Physical mechanism of ripple resonance diffusion of alpha particles in tokamaks

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Introduction

The confinement of fusion produced α particles is important to maintain burning plasmas in tokamaks. Although α particles are well confined in axisymmetric fields, it has been shown that the loss of α particles due to magnetic field ripple caused by discrete toroidal field (TF) coils is dominant in the diffusion process in actual tokamaks with orbit following Monte Carlo simulations [1, 2]. However the understanding of the loss processes in detail is not sufficient.

Since the radial displacement of a banana orbit by ripples depends on the toroidal phase at the banana tip [3], the cumulated radial displacement becomes resonantly large when the difference in the toroidal angles of successive banana tips is a multiple of the toroidal angle of adjacent TF coils (the ripple resonance). This toroidal distance of successive banana tips is determined by toroidal precession. Yushmanov theoretically analyzed this ripple resonance diffusion by means of the banana-drift kinetic equation without the radial change of the toroidal precession because it is much less than that of the toroidal length of the banana orbit [4]. Since the toroidal precession, however, strongly depends on the radial position in an actual tokamak, we investigate the ripple resonance diffusion in a realistic system with the radial change of the toroidal precession by numerical calculations based on orbit following Monte Carlo simulation.

Calculation Model

In this paper, the trajectory of an α particle is numerically evaluated with guiding center equations, in the same way as OFMC code [1, 2]. In an axisymmetric vacuum toroidal field, the guiding center equations conserve the toroidal canonical momentum $P_\phi = mRv_\phi + e\psi$, where R, v_ϕ and ψ are the major radius, the toroidal component of velocity and the poloidal magnetic flux, respectively. The velocities of α particles are also changed by Coulomb collisions with a bulk plasma. Then the velocity of α particles is changed according to both guiding center equations and Coulomb collisions. Diffusion coefficients are evaluated by a rate of change in a variance of the toroidal canonical momentum P_ϕ which corresponds to the poloidal magnetic flux at a banana tip, $e\psi_\mathrm{b} = P_\phi|_{v_\phi=0}$.

Numerical Results on Ripple Diffusion

The dependence of the diffusion coefficients on energy, ripple strength and other parameters are calculated with test particles which are launched from the same banana tip position. All α particles with kinetic energy W and magnetic moment μ_{m} start from banana tips whose poloidal magnetic flux is fixed while toroidal angles are randomly given.

Figure 1 (red line) shows the energy dependence of diffusion coefficients in rippled fields. The radial displacement by the ripple depends on the toroidal phase of the banana tip [3]. Generally, this radial displacement is canceled during several bounce since the toroidal phase of every banana tip varies. However, when the toroidal angle difference between its successive banana tips equals a multiple of the toroidal angle between adjacent TF coils, it continues having the same radial displacement at every banana tip and its cumulative radial displacement becomes large resonantly (ripple resonance). This ripple resonance condition depends on the energy of an α particle. In the magnetic configuration used in this paper, ripple resonance energies are 0.4, 1.5, and 2.9 MeV.

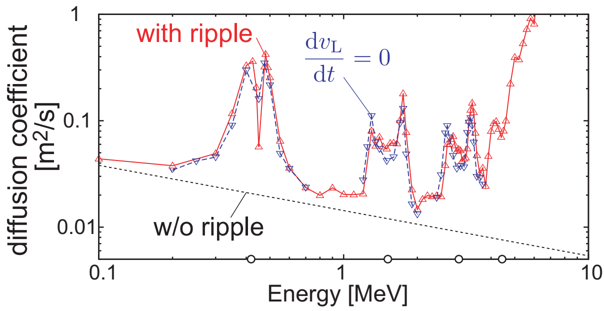

Figure 1: Energy dependence of the diffusion coefficient in rippled field.

The diffusion coefficients do not peak at the resonance energy but make an M-shaped dependence around the resonance energy. Because the ripple resonance of fast ions is a collisionless phenomena, collisionless orbits play an important role in the diffusion process in this energy range. Then we investigate collisionless orbits in rippled fields to understand the M-shaped dependence of diffusion coefficients in the next section.

Collisionless Orbit of Ripple Resonance Particle

The toroidal canonical momentum P_ϕ is conserved in axisymmetric fields and corresponds to the poloidal magnetic flux at the banana tip, ψ_{b}. However it is not conserved in rippled fields and has displacements $\Delta\psi$ mainly near banana tips. Since this displacement occurs mainly just near banana tips, the banana motion is described approximately by a mapping method which follows just banana tips.

To analyze a Poincaré map of (ϕ, ψ), we convert the mapping to differential equations whose solution is,

$$H(N\phi, \psi) \equiv \frac{N\phi'_{\mathrm{p}}(\psi - \psi_k)^2}{2\Delta_{\mathrm{b}}\cos(\Phi_k - \pi/4)} + \cos(N\phi + \Phi_k) = C, \tag{1}$$

where C is the integration constant. The structure of collisionless orbits around the resonance energy is, therefore, determined by these two parameters ψ_k and Φ_k. Since the characteristic times based on the collision are long enough compared to the bounce time, we can consider the following diffusion processes in the phase space:

1. orbits, on which particles initially exist, are determined by the initial condition,
2. particles are uniformly distributed on each orbit,
3. the diffusion is represented as the change in Hamiltonian H ($\Leftrightarrow C$), which is caused through the change in constants ψ_k and Φ_k by collisions.

