Symplectic Maps for Divertor Configurations

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The objective of this work is to introduce a symplectic mapping to study magnetic field lines near the separatrix of a tokamak with a single-null poloidal divertor.

We apply a methodology proposed in [1,2] to obtain an integrable map for an MHD equilibrium, with an arbitrary topology [1], perturbed by helical resonances due to an ergodic limiter described by the Martin-Taylor map [3].

Following the mentioned methodology, we choose a potential V(x) that produces a topology with an X-point, with a flux function (Hamiltonian) ψ given by

$$\Psi = \frac{y^2}{2} + V(x) \qquad (1)$$

with

$$V(x) = \begin{cases} x^2/2 & x < x_1 \quad \text{(I)} \\ -(x-c)^2/2 + d & x_1 \le x \le x_2 \quad \text{(II)} \\ (x-a)^2/2 + b & x \ge x_2 \quad \text{(III)} \end{cases} \qquad (2)$$

The continuous equations are transformed in a discrete map, where the continuous time parameter t is turned into a discrete time step:

$$\begin{aligned} x(x_0, y_0, t) &\to x_{n+1}(x_n, y_n, \Delta) \\ y(x_0, y_0, t) &\to y_{n+1}(x_n, y_n, \Delta) \end{aligned} \qquad (3)$$

We associate the time step $\Delta(\psi)$ to the safety factor q (ψ) obtained by numerically integrating the field lines.

The plasma magnetic field is a superposition of a uniform toroidal field and a poloidal field created by the current density j_Z:

$$j_z = \frac{I_p(\gamma+1)}{\pi a^2}\left(1 - \frac{r^2}{a^2}\right)^{\gamma} \qquad (4)$$

An infinite wire carrying a current I_d outside the cylinder gives the divertor field. Given these fields, we calculate the safety factor q to obtain Δ from

$$\Delta = \frac{2\pi}{q(\psi)} \quad (5)$$

To obtain the map, we consider lines starting at (x_0, y_0) with ψ given by $\psi = \psi(x_0, y_0)$ and $\Delta = \Delta(\psi)$.

The symplectic equilibrium map

$$M_\Delta(x_n, y_n) = (x_{n+1}, y_{n+1}) \quad (6)$$

gives the Poincaré map of Fig. 1.

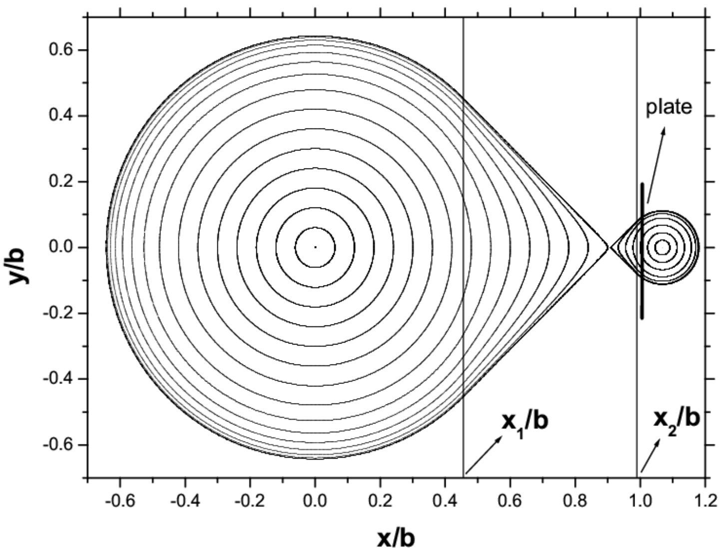

Fig. 1. Invariant surfaces obtained from the divertor map of Eq. (6).

To take into account perturbations that destroy magnetic surfaces at the SOL, we consider the symplectic Martin-Taylor map that describes perturbations created by an ergodic magnetic limiter [3]. Thus, this map describes the perturbation applied to the resulting coordinates (x*, y*) obtained by applying the divertor map to the coordinates (x_n, y_n), after the n-th toroidal turn. Then, the perturbed map can be represented by [1]:

$$\begin{aligned} (x^*, y^*) &= M_{\Delta_n}(x_n, y_n) \\ (x_{n+1}, y_{n+1}) &= P(x^*, y^*) \end{aligned} \quad (7)$$

