Ion acoustic wave in collisional plasmas

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The friction force effects on the ion acoustic (IA) wave are discussed. In a partially ionized plasma, the collisions with neutrals modify the IA wave beyond recognition. For a low density of neutrals the mode is damped. Upon increasing the neutral density, the mode becomes first evanescent and then reappears for a still larger number of neutrals. A similar behavior is obtained by varying the mode wave-length. The explanation for this behavior is given.

The starting set of equations includes the momentum equations for the ions, the electrons and the neutral particles, respectively,

$$m_i n_i \left(\frac{\partial}{\partial t} + \vec{v}_i \cdot \nabla\right) \vec{v}_i = -e n_i \nabla\phi - \kappa T_i \nabla n_i - m_i n_i \nu_{ie}(\vec{v}_i - \vec{v}_e) - m_i n_i \nu_{in}(\vec{v}_i - \vec{v}_n), \quad (1)$$

$$m_e n_e \left(\frac{\partial}{\partial t} + \vec{v}_e \cdot \nabla\right) \vec{v}_e = e n_e \nabla\phi - \kappa T_e \nabla n_e - m_e n_e \nu_{ei}(\vec{v}_e - \vec{v}_i) - m_e n_e \nu_{en}(\vec{v}_e - \vec{v}_n), \quad (2)$$

$$m_n n_n \left(\frac{\partial}{\partial t} + \vec{v}_n \cdot \nabla\right) \vec{v}_n = -\kappa T_n \nabla n_n - m_n n_n \nu_{ni}(\vec{v}_n - \vec{v}_i) - m_n n_n \nu_{ne}(\vec{v}_n - \vec{v}_e), \quad (3)$$

and the continuity equation

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \vec{v}_j) = 0, \quad j = e, i, n. \quad (4)$$

This set of equations is closed either by using the quasi-neutrality or the Poisson equation. The electron-ion friction terms in Eqs. (1) and (2) will cancel each other out [1] and in a few steps one derives the following dispersion equation containing the collisions of plasma species with neutrals and vice versa:

$$\omega^3 + i\omega^2 \left(\nu_{en}\frac{m_e}{m_i} + \nu_{in}\right)\left(1 + \frac{m_i}{m_n}\frac{n_0}{n_{n0}}\right) - k^2 c_s^2\, \omega - ik^2 c_s^2 \frac{m_e}{m_n}\frac{n_0}{n_{n0}}\left(\nu_{en} + \frac{m_i}{m_e}\nu_{in}\right) = 0. \quad (5)$$

In the derivation, the ion and neutral thermal terms are neglected. The ion thermal terms would give the modified mode frequency $\omega^2 = k^2 c_s^2(1 + T_i/T_e)$. Hence, even if $T_e = T_i$ the wave frequency is only modified by a factor $2^{1/2}$. The neutral thermal terms are discussed elsewhere [1].

Equation (5) is solved numerically for a plasma containing electrons, protons, and neutral hydrogen atoms using the following set of parameters: $T_e = 4$ eV, $n_0 = 10^{18}$ m^{-3}, $k = 10$ m^{-1}, with $\sigma_{en} = 1.14 \cdot 10^{-19}$ m^{-2}. The neutral density is varying in the interval $10^{16} - 10^{23}$ m^{-3}. The ion and hydrogen temperatures are taken $T_i = T_n = T_e/20$, satisfying the condition of their small thermal effects. This also gives $\sigma_{in} = 2.24 \cdot 10^{-18}$ m^{-2}. The results are presented in Fig. 1. The IA mode propagates in two distinct regions A and B.

