Neoclassical Plateau Regime Transport in a Tokamak Pedestal

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In tokamak pedestals with subsonic flows the radial scale of plasma profiles can be comparable to the ion poloidal Larmor radius, thereby making the radial electrostatic field so strong ($\sim 100\,\mathrm{kV/m}$ [1]) that the $\mathbf{E}\times\mathbf{B}$ drift has to be retained in the ion kinetic equation in the same order as the parallel streaming. By adopting the approach of Ref. [2], the modifications of neoclassical plateau regime transport [3] – such as the ion heat flux, and the poloidal ion and impurity flows – are evaluated in the presence of a strong radial electric field, allowing for $U = v_{\mathbf{E}\times\mathbf{B}}B/(v_i B_p) = \mathcal{O}(1)$, where $v_{\mathbf{E}\times\mathbf{B}}$ is the $\mathbf{E}\times\mathbf{B}$ velocity, $v_i = (2T_i/M)^{1/2}$ is the ion thermal speed, and B and $B_p(\ll B)$ are the magnitudes of the total and poloidal magnetic fields. The altered poloidal ion flow is most pronounced in the region of the strongest radial electric field where it modifies the friction of the electrons with the ions and can lead to an increase in the bootstrap current, by enhancing the coefficient of the ion temperature gradient term.

Ion transport and parallel ion flow

The magnetic field is represented as $\mathbf{B} = I\nabla\zeta + \nabla\zeta\times\nabla\Psi$, where ζ is the toroidal angle, $2\pi\Psi$ is the poloidal flux, and $I(\Psi)$ is defined by $\mathbf{B}_t = I\nabla\zeta$ where $\mathbf{B}_t$ is the toroidal magnetic field and $|\nabla\zeta| = 1/R$. We assume a quadratic electric potential well $\Phi(\Psi) = \Phi(\Psi_*) + (\Psi - \Psi_*)\Phi'(\Psi_*) + 1/2(\Psi-\Psi_*)^2\Phi''(\Psi_*)$, where $\Psi_* = \Psi - \frac{Mc}{Ze}R\mathbf{v}\cdot\hat{\zeta} \approx \Psi - Iv_\parallel/\Omega$ is the canonical angular momentum with M is the mass and Ze is the charge of the particle, R is the major radius, $\Omega = ZeB/Mc$ is the cyclotron frequency and $\mathbf{b} = \mathbf{B}/B$.

We introduce $u = cI\Phi'/B$, together with $u_* = cI\Phi'_*/B$, where $\Phi'_* = \Phi'(\Psi_*)$. The $\mathbf{E}\times\mathbf{B}$ drift competes with the parallel streaming when they have comparable projections in the poloidal plane - ths situation we considered here. The quantity u is sometimes referred to as the $\mathbf{E}\times\mathbf{B}$ drift in the poloidal magnetic field. We assume B is slowly varying with Ψ, so that $B(\Psi_*,\theta)\approx B(\Psi,\theta)$. The orbit squeezing factor $S = 1 + cI^2\Phi''_*/(B\Omega)$ is considered to be constant except for its B dependence. Using the preceding notation the poloidal motion of the particles is given by $\dot{\theta} = (v_\parallel\mathbf{b} + \mathbf{v}_{\mathbf{E}\times\mathbf{B}})\cdot\nabla\theta = (v_\parallel + u)\mathbf{b}\cdot\nabla\theta \approx (Sv_\parallel + u_*)/(qR)$, where θ is the poloidal angle and q is the safety factor. We use $\mathcal{E} = E - Ze\Phi(\Psi_*)/M = Sv_\parallel^2/2 + \mu B + v_\parallel u_*$ as an energy variable, where $E = v^2/2 + Ze\Phi/M$ is the total energy and $\mu = v_\perp^2/2B$. Note that $\mathcal{E}$ is conserved by the Vlasov operator.