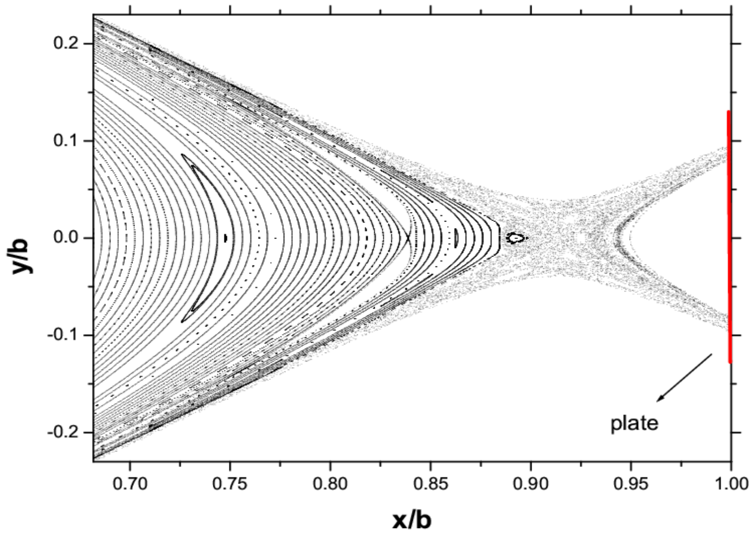

Fig. 2 : Poincaré map for perturbation current $I_h = 0.2\ I_p$. The vertical red line represents the divertor collector plate.

One example of a chaotic region obtained around the X-point is shown in Fig. 2. One consequence of the obtained chaotic configuration is a fractal like structure of the connection lengths calculated to respect to the divertor plate, as shown in Fig. 3. The connection lengths are calculated for field lines from the lower part of the divertor plate (negative values of y) to the upper part of the plate (positive values of y). We consider an Y_0 position, which represents the initial condition, takes from the plate around Y_{S-} position, which is the lowest vertical position where the separatrix reaches the plate. We take initial conditions close to Y_{S-} and write $\Delta y_0 = y_0 - y_{S-}$ as the displacement of initial condition with respect to the lower branch of which the unperturbed separatrix intersects the plate.
We can see the fractal distribution of connection lengths for lines in the chaotic (laminar) region.

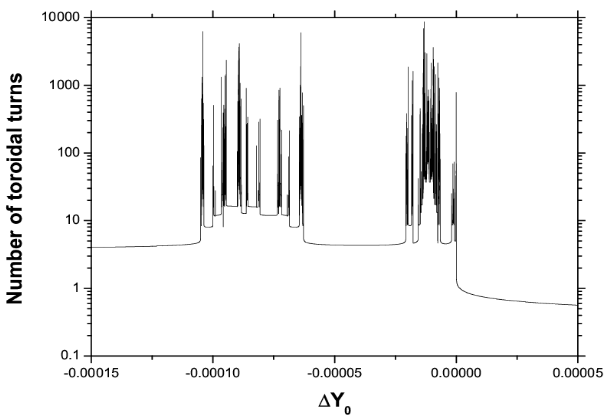

Fig.3. Connection length as a function of Δy_0 for $I_h = 0.2$ Ip.

To incorporate elongation and triangularity to the equilibrium configuration described by our model, we consider another potential V(x) and calculate a new Δ(ψ) profile, which gives the map presented in Fig.4 .

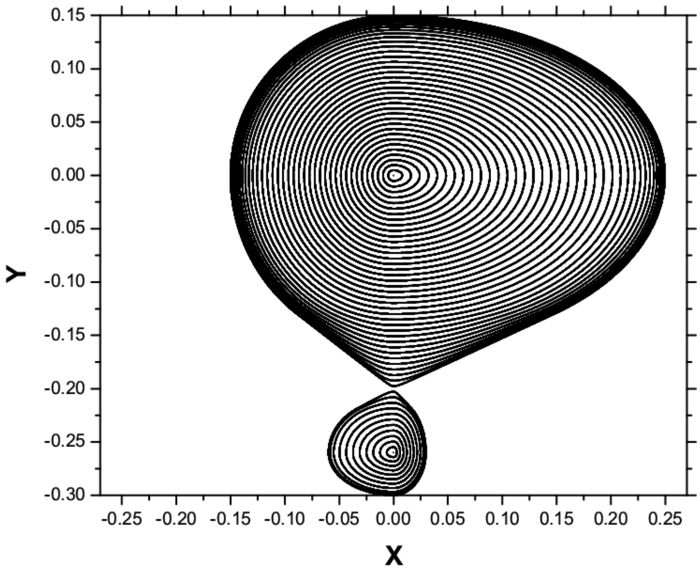

Fig.4. Magnetic surfaces obtained from the map including elongation and triangularity.

Conclusions

We present a symplectic map for field lines in tokamaks that describes a single null-divertor and resonant perturbations due to an ergodic limiter. This map can be applied to reproduce qualitatively fractal connection lengths, escape line distribution, and footprints observed in divertor experiments.

References

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