Coherent structures in multipy-charged dusty plasma

K. Annou, S. Bahamida and R. Annou.

Theoretical physics laboratory, faculty of physics, USTHB,

Abstract

Finite amplitude localised electrostatic solitons in a multi-component dusty plasma are presented. Assuming that the constituents of dusty plasma are warm electrons, nonthermal ions, and an admixture of cold dust grains with negative and positive charges ,it is shown that stationary solutions of the fluid equation combined with Poisson's equation can be expressed in terms of the energy integral of a classical particle with a Sagdeev Potential. Furthermore, the fore-fluid dusty plasma system, with both negative and positively charged dust grains, and nonthermal ions, also provides the possibility of double layers.

Introduction

Since the discovery of dust acoustic waves (DAW) by Rao *et a*l., **[1],** there has been a great deal of interest in understanding different types of collective presses in dusty plasma, because of its vital roles in the study of astrophysical and space environments. In their paper Rao et al. also introduced a theory for dust acoustic solitons in three-component dusty plasma with negatively charged dust grains. They pointed out the possibility of a finite-amplitude rarefactive dust-acoustic potential, in contrast to a compressional potential that is associated with the usual ion-acoustic soliton in plasma without the dust component. Recently it has been suggested that positively and negatively charged dust grains can co-exist in space **[2,4]** and laboratory plasma. On the other hand, it has been found that the presence of nonthermal ions provides the possibility of co-existence of positive and negative dust-acoustic solitary waves [5] .The present study has assumed dusty plasma consisting of warm electrons, nonthermal ions, and an admixture of cold dust grains with negative and positive charges and has investigated the pseudo-potential approach, which is valid for arbitrary amplitude solitary waves. It has been found that the plasma system, under consideration, gives rise to such interesting features of the nonlinear structures as the compressional DA potential distribution and the monotonic double layer, which otherwise are absent.

Model equations

We consider four-component dusty plasma with extremely massive, micron-sized, negatively and positively charged inertial dust grains, boltzmannian electrons and nonthermally distributed ions. Thus at equilibrium, we have $N_{e0} + Z_n N_{n0} = N_{i0} + Z_p N_{p0}$ where,

N_{i0}, N_{e0} are the ions, electrons number densities, Z_{p0}, Z_{n0} are the positive and negative dust particle charge, N_{p0}, N_{n0} dust particle number densities respectively.

The dust particles are assumed to be point charged and their sizes are much smaller than the Debye length. For low velocity dust ion acoustic waves, electrons are Boltzmann distributed and the ions are nonthermaly distributed.

$N_e = N_{e0} e^{e\phi/T_e}$; $N_e = N_{e0} e^{e\phi/T_e}$.The dynamics of dust grains are governing by

$\frac{\partial N_n}{\partial t} + \frac{\partial}{\partial x}(N_n V_n) = 0$, $\frac{\partial V_n}{\partial t} + V_n \frac{\partial V_n}{\partial x} = \frac{Z_n e}{M_n}\frac{\partial \phi}{\partial x}$ for negative dust grains

and $\frac{\partial N_p}{\partial t} + \frac{\partial}{\partial x}\left(N_p V_p\right) = 0$, $\frac{\partial V_p}{\partial t} + V_P \frac{\partial V_p}{\partial x} = -\frac{Z_p e}{M_p}\frac{\partial \phi}{\partial x}$ for positive grains.

Here are the fluid velocities and mass of the positively and negatively charged dust grains, respectively.The system of equations is closed by the Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi\left(N_e - N_i + Z_n N_n - Z_p N_p\right)$$

Normalization

Where n_d is the dust particle number density normalized to n_{d0}; μ_d is the dust fluid velocity normalized to the dust acoustic speed $C_d = (Z_d T_i / m_d)^{1/2}$, with T_i being the ion temperature in units of the Boltzmann constant and m_d being the mass of negatively charged dust particles; ϕ is the electrostatic wave potential normalized to T_i/e, with e being the magnitude of the electron charge. The time space variables are in the units of dust plasma period $\omega_{dp}^{-1} = \left(m_d / 4\pi Z_d^2 e^2\right)^{1/2}$ and the Debye length $\lambda_{Dd} = \left(T_i / 4\pi Z_d n_{d0} e^2\right)^{1/2}$, we define then

$\Phi = \frac{e\phi}{T}$, and $V(\Phi) = \frac{V(\phi)}{4\pi N_0 T_0}$, $n_{e0} = \frac{N_{e0}}{N_0}$, $n_{i0} = \frac{N_{i0}}{N_0}$, $a_e = \frac{T_0}{T_e}$, $a_i = \frac{T_0}{T_i}$

Arbitrary large amplitude non-linear dust acoustic waves

In this section we'll be looking for arbitrary large amplitude solutions of the nonlinear equations in the stationary frame $\xi = x - V_0 t$ (unnormalized). With the conditions that at $\xi \to -\infty$, $N_p \to N_{p0}$ and $V_p \to 0$, we obtain For positive (resp. Positive) dust particle number density: $N_p = N_{p0} \Big/ \sqrt{1 - \frac{2 Z_p e}{M_p V_0^2}\Phi}$, $N_n = N_{n0} \Big/ \sqrt{1 - \frac{2 Z_n e}{M_n V_0^2}\Phi}$

