Effect of ionization-absorption balance processes on the potential distribution around a test particle in isotropic complex plasma

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A complex plasma is a plasma containing charged dust particles. The proper characterization of the electric potential distribution around these charged particles is considered to be one of the fundamental problems in the field of complex plasmas. The point is that the shape of the potential distribution and the interaction potential between charged grains is not fixed but depends considerably on surrounding complex plasma conditions. Diverse mechanisms of attractive and repulsive interactions have been discussed in the literature depending on the complex plasma conditions in different regimes [1, 2].

However, it is to be noted that in all these studies the effect of collective plasma loss (due to absorption on many dust particles) on the interaction potential between dust grains has been ignored. The importance of collective plasma loss has been discussed recently in the literature which leads to the possibility of collective attraction in complex plasmas [3, 4, 5]. It was suggested that two dust particles in a complex plasma could exhibit electrostatic attraction as a result of their self-consistent perturbation of the balance between plasma production due to ionization and collective plasma loss due to absorption on all other dust particles in the plasma. However all these studies are based on an implicit assumption that such an ionization-absorption balanced complex plasma is stable with respect to ion perturbations. Indeed, these investigations were performed by analyzing the test grain potential which was considered as a static perturbation (in most studies as a linear static perturbation) of a homogeneous static solution of equations governing the dynamics of a complex plasma at ion time scales. This unperturbed state cannot be realized if it is not stable.

This implicit assumption has been recently shown to be invalid in the case where the ionization source is proportional to the electron density [6]. Moreover, the instability length was found to be exactly equal to the length of the collective attraction [6]. However, this instability can disappear in the case of a constant ionization source [6], but in this case the functional form of the potential can be different [5]. The objective of the present paper is to explore the possibility of inter particle attraction *in the stability regime* of the ionization-absorption balance.

In our model we consider an individual point-like test charge $Q(= eZ_d)$ in a weakly ionized complex plasma consisting of neutral particles, positive ions, electrons, and negatively charged dust grains. Here, $Z_d > 0$ is the number of elementary charges on individual dust grain. The dust grains are considered as a continuous, immovable, homogeneous "dust medium" with constant number density n_d, whereas the ions and electrons have number densities n_i and n_e, respectively, which may vary with space. The dust charge Q is governed by absorption of ions and electrons and hence may vary with space.

For ions we use the static continuity and momentum equations:

$$\nabla \cdot (n_i \mathbf{v}) = -\alpha(Z_d) n_i + S(n_e), \tag{1}$$

$$(\mathbf{v} \cdot \nabla)\mathbf{v} = -(e/m_i)\nabla\phi - (T_i/m_i)(\nabla n_i/n_i) - \nu\mathbf{v}. \tag{2}$$

Here, T_i is the (constant) ion temperature, m_i is the ion mass, $\mathbf{v}$ is the ion fluid velocity, and ϕ is the electric potential. As regards the right-hand side of Eq. (1), the ions are produced by ionization at a rate per unit volume $S(n_e)$ which we take to depend generally on n_e, and they are absorbed by the dust at a rate per unit volume $\alpha(Z_d)n_i$ where we take the coefficient $\alpha(Z_d)$ to depend generally on Z_d. In Eq. (2) the friction coefficient $\nu = \nu_{\text{in}} + \nu_{\text{id}} + S(n_e)/n_i$ is the sum of the effective ion-neutral collision frequency ν_{in}, effective ion-dust collision frequency ν_{id}, and the ionization term $S(n_e)/n_i$. The dependence of the friction coefficient ν on plasma parameters, as well as the dependence of the absorption coefficient α on the ion fluid velocity $\mathbf{v}$, is not important to our investigation because we will consider here only first-order perturbations of a state with $\mathbf{v} = \mathbf{0}$.

For electrons we use the Boltzmann distribution:

$$e\nabla\phi - (T_e/n_e)\nabla n_e = \mathbf{0}. \tag{3}$$

For the dust charge we use the equation of the balance of the ion and electron fluxes:

$$\alpha(Z_d)n_i = \sqrt{\frac{8\pi T_e}{m_e}} a_d^2 n_e n_d \exp\left(-\frac{Z_d e^2}{a_d T_e}\right), \tag{4}$$

where a_d is the dust radius and m_e is the electron mass. Here we have used the Orbit Motion Limited (OML) expression for the electron flux [1]. For ions OML does not work well in many cases so that we assume an arbitrary dependence of ion flux on $\alpha(Z_d)$.

The above set of equations is closed with the Poisson equation

$$-\nabla^2\phi = 4\pi e(n_i - n_e - n_d Z_d) + 4\pi Q\delta(\mathbf{r}). \tag{5}$$

The potential distribution around a charged particle is obtained using linear response method,

$$\phi = \frac{Q}{2\pi^2}\int \frac{\exp(i\mathbf{k}\cdot\mathbf{r})d\mathbf{k}}{k^2 D(k)}, \tag{6}$$

where $D(k)$ is the static dielectric function and can be written in simplified form as,

$$\frac{1}{k^2 D(k)} = \frac{k^2 + W k_{\rm Di}^2}{(k^2+k_+^2)(k^2+k_-^2)}, \tag{7}$$

where

$$k_\pm^2 = (1/2)\left(X \pm \sqrt{X^2-4Y}\right) k_{\rm Di}^2 \tag{8}$$

and

$$X = 1+\tau(1-P) + \frac{(1+\tau)P+\eta\zeta z}{\beta+z}, Y = -\frac{\tau\eta\zeta\left[(z+P+\beta)(\gamma-1)+Pz\right]}{\beta+z}, W = \frac{\eta\zeta z}{\beta+z}.$$