Ripple Resonance Diffusion and its Energy Dependence

In a collisionless case, although particles spread in the phase space, they just keep periodic motion with $H = C$(const.), and diffusion does not occur. The diffusion is caused by collisions through the changes of ψ_k, Φ_k and H. Since particles spread along a collisionless trajectory in much shorter time than the collision time, we evaluated the diffusion coefficients of each trajectory labeled by $H = C$ on which initial positions of test particles were randomly distributed. Although H in the analytic representation (1) is constant in collisionless cases, H obtained from the guiding center orbit is not constant. Then, we introduce a new label of each trajectory, $\tilde{C}$, defined by

$$\tilde{C} \equiv \left(\frac{\psi - \psi_k}{\tilde{\Delta}_{\rm p}}\right)^2 - 1 = H(\pi - \Phi_k, \psi_k), \tag{2}$$

where $2\sqrt{2}\tilde{\Delta}_{\rm p}$ is the width of the island skimmed from the Poincaré map of the guiding center orbits. Diffusion coefficients on the evey orbit are calculated in Fig. 2(A). At $\tilde{C} > 1$ there are two plots corresponding to the upper and the lower side of the island. The Poincare plot of trajectories of particles with a fixed energy 1.5 MeV is shown in Fig. 2(B). The diffusion coefficients are large just outside the separatrix and become smaller with the distance from the separatrix. On the other hand, the diffusion coefficients inside the separatrix are small. This behavior can be interpreted as follows. When the particles outside the separatrix enter the separatrix by collisions, the averaged radial positions jump by about one half of the island width and hence the diffusion is enhanced.

When the energy and the magnetic moment of particles vary by collisions, the structure of the Poincaré map changes. Thus, the diffusion is caused not only by the transition of trajectories in Fig. 2(B) but also by the change of the structure of the Poincaré map. Even in that case, diffusion coefficients have maximum just outside of the $\tilde{C} = 1$ surface, because the separatrix is always on $\tilde{C}(= H) = 1$

This M-shaped dependence does not appear in Yushmanov's work [4] because of the neglect of the radial change in ϕ_p, although this radial change creates the island structure, which contributes to the ripple diffusion.

Conclusions

The energy dependences of the diffusion coefficients of α particles in rippled fields were investigated by using an orbit-following Monte-Carlo code. Concluding remarks are made as follows.

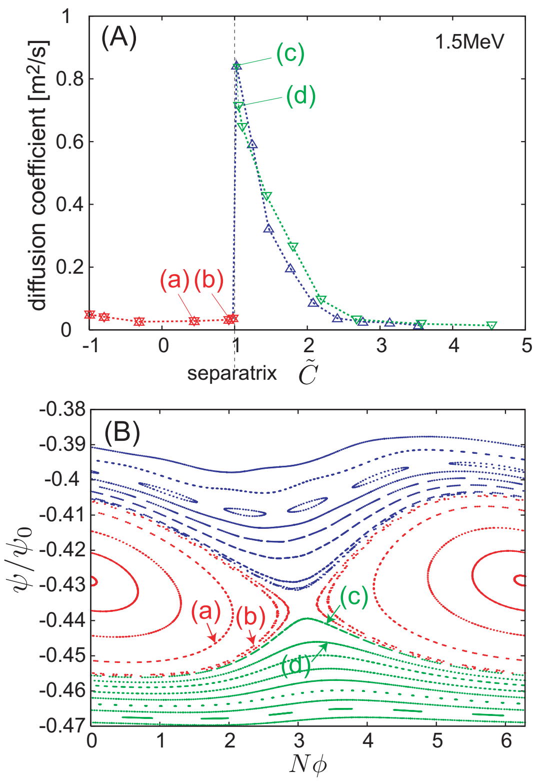

Figure 2: (A) $\tilde{C}$-dependence of diffusion coefficients at 1.5 MeV from which the neoclassical diffusion is subtracted. (B) Initial trajectories of the calculation of each diffusion coefficient in (A).

1. The diffusion coefficients are enhanced around the ripple resonance energy while they has a minimum near the resonance energy, and hence have an M-shaped dependence on the energy.

2. Analytical studies using a mapping method show that the particles just outside the separatrix mainly contribute to the ripple resonance diffusion, since they can enter the separatrix by collisions and spread over the island. Accordingly the diffusion coefficients are enhanced at both sides of the resonance energy where the ratio of the particles just outside the separatrix is high, and the diffusion coefficients have the M-shaped dependence on the energy.

In these processes, the radial changes in the toroidal precession was found to play a very important role. It is, therefore, clarified that the toroidal precession can not be neglected to investigate the ripple diffusion.

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