimental and simulation results. Because of its simplicity, it is straightforward to extend it by including several impurity species or include it in transport simulations. However, due to the model electrostatic potential used in the calculations, reliable quantitative predictions can only be obtained in the moderate shear region.

References

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