Inserting densities expressions in equation (Poisson), we obtain, in the stationary frame

$$\frac{\partial^2\phi}{\partial\xi^2}=4\pi e\left\{N_{e0}\exp\left(\frac{e\phi}{T_e}\right)-N_{i0}\left[1+\beta\left(-\frac{e\phi}{T_i}\right)+\beta\left(\frac{e\phi}{T_i}\right)^2\right]\exp\left(-\frac{e\phi}{T_i}\right)+N_nZ_n-Z_pN_p\right\}$$

Multiplying both sides of equation by $\partial\phi/\partial\xi$ and integrating once from $-\infty$ to ξ, with the conditions $\phi\to 0$ and $\partial\phi/\partial\xi\to 0$ at $\xi\to-\infty$, we obtain the energy integral equation's

$\frac{1}{2}\left(\frac{\partial\Phi}{\partial\xi}\right)^2+V(\Phi)=0$. Where

$$-V(\Phi)=\frac{n_{e0}}{a_e}\left(\exp(a_e\Phi-1)\right)+\frac{n_{i0}}{a_i}\left(\exp(-a_i\Phi)\left[1+3\beta-\beta a_i\Phi-\beta a^2{}_i\Phi^2\right]-(1+3\beta)\right)+\frac{n_n}{a_n}\left(\sqrt{1+2a_n\Phi}-1\right)$$
$$+\frac{n_p}{a_p}\left(\sqrt{1-2a_p\Phi}-1\right)$$

$$\frac{dV(\Phi)}{d\Phi}=-\left[n_{e0}\exp(a_e\Phi)-n_{i0}\exp(-a_i\Phi)+\frac{n_n}{\sqrt{1+2a_n\Phi}}-\frac{n_p}{\sqrt{1+2a_p\Phi}}\right].$$

Results and figures of large amplitude solutions

Condition under which soliton allows

i/ $V(\Phi)=V'(\Phi)=0$ pour $\Phi=0$;

ii/ $V(\Phi)=0$ pour $\Phi=\Phi_1\neq 0$;

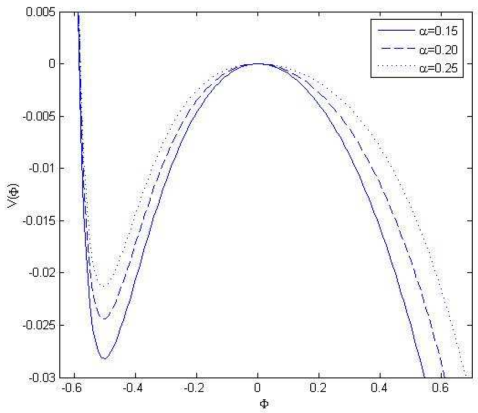
iii/ $V(\Phi)<0$ pour $0<|\Phi|<|\Phi_1|$.

Condition under which Double Layer allows:

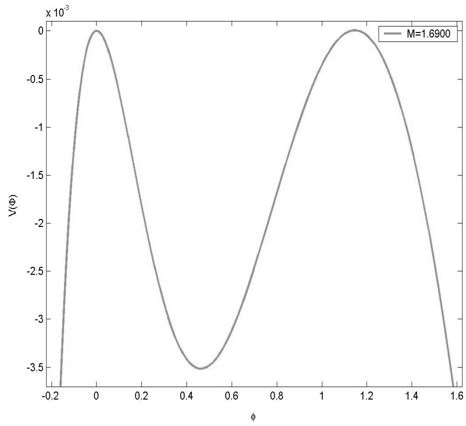
i/ $V(\Phi)=V'(\Phi)=0$ pour $\Phi=0$;

ii/ $V(\Phi)=V'(\Phi)=0$ pour $\Phi=\Phi_1\neq 0$;

iii/ $V(\Phi)<0$ pour $0<|\Phi|<|\Phi_1|$.



Potentiel de Sagdeev $V(\Phi)$ vs. Φ .
$M=1.40$, $a_e=0.102$ $a_{dp}=0.016$ $a_{dn}=0.200$



Potentiel de Sagdeev $V(\Phi)$ vs. Φ .
for $\alpha=0.20$, $a_e=0.100$, $a_{dp}=0.163$, $a_{dn}=0.200$

Small amplitude limit: K dV solutions

To study the dynamics of small amplitude dust-acoustic solitary waves, we derive the Korteweg de Vries equation (k-dv) equation from our basic equation by employing the reductive perturbation technique and the stretched coordinate $\xi = \varepsilon^{1/2}(x - V_0 t)$ and $\tau = \varepsilon^{3/2} t$ where ε is a smallness parameter measuring the weakness of the amplitude or dispension and v_0 is the soliton velocity (normalized to C_d)We can then expend our variables about the unperturbed states in power series of ε and develop the equations in various power of ε ; $\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \ldots$, ; $N_{dn} = N_{dn0} + \varepsilon N_{dn1} + \varepsilon^2 N_{dn2} + \varepsilon^3 N_{dn3} + \ldots$,

