

# Nonlinear Dynamics of Magnetic Electron Drift Modes and Self-Organization Phenomena in Turbulent Plasma

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**I.** It is already well accepted, that nonlinear interaction of short-scale fluctuations in magnetized plasma can generate low-frequency, large-scale nonlinear structures (so called zonal flows and streamers) that play an important role, for example, in controlling plasma transport properties in magnetic confinement systems. Here we investigate the generation of large-scale magnetic structures in magnetic electron drift mode (MEDM) turbulence. These modes are of interest in, e.g., laser fusion experiments, where they are thought to be responsible for the very strong self-generated magnetic fields which have been observed since the early 1970s. The driving mechanism is the baroclinic effect, i.e., the fluctuating electron temperature and density gradients should align. To understand nonlinear dynamics of these modes we employ the ansatz of scale separation to distinguish between the small-scale fluctuations of magnetic electron drift modes and the large-scale shear flow pattern generated by turbulence. Then, the evolution equations for mean flow generation are obtained by averaging the model equations for magnetic electron drift modes over fast small-scales. When the large-scale structures are excited, they form an environment for the parent drift waves. The propagation of drift modes in such weakly inhomogeneous media with slowly varying parameters can be conveniently described with the help of a wave kinetic equation for the wave action density in  $\mathbf{r}$ - $\mathbf{k}$  space. The sources of these slow spatial and temporal variations are flow induced velocity perturbations. Finally, the evolution of nonlinearly interacting MEDM is illustrated by a simulation study of the model equations for the different set of parameters.

**II.** We consider a nonuniform unmagnetized plasma, and fluctuations on a space scale much smaller than that of the equilibrium density and temperature inhomogeneities, which are taken to be in the  $x$ -direction. The time scale is faster than the ion and slower than the electron plasma frequency, and hence we consider an unpolarized electron fluid and immobile ions. We confine our analysis to two-dimensional solutions where all quantities are independent on  $z$ . The temperature and density gradients of the fluctuations are in general not collinear, and this generates a vorticity in the electron fluid. The consequent motion generates a perpendicular magnetic field,  $B(x,y)\mathbf{z}$  say, which actually plays role of a stream function. Then, the simplest energy and momentum equations along with Faraday's and Ampere's laws can be reduced to a pair of coupled non-linear equations for  $B$  and perturbed electron temperature  $T$

$$\frac{\partial}{\partial t} (B - \lambda^2 \nabla^2 B) + \beta \frac{\partial T}{\partial y} - \lambda^4 \frac{e}{mc} \{B, \nabla^2 B\} = 0, \quad (1a)$$

$$\frac{\partial}{\partial t} T + \alpha \frac{\partial B}{\partial y} + \lambda^2 \frac{e}{mc} \{B, T\} = 0, \quad (1b)$$

where  $\alpha = \lambda^4 \frac{eT_0}{mc} (\kappa_T - \frac{2}{3} \kappa_n)$ ,  $\beta = \frac{c}{e} \kappa_n$ ,  $\lambda = \frac{c}{\omega_{pe}}$ , and  $\kappa_n, \kappa_T$  being the inverse length

scales of the equilibrium density and temperature inhomogeneities. The Jacobian, or Poisson bracket,  $\{a, b\}$  is defined as  $(\nabla a \times \nabla b) \cdot \mathbf{z}$ . The system (1) resembles models describing various low frequency electrostatic modes in magnetized plasmas, as well as shallow water model. Linear analysis for small perturbations,  $(B, T) \sim \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r})$

yields  $\omega_k = k_y \left( \frac{\alpha\beta}{1 + k^2 \lambda^2} \right)^{1/2}$ ,  $\alpha\beta = V_T^2 \lambda^2 \kappa_n \left( \kappa_T - \frac{2}{3} \kappa_n \right)$ . It is clear that magnetic streamers ( $k_y \neq 0$ ) that have short extent in the direction of translation symmetry can be generated even by the linear instability mechanism, whereas zonal flows ( $k_y = 0$ ) can be generated only through some nonlinear interaction mechanism.