We write the gyro-averaged distribution function as $\bar{f} = f_*(\Psi_*,\mathcal{E}) + h(\Psi_*,\mathcal{E},\mu,\theta,t)$ where

$$f_* \approx f_{Mi}\left\{1 - \frac{Iv_\parallel}{\Omega_i}\left[\frac{\partial \ln p_i}{\partial\Psi} + \frac{Ze}{T_i}\frac{\partial\Phi}{\partial\Psi} + \left(\frac{mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial \ln T_i}{\partial\Psi}\right] + \dots\right\}, \quad (1)$$

We consider subsonic flows, so that in the pedestal $\partial_\Psi \ln n_i \approx -(Ze/T_i)\partial_\Psi\Phi$. The governing equation for a time independent perturbed ion distribution, in accordance with Ref. [2], is

$$\dot{\theta}\frac{\partial h_{1i}}{\partial\theta} - C_{ii}^l\left\{h_{1i} - \frac{Iv_\parallel f_{Mi}}{\Omega_i}\left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial \ln T_i}{\partial\Psi}\right\} = 0, \quad (2)$$

where C^l_{ii} is the linearized ion-ion collision operator, which is momentum conserving, and the θ derivative is taken keeping $\mathcal{E}$, Ψ_* and μ fixed. The kinetic equation Eq. (2) can be written as

$$\left(Sv_\| + u_*\right)\mathbf{b}\cdot\nabla\left[H_i + \frac{Iv_\| f_{Mi}}{\Omega_i}\left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial \ln T_i}{\partial \Psi}\right] - C^l_{ii}\left\{H_i\right\} = 0, \tag{3}$$

where we have introduced $H_i = h_{1i} - (Iv_\| f_{Mi}/\Omega_i)\left[Mv^2/(2T_i) - 5/2\right]\partial_\Psi T_i$.

In the plateau regime, the form of the collision operator cannot affect the transport when the kinetic equation is written in the form of (3). Therefore we can use a simple Krook operator to model the collisions. However, the replacement $C^l_{ii}\{H_i\} \to -\nu H_i$ destroys the momentum conserving property of the operator. This defect is remedied by adding a homogeneous solution to H_i, making use of $C^l_{ii}\{v_\| f_M\} = 0$. Accordingly $H_i \to H_i + MBkv_\| f_{Mi}/T_i$ where the unknown k is to be determined by requiring that the solution gives no radial particle flux.

After the replacements the kinetic equation becomes

$$\left(Sv_\| + u_*\right)\mathbf{b}\cdot\nabla H_i + \nu H_i = -\left(Sv_\| + u_*\right)\mathbf{b}\cdot\nabla\left\{\frac{Iv_\| f_{Mi}}{\Omega_i}\left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial \ln T_i}{\partial \Psi} + \frac{MBkv_\| f_{Mi}}{T_i}\right\}. \tag{4}$$

Employing $\mathbf{b}\cdot\nabla|_{\mathcal{E},\mu,\Psi_*}\mathcal{E} = 0$ we find the relations $Sv_\| + u_*)\mathbf{b}\cdot\nabla\left(v_\|/\Omega_i\right) = 1/(2\Omega_i)(2v_\|^2 + v_\perp)\mathbf{b}\cdot\nabla \ln R$ and $(Sv_\| + u_*)\mathbf{b}\cdot\nabla(v_\| B) = B/2[v_\perp^2 - (4S-2)v_\|^2 - 4v_\| u_*]\mathbf{b}\cdot\nabla \ln R$.

The plateau regime ($\epsilon^{1/2} \ll \nu_i qR/v_i \ll 1$) solution for large aspect ratio ($\epsilon \ll 1$, where $\epsilon = r/R_0$ with the minor radius r) is

$$H \approx Q_i\left[\pi\delta\left(Sx_\| + \frac{u_*}{v_i}\right)\sin\theta - \frac{\cos\theta}{Sx_\| + \frac{u_*}{v_i}}\right], \tag{5}$$

where $x = v/v_i = (x_\perp^2 + x_\|^2)^{1/2}$ and

$$Q_i = \epsilon v_i f_{Mi}\left\{\frac{2x_\|^2 + x_\perp^2}{2\Omega_i} I\left(x^2 - \frac{5}{2}\right)\frac{\partial \ln T_i}{\partial \Psi} + \frac{MBk}{2T_i}\left[x_\perp^2 - (4S-2)\,x_\|^2 - 4x_\| \frac{u_*}{v_i}\right]\right\}. \tag{6}$$

The full gyro-averaged perturbed distribution $\bar{f}_{1i} = \langle f_i - f_{Mi}\rangle_\varphi$ is given by

$$\bar{f}_{1i} = h_i - \frac{Iv_\|}{\Omega}\frac{\partial f_{Mi}}{\partial \Psi} = H_i + \frac{MBkv_\| f_{Mi}}{T_i} - \frac{Iv_\| f_{Mi}}{\Omega_i}\left(\frac{\partial \ln p_i}{\partial \Psi} + \frac{Ze}{T_i}\frac{\partial \Phi}{\partial \Psi}\right). \tag{7}$$

