The floating potential of large dust grains in a collisionless, flowing plasma

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Abstract

Many dusty plasma environments have some degree of flow associated with them. Shifted orbital motion limited (SOML) is the most widely used theory for predicting the floating potential of dust in a flowing plasma. The applicability of SOML is investigated via PIC simulation. Flow velocities up to five times the cold ion acoustic speed are simulated for dust grains of varying size.

Introduction

Predicting dust behaviour in any plasma requires information about the floating potential, the potential at which ion and electron currents to the grain balance. Providing the dust grain is small with respect to the electron Debye length (λ_{D_e}) the floating potential in a stationary plasma can be predicted accurately using orbital motion limited (OML) [1] theory.

OML assumes that the potential is spherically symmetric and is accurate only when $\rho < 1$ [2], where ρ is the dust grain radius normalised by λ_{D_e}. Ions and electrons approach the dust grain from infinity and are attracted or repelled depending on the sign of the potential. A cross-section is defined with a dependence on the potential and conservation of energy and angular momentum are used to track the particle orbits. The ion and electron distributions far from the dust grain chosen are typically Maxwellian and an integration is performed over the particle velocities. Particles striking the dust are captured and contribute to the charge, particles missing the dust escape. In the case of a flowing plasma the conventional method for calculating the dust floating potential is shifted OML (SOML)[3]. This is simply OML but with the static Maxwellian ion distribution replaced with a shifted Maxwellian distribution of the form

$$f_i(v,\theta) = n_i\left(\frac{m_i}{2\pi k_B T_i}\right)^{3/2} \exp\left(-\frac{m_i}{2k_B T_i}(v^2+w^2-2vw\cos\theta)\right) \tag{1}$$

where w is the bulk speed of the plasma relative to the dust grain.

Since OML assumes angular momentum conservation it is only valid in spherically symmetric situations. Clearly the presence of flow breaks the spherical symmetry and it is therefore interesting to investigate the accuracy of SOML .

In this paper we investigate the effect of flow on floating potential for a wide range of dust grain size ($\rho = 0.01$ to 100). We consider flow speeds up to u = 5, where u is the flow speed normalised by the cold ion acoustic speed $\sqrt{k_B T_e/m_i}$. The floating potential is calculated via the PIC code SCEPTIC [4 $\rightarrow$ 8]. SCEPTIC is ideal for studying this problem as it assumes a shifted Maxwellian distribution at infinity, and no assumptions about the form of the potential are made in the computational domain [6]. Hutchinson [5] has already used SCEPTIC to investigate flowing plasmas, with particular focus on fixed potentials rather than floating. For ρ between 0.01 and 1.00 Hutchinson has also investigated SOML for the ion to electron temperature temperature ratio ($\beta = T_i/T_e$) of 0.1 and 1.0 (figure 2 of [6]) and finds SCEPTIC agrees well with SOML for flow velocities up to $u = 5$.

Validity of shifted orbital motion limited

Investigated first is the effect of strong flows on the validity of SOML with respect to dust grain size. As seen in fig.1(a), flow speeds of $u = 1.85$ have little effect on the symmetry of the potential for grains small with respect to the Debye length. The flow has a much more significant effect on the large dust grain shown in figure 1(b), particularly downstream.

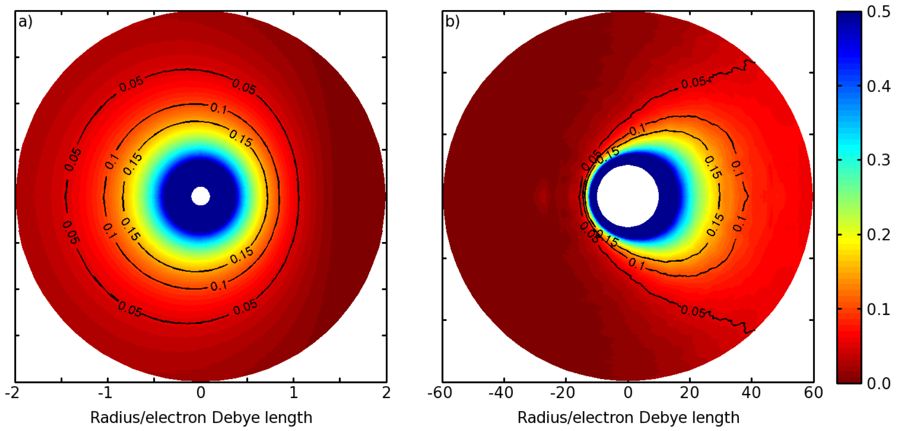

Figure 1: The potential (normalised by $-k_BTe/e$) around two dust grains in a flow of $u = 1.85$ from left to right. a) A small dust grain, $\rho = 0.1$ b) A large dust grain, $\rho = 10.0$. $\beta = 1.0$ in both cases.

As mentioned in the introduction, OML is accurate only for dust grains small compared to λ_{D_e}. In the event that for large dust grains the SOML potential is found to be significantly wrong, new expressions are required to predict the potential. In figure 2 the potential from simulation is seen to agree with SOML for dust grains up to $\rho \approx 1$. Significant differences are found for large particles however, these differences are reduced by increasing flow velocity. This is attributed to sheath becoming thin for large dust grains and ion collection being dominated by flow rather than electrostatic attraction. As seen in figure 1(a), an appreciable potential still exists over a length of a few dust radii for small dust grains.