**III.** To describe the evolution of the coupled system (wave turbulence + large scale plasma flows) we represent the perturbed magnetic field  $B$  as a sum of a large scale flow  $\bar{B}$  quantity and a small scale turbulent part  $\tilde{B}$ . A similar representation has been chosen for the electron temperature  $T$ . The large scale plasma flow varies on longer time scale compared to the small scale turbulent fluctuations, so we may employ a multiple scale expansion, thus assuming that there is a sufficient spectral gap separating large scale and small scale motions. Averaging (1) over fast small scales, we obtain the evolution equations for the large-scale flow:

$$\frac{\partial}{\partial t} (\bar{B} - \lambda^2 \nabla_{\perp}^2 \bar{B}) + \beta \frac{\partial}{\partial y} \bar{T} = \lambda^4 \frac{e}{mc} \overline{\{\tilde{B}, \nabla_{\perp}^2 \tilde{B}\}} = \overline{R^B}, \quad (2a)$$

$$\frac{\partial}{\partial t} \bar{T} + \alpha \frac{\partial}{\partial y} \bar{B} = -\lambda^2 \frac{e}{mc} \overline{\{\tilde{B}, \tilde{T}\}} \equiv -\overline{R^T}. \quad (2b)$$

It is seen that a small-scale turbulence can drive large-scale flows via Reynolds stresses  $R^B$  and  $R^T$ . When the large-scale flows are excited, they form an environment for the parent waves, providing the modulation of the turbulence dynamics.

**IV.** Coupling of small-scale fluctuations to the mean flow can be described by the wave kinetic equation for the wave packets, and corresponds to the conservation of an action-like invariant of the wave turbulence,  $N_k(\mathbf{r}, t)$ , with slowly varying parameters due to the large-scale flow:

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega_{k,r}}{\partial \mathbf{k}} \frac{\partial N_k}{\partial \mathbf{r}} - \frac{\partial \omega_{k,r}}{\partial \mathbf{r}} \frac{\partial N_k}{\partial \mathbf{k}} = St\{N_k\}, \quad (3)$$

where the adiabatic action invariant is given by  $N_k = 4 \frac{\alpha}{\beta} (1 + k^2 \lambda^2) |B_k|^2$ . The source term  $St\{N_k\}$  describes the wave growth and damping due to linear and non-linear mechanisms.

We assume that small-scale turbulence is close to the stationary state, so that  $St\{N_k\} \rightarrow 0$ .

The linear frequency of magnetic electron drift modes is now modified by the flows

$$\omega_{k,r} = \omega_k + \mathbf{k} \mathbf{V}_0 \frac{1 + 2k^2 \lambda^2}{1 + k^2 \lambda^2} - \mathbf{k} \mathbf{V}_1 (1 + k^2 \lambda^2)^{-1/2}, \quad (4a)$$

$$\mathbf{V}_0 = -\frac{e}{4mc} \lambda^2 [\nabla \bar{B} \times \mathbf{z}], \quad \mathbf{V}_1 = -\frac{e}{4mc} \lambda^2 \left( \frac{\beta}{\alpha} \right)^{1/2} [\nabla \bar{T} \times \mathbf{z}]. \quad (4b)$$

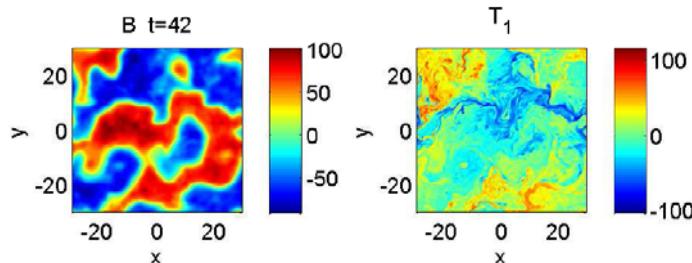
Thus, the small-scale turbulence will be sheared by both the magnetic and electron temperature shear flows.

**V.** The wave kinetic equation (3) describes the dynamics of MEDM in a prescribed large-scale structure spectrum. The broad spectrum of large-scale structures regulates turbulence by the process of random shearing, which may be represented, in the spirit of quasilinear theory, by diffusion in  $\mathbf{k}$  space. Assuming that the wave spectrum consists of an equilibrium and a perturbed part,  $N_k = N_k^0 + \tilde{N}_k$ , and the temporal and spatial variations for small perturbation in the form  $(\tilde{N}_k, B_q, T_q) \sim \exp(-i\Omega T + i\mathbf{q} \cdot \mathbf{r})$ , the  $\mathbf{k}$ -space diffusion coefficient for magnetic zonal flows ( $q_x \neq 0, q_y = 0$ ) can be calculated:

$$D_{k_x} = \left( \frac{e}{4mc} \right)^2 \sum_q k_y^2 q_x^4 \left[ \left( \frac{1 + 2k^2 \lambda^2}{1 + k^2 \lambda^2} \right)^2 |B_q|^2 + \frac{\lambda^4}{1 + k^2 \lambda^2} \frac{\beta}{\alpha} |T_q|^2 \right] R(\Omega, q). \quad (5)$$