In order to determine the unknown k, we now make the radial ion particle transport vanish $0 = \langle \mathbf{\Gamma}_i \cdot \nabla\Psi\rangle = \left\langle \int d^3v \bar{f}_{1i}\mathbf{v}_d\cdot\nabla\Psi\right\rangle \approx -\left\langle I\epsilon/(2\Omega qR)\int d^3v(2v_\|^2 + v_\perp^2)H_i \sin\theta\right\rangle$, where $\mathbf{v}_d$ is the magnetic drift velocity. Note that from all the terms in $\bar{f}_{1i}$ only the $\propto \sin\theta$ resonant part of H_i has a finite contribution to the cross-field transport fluxes. The velocity integral is to be performed holding Ψ constant, thus $H_i(\Psi_*)$ needs to be transformed back to flux surfaces $u_*(\Psi_*) \to u(\Psi) + (1-S)v_\|$. Accordingly, $\delta(Sx_\| + u_*/v_i) \to \delta(x_\| + U)$ and $x_\perp^2 - (4S-2)\,x_\|^2 - 4x_\| u_*/v_i \to x_\perp^2 - 2x_\|^2 - 4x_\| U$, where we introduced $U = u/v_i$. This shows that the resulting transport is insensitive to the orbit squeezing. Now the integrals can be evaluated yielding

$$\langle \boldsymbol{\Gamma}_i \cdot \nabla\Psi \rangle \approx -\sqrt{\frac{\pi}{2}} \frac{I^2 \epsilon^2 n_i}{\Omega_i^2 q R_0} \left(\frac{T_i}{M}\right)^{3/2} \times e^{-U^2} \left\{ \left(\frac{1}{2} - U^4 + 2U^6\right) \frac{\partial \ln T_i}{\partial \Psi} + \left[1 + 2\left(U^2 + U^4\right)\right] \frac{Zek\langle B^2 \rangle}{IT_i c} \right\}, \quad (8)$$

where n_i is the ion density. The ambipolarity condition $0 = \langle \boldsymbol{\Gamma}_i \cdot \nabla\Psi \rangle$ requires that

$$k = -\frac{J(U^2)}{2} \frac{\partial \ln T_i}{\partial \Psi} \frac{IT_i c}{Ze\langle B^2 \rangle}, \quad \text{with} \quad J(U^2) = \frac{1 - 2U^4 + 4U^6}{1 + 2\left(U^2 + U^4\right)}. \quad (9)$$

The preceding calculation of J is based on the observation that if we artificially set $k = 0$ in Eq. (8) the resulting ion particle flux would be much higher than the electron particle flux. However, for higher values of U the $\exp(-U^2)$ factor appearing in the expression for the ion particle flux (8) reduces it to the level of neoclassical electron transport. Therefore, our ambipolarity assumption must be modified to include the electrons. As this does not happen until around $U = 3.5$, it need not concern us here.

Having calculated the full H_i distribution, we can evaluate the radial ion heat flux

$$\langle \mathbf{q}_i \cdot \nabla\Psi \rangle = \left\langle \int d^3v \frac{Mv^2}{2} \bar{f}_{1i} \mathbf{v}_d \cdot \nabla\Psi \right\rangle \approx -3\sqrt{\frac{\pi}{2}} \frac{I^2 \epsilon^2 p_i}{\Omega_i^2 q R_0} \left(\frac{T_i}{M}\right)^{3/2} \frac{\partial \ln T_i}{\partial \Psi} L(U^2), \quad (10)$$

with

$$L(U^2) = e^{-U^2} \frac{1 + 4\left\{U^2 + 2U^4 + \left[(4U^6 + U^8)/3\right]\right\}}{1 + 2(U^2 + U^4)}. \quad (11)$$

The preceding reduces to the standard plateau result [5] in the $U \to 0$ limit.

To calculate the bootstrap current the ion parallel flow needs to be evaluated. Neglecting the small local contribution from H_i we obtain

$$n_i V_{\|i} = \int d^3v v_\| \bar{f}_{1i} \approx -\frac{Ip_i}{M\Omega_i} \left[\frac{\partial \ln p_i}{\partial \Psi} + \frac{Ze}{T_i} \frac{\partial \Phi}{\partial \Psi} + \frac{J(U^2)}{2} \frac{B^2}{\langle B^2 \rangle} \frac{\partial \ln T_i}{\partial \Psi} \right]. \quad (12)$$