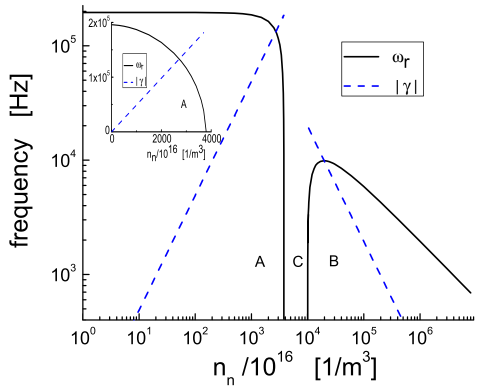

Figure 1: Frequency ω_r and absolute value of the IA mode damping $|\gamma|$ in terms of the number density of neutrals. The mode behavior in the region A, in the linear scale, is given in small figure inside.

Only a limited left part of the region A would correspond to the 'standard' IA wave behavior in a collisional plasma: the mode is damped and the damping is proportional to the neutral number density. Hence, in this region it may be more or less appropriate to use the approximate expressions for the friction force, like (in the case of electrons) $F_e \simeq m_e n_0 \nu_{en} v_{e1}$. However, this domain is very limited because in the rest of the domain the frequency drops and the mode becomes non-propagating for $n_{n0} \geq 3.8 \cdot 10^{19}$ m^{-3} (this is the lower limit of the region C in Fig. 1).

Increasing the neutral number density, after some critical value (in the present case this is around $n_{n0} \simeq 10^{20}$ m^{-3}) the IA mode reappears again in the region B, with a frequency starting from zero. For even larger neutrals number densities, the mode damping in fact vanishes completely and the wave propagates freely but with a frequency that is many orders of magnitude below the ideal case $kc_s \simeq$ 196 kHz. This behavior can be explained in the following manner. For a relatively small number of collisions the IA mode is weakly damped because initially neutrals do not participate in the wave motion and do not share the same momentum. Increasing the number of neutrals, the damping may become so strong that the wave becomes evanescent.

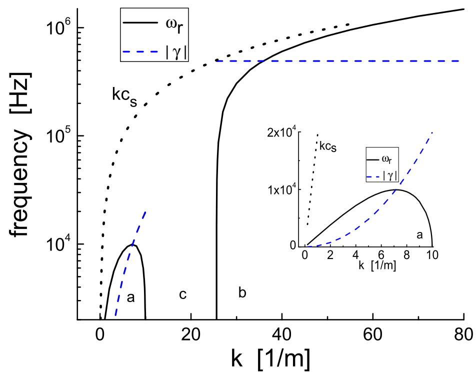

Figure 2: Frequency ω_r and the damping $|\gamma|$ in terms of the wave number. The line kc_s shows part of the graph of the ideal mode. The domain *a* is better understood in the linear scale (small figure).

However, for much larger collision frequencies (i.e., for a lower ionization ratio), the tiny population of electrons and ions is still capable of dragging neutrals along and all three components move together. The plasma and the neutrals become so strongly coupled that the two essentially different fluids participate in the electrostatic wave together. In this regime, the stronger the collisions are, the less wave damping there is! Yet, this a bit counter-intuitive behavior comes with a price: the wave frequency and the wave energy flux becomes reduced by several orders of magnitude.

Similar effects may be expected by varying the wave-length. The previous role of the varying density of neutrals is now replaced by the the ratio of the mean free path of a species $\lambda_{fj} = v_{Tj}/\nu_j$ (with respect to their collision with neutrals) and the wavelength. This ratio now determines the coupling between the plasma and the neutrals. The mode behavior is directly numerically checked by fixing $n_{n0} = 10^{20}$ m^{-3}, $n_0 = 10^{18}$ m^{-3}, and for other parameters same as above. For these parameters we have $\lambda_{fe} = v_{Te}/\nu_{en} = 0.09$ m, and $\lambda_{fi} = v_{Ti}/\nu_{in} = 0.004$. The numerical results are presented in Fig. 2 for k varying in the interval $0.2 - 80$ m^{-1}. The mode vanishes in the interval c, between $k \simeq 10$ m^{-1} and $k \simeq 25.6$ m^{-1}. The explanation is similar as before. Note that for $k = 0.2$ m^{-1} (in the region a) we have $\omega_r \simeq 390$ Hz, and this is about one order below kc_s. Compared to the mode behavior in Fig. 1, this implies that the mode in the present domain a is in the regime equivalent to the domain B from Fig. 1; here, in Fig. 2, these large wave-lengths imply well coupled plasma-neutrals, where the frequency is reduced and the damping is small. The region a is also given separately in linear scale together with the dotted line describing the ideal mode kc_s. Clearly, in general the realistic behavior of the wave is beyond recognition and completely different as compared to the ideal case.