This effect is investigated further in figure 3(a) and increasing flow speed is seen to have a flattening effect on the potential with increasing dust grain radius. The low flow case of $u = 0.5$ has no discernible effect on the potential as it agrees closely with the stationary values over the whole range of ρ. The more appreciable flow case of $u = 1.5$ however shows a clear flattening of the potential as the flow begins to dominate over the thermal motion. For the strong flow case, $u = 5.0$, even for very large dust grains the potential deviates from the SOML potential by less than 5%.

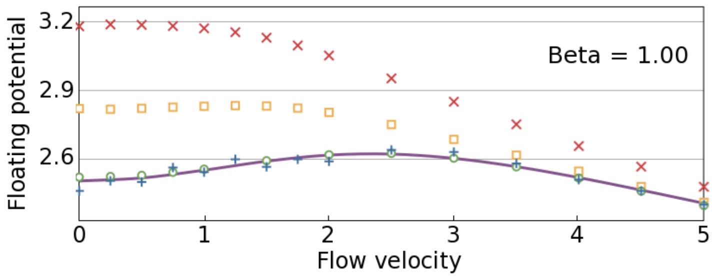

Figure 2: Floating potential as a function of flow velocity for different size dust grains. The solid line is SOML. The potential is normalised by $-k_B T_e/e$.
ρ: $+ = 0.1$, $\circ = 1.0$, $\square = 10.0$ and $\times = 100.0$.

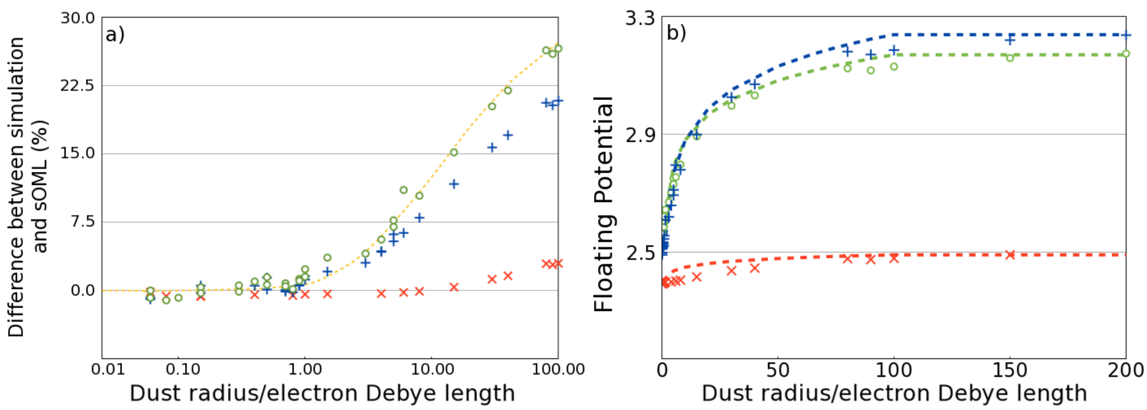

Figure 3: a) % difference between SOML and simulation for various u. The dashed line is the floating potential of dust in a stationary plasma. b) The large dust grain asymptotes, dashed lines are expressions 2 and 3.
u: $\circ = 0.5$, $+ = 1.5$ and $\times = 5.0$.

Numerical fits are made to the data in order to find scalings and describe the potential's behaviour. As the potential is dependent on numerous parameters (β, ρ, flow speed and ion species) a general formula is not easily found. For the present, the analysis is restricted to

$\beta = 1.0$ and hydrogen.

As seen in figure 3(a), SOML predicts accurate floating potentials up to $\rho \approx 1.0$. The value of ρ at which simulation departs from SOML has a weak dependence on flow speed. For $\rho \gtrapprox 100$ a "large dust grain limit" floating potential is approached asymptotically, as seen in figure 3(b), this large grain potential again has a complex dependence on a number of parameters, the following polynomial fits the data closely

$$\phi_{LD} = -0.029u^2 - 0.007u + 3.249, \qquad \rho \geq 100 \qquad (2)$$

the subscript LD indicates the large dust grain limit.

For the transition region between the two limits the potential rises with a logarithmic dependence on ρ. To estimate the potential in this region a line of the form $\phi = m \ln \rho + c$ is fit between the limiting cases giving

$$\phi(\rho) = \left(\frac{\phi_{LD} - \phi_{SOML}}{\ln \phi_{LD}} \right) \ln \rho + \phi_{SOML}. \qquad 1 < \rho < 100 \qquad (3)$$

Expressions 2 and 3 fit the data well (within 3%) over $\rho = 1 \rightarrow 100$, shown in figure 3(b), and SOML is accurate for $\rho \leq 1$.

Conclusion

Initial investigation shows SOML to be accurate for small dust grains. Increasing flow causes asymmetries in the potential but does not dramatically affect the accuracy of SOML for small dust grains. For dust grains large with respect to the electron Debye length SOML does not predict correct values however, the difference between SOML and simulation is reduced as flow speed is increased, this is attributed ion collection being dominated by flow rather than long range interactions. Expressions for the floating potential as a function of dust grain size and flow speed are presented for a fully ionised hydrogen plasma with $T_e = T_i$, more work is needed to generalise the expression.

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