Thus, the zonal flows constitute a random strain field which randomly refracts MEDM, causing a diffusive increase in  $k_x$  which in turn enhances their coupling to small-scale (high- $k_x$ ) dissipation. Here  $R(\Omega, \mathbf{q}) = i/(\Omega - \mathbf{q} \cdot \mathbf{V}_g + i\Delta\omega_k)$  is the response function,  $\mathbf{V}_g$  is the MEDM group velocity,  $\Delta\omega_k$  is the total decorrelation frequency. In a weakly nonlinear regime  $R(\Omega, \mathbf{q}) \rightarrow \pi\delta(\Omega - \mathbf{q} \cdot \mathbf{V}_g)$ , for a wide spectrum of fluctuations  $R(\Omega, \mathbf{q}) \rightarrow 1/\Delta\omega_k$ .

**VI.** A simulation study of the Eqs. (1) for different sets of parameters has been performed. The simulation code is based on a pseudospectral method to resolve derivatives in space with periodic boundary conditions, with random fluctuations as initial conditions.



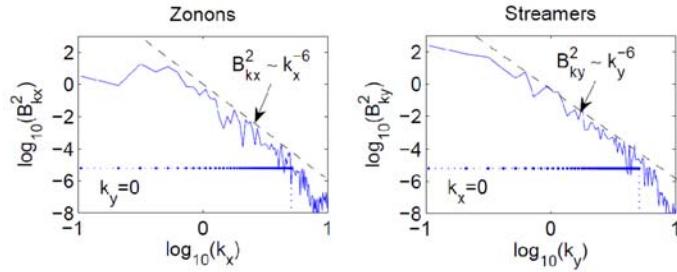


Figure 1: Linearly unstable regime. Top panels: Magnetic field and temperature fluctuations  
Bottom panels: Zonon and streamer energy spectra of the magnetic field.

In the unstable regime ( $\alpha\beta < 0$ ), displayed in Fig. 1, we could observe magnetic field generation and the formation of large scale magnetic structures, accompanied by small-scale turbulence visible in the temperature fluctuations. The energy spectra are non-Kolmogorov and concentrated to streamers at small wave numbers.

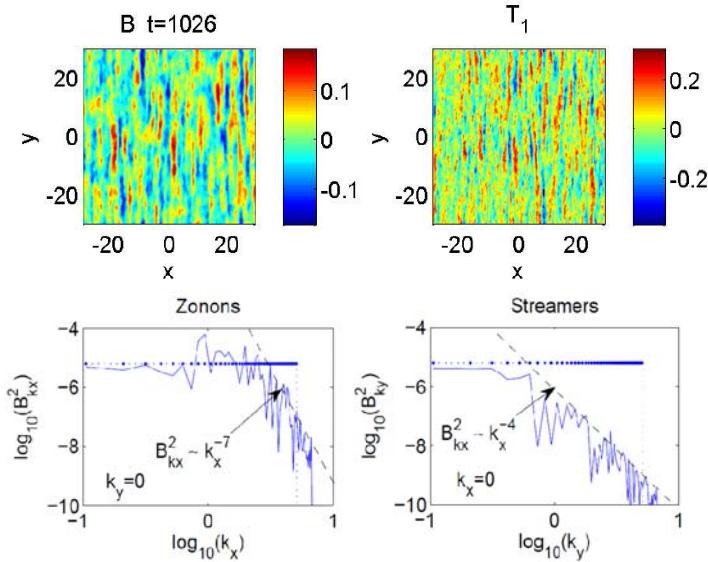


Figure 2: Linearly stable, small amplitude regime. Top panels: Magnetic field and temperature fluctuations  
Bottom panels: Zonon and streamer energy spectra of the magnetic field.

In the linearly stable regime ( $\alpha\beta > 0$ ) in Fig. 2, we observe small-scale turbulence and the formation of zero-frequency zonal flows (zonons). The energy spectra are strongly anisotropic with magnetic wave energy concentrated at zonons.

**YII.** The self-consistent theory of large-scale structure generation by the MEDM turbulence is presented. A coupled system of equations has been derived to self-consistently describe these two components of wave turbulence. It is shown that a small-scale turbulence drives, via Reynolds stresses, a large-scale flow. The pattern of the flow, in turn, modulates and regulates the turbulence dynamics by random shearing it. This process is described by the diffusion in  $\mathbf{k}$  – space and corresponding diffusion coefficients are calculated. The obtained analytical results are illustrated by numerical studies of the model equations which exhibit the excitation of the large-scale magnetic structures by the MEDM turbulence.