To relate the poloidal flow of a collisional trace impurity to the poloidal flow of a background ion in the plateau regime we note that the flux surface average of B times their parallel flows must be related by $\langle BV_{\|i} \rangle = \langle BV_{\|Z} \rangle$ [4]. Using radial pressure balance for the ions and impurities along with the preceding result for $V_{\|i}$ gives the impurity poloidal flow to be

$$V_{Z,\theta} = \frac{cIT_i B_\theta}{eZ_i \langle B^2 \rangle} \left[\frac{T_z Z_i}{T_i Z_Z} \frac{\partial \ln p_Z}{\partial \Psi} - \frac{\partial \ln p_i}{\partial \Psi} - \frac{J(U^2)}{2} \frac{\partial \ln T_i}{\partial \Psi} \right]. \quad (13)$$

Bootstrap Current

Electron orbits are practically unaffected by the strong radial electric field due to their large thermal speed. However, electron-ion collisions depend on the ion mean flow, the electron distribution experiences this friction and is thereby indirectly influenced by the presence of the electric field.

Simply calculating the resonant part of the distribution function from the electron drift-kinetic equation ($v_\parallel \mathbf{b}\cdot\nabla h_{1e} + e f_{Me} v_\parallel E_I/T_e = C_e^{(1)}\{\bar{f}_{1e}\}$, with $E_I = \mathbf{b}\cdot\nabla(\mathbf{E}+\nabla\Phi)$ and $\bar{f}_{1e} = h_{1e} - (Iv_\parallel/\Omega_e)\partial_\Psi f_{Me}$ [5]) and evaluating the parallel current as $\langle j_\parallel B\rangle = e\langle B\int d^3v\, v_\parallel(Z\bar{f}_{1i} - \bar{f}_{1e})\rangle$ would not give the correct bootstrap current J_{BS} since h_e is not accurate enough. Instead, it is convenient to use an adjoint method [6] to find $\langle j_{BS}B\rangle$. The method relies on the solution of the Spitzer problem $C_e^{(1)}\{f_S\} = eE_I v_\parallel f_{Me}/T_e$, which can be calculated in terms of generalized Laguerre polynomials, using a variational method. For the bootstrap current we obtain [7]

$$\langle j_{BS}B\rangle = -\sqrt{\frac{\pi}{2}}\frac{\epsilon^2 cIp_e v_e}{\nu_e q R_0}\frac{\sqrt{2}+4Z}{Z(2+\sqrt{2}Z)}\left[\frac{1}{p_e}\frac{\partial p}{\partial\Psi} + \frac{\sqrt{2}+13Z}{2\left(\sqrt{2}+4Z\right)}\frac{1}{T_e}\frac{\partial T_e}{\partial\Psi} + \frac{J(U^2)}{2ZT_e}\frac{\partial T_i}{\partial\Psi}\right]. \quad (14)$$

Discussion

As the electric field increases, the resonance causing the plateau transport, which would be at $v_\parallel \approx 0$ for $U = 0$, is now shifted towards the tail of the distribution. For strong electric field $U \gg 1$ this leads to an exponential reduction of the ion heat flux (See Fig. 1). However, for moderate values of U the ion heat diffusivity is enhanced $L(|U| \approx 0.91) \approx 1.46$.

The temperature gradient driven part of parallel ion flow is multiplied by factor $J(|U|)$ that decreases until $J(|U| \approx 0.76) \approx 0.39$ then it starts to increase approaching an asymptote of $2U^2 - 3$. The same factor appears also in the expressions for the poloidal impurity rotation and the bootstrap current multiplying the ion temperature gradient term.

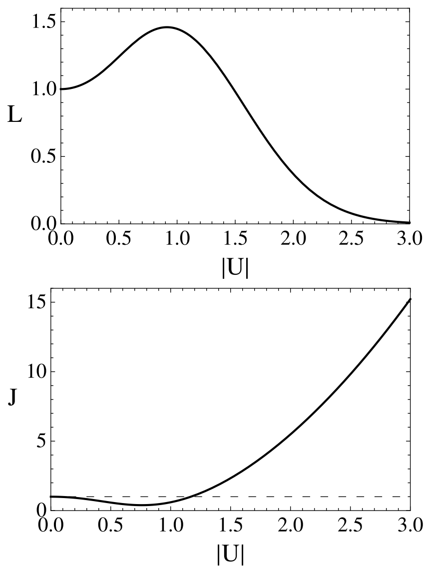

Figure 1: The $L(|U|)$ factor of the ion heat flux, and the $J(|U|)$ factor of the $\partial_\Psi \ln T_i$ term in the ion flows and the bootstrap current.

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