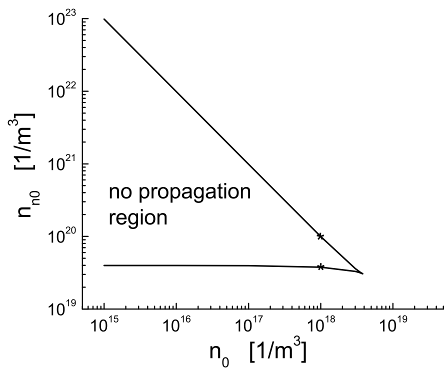

Figure 3: The two lines give the lower and upper values of the neutrals' density n_{n0} between which, for the given plasma density n_0, the IA mode does not propagate.

After checking for various sets of plasma densities, it appears that the evanescence region reduces and vanishes for larger plasma densities n_0. This is presented in Fig. 3 for the same parameters as above, by taking $k = 10$ m^{-1}, but for a varying plasma density n_0. The two lines represent boundary values of the number densities of neutrals, for the given plasma density, at which the IA mode vanishes; for the neutrals densities between the two lines the IA mode does not propagate. The symbols $*$ on the two lines denote the boundaries of the region C from Fig. 1. It is seen that for the given case the IA mode propagates without evanescence for the plasma densities above $n_0 = 3.8 \cdot 10^{18}$ m^{-3}. Physical reason for a larger non-propagating domain for low plasma density is obvious, namely the tiny plasma population is less efficient in inducing a synchronous motion of neutrals. In the other limit, the opposite happens and the forbidden region eventually vanishes.

A similar check is done by varying the wave-number and the plasma density, and the result is presented in Fig. 4 for a fixed $n_{n0} = 10^{20}$ m^{-3}. The lines represent the values (n_0, k) at which the IA wave becomes evanescent. There can be no wave in the region between the lines. On the other hand, there is no evanescence for the plasma density above $n_0 = 1.2 \cdot 10^{19}$ m^{-3}. Here $*$ denote the boundaries of the region c from Fig. 2.

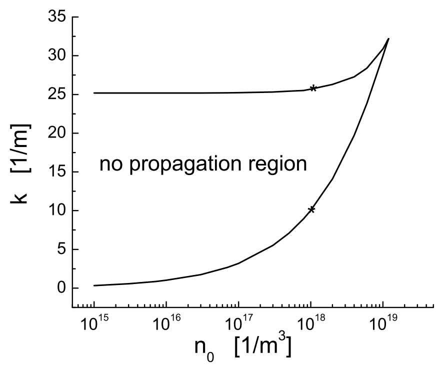

Figure 4: Values of the wave-number, in terms of the plasma density, for which the IA wave becomes evanescent. In the region between the lines the mode does not propagate.

All these results clearly indicate that without a proper analytical description, the identification of the mode in the laboratory and space observations may be rather difficult because one might fruitlessly search for the wave in a very inappropriate domain, as can be concluded from the graphs presented here, and in particular from Fig. 2. Not only the wave frequency may become orders of magnitude below an expected ideal value, but also the mode may completely vanish. The impression is that these effects are frequently overlooked in the literature, hence the necessity for the quantitative analysis given in the present work that can be used as a good starting point for an eventual experimental check of the wave behavior in collisional plasmas.

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References

[1] J. Vranjes and S. Poedts, Phys. Plasmas **17**, 022104 (2010).