$N_i = N_{i0} + \varepsilon N_{i1} + \varepsilon^2 N_{i2} + \varepsilon^3 N_{i3} + \ldots$, ; $V_{dn} = \varepsilon V_{dn1} + \varepsilon^2 V_{dn2} + \varepsilon^3 V_{dn3} + .$,

Results and figures for small amplitude solutions

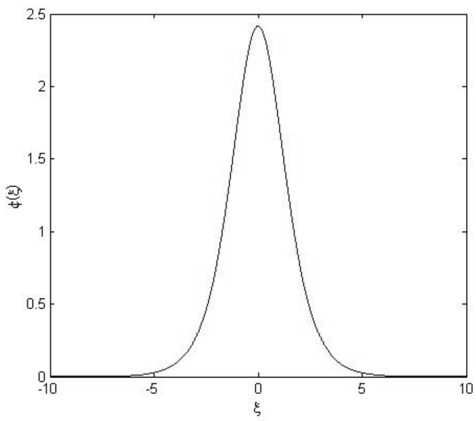
For K-Dv equation

$\xi = \varepsilon^{1/2}(x - V_0 t), \tau = \varepsilon^{3/2} t$,

$$\frac{\partial\Phi_1}{\partial\tau} + \frac{\eta}{2}\Phi_1\frac{\partial\Phi_1}{\partial\xi} + \frac{1}{2}\frac{\partial^3\Phi_1}{\partial\xi^3} = 0 .$$

$$\eta = \frac{\mathbf{1}}{\mathbf{2}}\begin{bmatrix} \left(\mathbf{3}a_{dp\mathbf{1}}^{\mathbf{2}} n_{dp} + a_i^{\mathbf{2}} n_{i\mathbf{0}}\left(\mathbf{1} + \mathbf{2}\beta\right)\right) + \\ -\left(\mathbf{3}a_{dn\mathbf{1}}^{\mathbf{2}} n_{dn} + a_e^{\mathbf{2}} n_{e\mathbf{0}}\right) \end{bmatrix}$$

$$\boxed{\Phi_1 = \frac{3\tilde{M}}{\eta} Sech^2\left[\left(\xi - \tilde{M}\tau\right)/\omega\right]}.$$



Φ Vs $\zeta = \xi - M\tau$

for $M = 1.40$, $a_e = 0.102$, $a_{dp} = 0.016$,

$a_{dn} = 0.200$ et $\alpha = 0.20$.

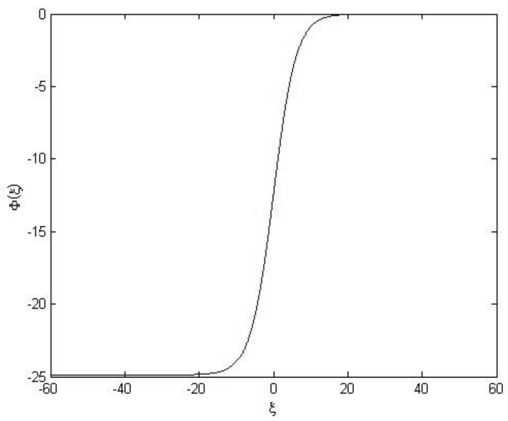
For mK-dV equation

$\xi = \varepsilon(x - V_0 t)$ and $\tau = \varepsilon^3 t$,

$$\frac{\partial\Phi_1}{\partial\tau} + \frac{\eta}{2}\Phi_1\frac{\partial\Phi_1}{\partial\xi} + \kappa\frac{\partial\Phi_1^3}{\partial\xi} + \frac{1}{2}\frac{\partial^3\Phi_1}{\partial\xi^3} = 0$$

$$\kappa = \frac{\mathbf{1}}{\mathbf{2}}\begin{bmatrix} -\frac{\mathbf{1}}{\mathbf{6}}\left(n_{e\mathbf{0}} a_e^{\mathbf{3}} + n_{i\mathbf{0}} a_i^{\mathbf{3}}\left(\mathbf{1} - \mathbf{3}\beta^{\mathbf{2}}\right)\right) + \\ + \frac{\mathbf{5}}{\mathbf{3}}\left(n_{dn} a_{dn}^{\mathbf{3}} + n_{dp} a_{dp}^{\mathbf{3}}\right) \end{bmatrix}$$

$$\boxed{\Phi_1(\xi) = \frac{\Phi_{11}}{2}\left[1 - \tanh(\omega\xi)\right]}$$



Φ Vs $\zeta = \xi - M\tau$

$\alpha = 0.20$, $a_e = 0.100$, $a_{dp} = 0.163$

$a_{dn} = 0.200$ et $M = 1.690$

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