Here, $P = n_d Z_d/n_i$ is the Havnes parameter, $\zeta = \nu/\omega_{\rm pi}$ is the friction parameter, $\eta = \alpha(Z_d)/\omega_{\rm pi}$ is the absorption parameter, $z = Z_d e^2/(a_d T_e)$ is the normalized dust charge, $\tau = T_i/T_e$ is the ion-to-electron temperature ratio, $\beta = \frac{d\alpha(Z_d)}{dZ_d}\frac{Z_d}{\alpha(Z_d)}$ characterizes the dependence of the ion absorption rate on the dust charge and the parameter $\gamma = \frac{dS(n_e)}{dn_e}\frac{n_e}{S(n_e)}$ characterizes the dependence of the ionization source on the electron density (e.g., $\gamma = 1$ for an ionization source proportional to the electron density, whereas $\gamma = 0$ for a constant ionization source). Here, $\omega_{\rm pi} = \sqrt{4\pi n_i e^2/m_i}$ is the ion plasma frequency and $k_{\rm Di} = \sqrt{4\pi n_i e^2/T_i}$ is the inverse ion debye length.

The functional form of the potential depends exclusively on $k_\pm^2$ which has been discussed as follows:

Case 1: $k_\pm^2$ are real and positive. In this case the potential is,

$$\phi = \frac{Q_+}{r}\exp(-r|k_+|) + \frac{Q_-}{r}\exp(-r|k_-|) \tag{9}$$

where

$$Q_\pm = \pm Q\left(\frac{k_\pm^2 - W k_{\rm Di}^2}{k_+^2 - k_-^2}\right). \tag{10}$$

In this case to explore the possibility of the *exponential attraction*, we consider the following parameter regime which is typical for laboratory and industrial complex plasmas [1]: $\eta\zeta \ll 1$, $\tau \le 1$, $P \sim 1$, $z \sim 1$, $\beta \sim 1$, and $0 \le \gamma \le 1$. Then the condition for the case 1 to apply (i.e., for $k_\pm^2$ to be real and positive) is

$$(z+P+\beta)(\gamma-1)+Pz < 0. \tag{11}$$

This condition is always satisfied for a constant ionization source ($\gamma = 0$) and is never satisfied for an ionization source proportional to the electron density (we assume negative dust charge and $\beta > 0$). If the case 1 applies [i.e., if condition (11) is satisfied], the condition for the potential (9) to have an attractive part (i.e., the condition for Q_-/Q to be negative) is

$$\beta - \gamma(\beta+z) > 0, \qquad \tau > \frac{z}{\beta-\gamma(\beta+z)} \tag{12}$$

To investigate the stability we consider the following numerical example in the dispersion relation [Eq. (12) of Ref. [6]]: $P = 0.5$, $\zeta = 1$, $\eta = 0.02$, $z = 0.5$, $\tau = 1$, $\beta = 1$, $\gamma = 0$. This example demonstrates the possibility of the exponential attraction in the stability regime. Note that the exponential attraction is impossible at $\tau \ll 1$.

Case 2: $k_{\pm}^2$ are real and k_{-}^2 is negative. In this case we obtain oscillatory potential,

$$\phi = \frac{Q_+}{r}\exp(-r|k_+|) + \frac{Q_-}{r}\cos(r|k_-|), \tag{13}$$

where $Q_{\pm}$ are defined by Eq. (10). In this case the inequality $(z+P+\beta)(\gamma-1)+Pz > 0$ is satisfied which makes the unperturbed state unstable and hence this situation is not possible in the regime of stability [6].

Case 3: $k_{\pm}^2$ have non-zero imaginary parts. In this case the potential is

$$\phi = \frac{1}{r}\exp\left[-r\mathrm{Re}(k_+)\right] \times \{Q\cos[\mathrm{Im}(k_+)r] + 2\mathrm{Im}(Q_+)\sin[\mathrm{Im}(k_+)r]\}, \tag{14}$$

where k_+ should be taken with positive real and imaginary parts and Q_+ is defined by Eq. (10) where k_+^2 and k_-^2 should be taken with positive and negative imaginary parts, respectively. This situation is applicable only under rather unusual conditions. Indeed, the condition for the case 3 to apply is $Y > X^2/4$ which implies $\eta\zeta \geq 1$ when $\tau \leq 1$, $P \sim 1$, $z \sim 1$, $\beta \sim 1$, $0 \leq \gamma \leq 1$. We provide a numerical example where the case 3 applies and the unperturbed state is stable: $P = 0.5$, $\zeta = 2$, $\eta = 1$, $z = 0.5$, $\tau = 1$, $\beta = 1$, $\gamma = 0$. We ascertained the stability by substituting the above parameters to the dispersion relation.

The possibility of attraction between negatively charged dust particles has been investigated using hydrodynamic model with arbitrary dependence of the ionization source on the electron density. It is found that the potential can have an attractive part in the *stable region* against ion perturbations only in very limited and specific circumstances (constant ionization source with comparable ion and electron temperatures).

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