Large Amplitude double layers in dusty plasma with size-distributed dust grains.

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Abstract

The investigation of the existence of arbitrarily large amplitude electrostatic double layers is conducted in a four-components plasma consisting of electrons, two distinct positive ion species of different temperatures, and massive negatively-charged dust particles that are assumed spheres of different radii, which are distributed according to a power-law[1,2]. Comparison is conducted between plasmas containing size-distributed dust grains and those containing monosize dust grains[3], while examining the criteria for the existence of double layers alongwith the dependence of their amplitudes and Mach numbers on plasma parameters. It is reveled that not only supersonic double layers are possible, but also subsonic ones when the size distribution of dust grains is taken into account.

1. Introduction

A double-layer (DL) is a structure consisting of two space-charge layers of opposite charges. Consequently, the potential experiences a drop which is necessarily greater than the thermal energy per unit of charge of the coldest plasmas bordering the layer. Hence, the electric field is stronger within the double layer where as, quasi-neutrality is violated in the space-charge layers[4]. These electrostatic structures (DLs) have a tremendous role to play in space plasmas as well as laboratory plasmas. Indeed, double layers are considered the appropriate candidate to interpret charged particles acceleration to high energies in plasmas, e.g., the auroral region of the ionosphere. The characteristics along with the existence criteria of the DLs may be affected by the presence of dust particulates having high charge and mass in the plasma. This type of plasmas is believed to be the rule, as they are encountered almost every where in situations spanning from astrophysical to industrial ones. Sofar, the dust particulates have been taken monosized, whereas in real situations they present a size distribution due to grain-grain collisions leading to fragmentation and coalescence. Hence, grain size-distribution is an additional element to be taken into account while modeling a plasma. Indeed, Ishak-Boushaki, et al, have investigated dust-acoustic solitons when ions are adiabatically heated and dust grains are size-distributed, and found that the solutions experienced translation from a solitary wave to a Cnoidal wave[2]. Moreover, the grain size-distribution affects the modes supported by the plasma alongwith the growth rate of some parametric instabilities[5]. In this paper, we consider a collisionless, unmagnetized plasma consisting of Boltzmannian electrons, size-distributed dust grains, and two types of Boltzmannian positive ions having different temperatures T_c (for the cool species) and T_h (for the hot species), where double layers formation is investigated.

II. Basic Equations

The number densities of electrons and ions are given by the Boltzmann distribution,

$n_e = n_{eo}\exp(e\phi/T_e)$ (1), $n_c = n_{co}\exp(-e\phi/T_c)$ (2), $n_h = n_{ho}\exp(-e\phi/T_h)$ (3).

where, n_{co}, n_{ho}, n_{jo} and n_{eo}, are the unperturbed, cold ion, hot ion, j^{th} dust grain and electron number densities respectively, whereas Z_{jo} is the unperturbed charge number of the j^{th} dust grain. The quasi-neutrality condition is given by [6], $n_{co} + n_{ho} = n_{eo} + \sum_{j=1}^{N} Z_{jo} n_{jo}$ (4).

Let us adopt the following normalization, viz., the space coordinate x is normalized by the effective Debye length $\lambda_{Dd} = \left(T_{eff}/4\pi Z_o n_{tot} e^2\right)^{1/2}$, the dust velocity u_j is normalized by the effective dust acoustic speed defined by $C_d = \left(Z_o T_{eff}/m_o\right)^{1/2}$, time t is normalized by the effective dusty plasma period defined as $\omega_{pd}^{-1} = \left[m_o/\left(4\pi Z_o^2 n_{tot} e^2\right)\right]^{1/2}$, the ion and electron densities are normalized by $n_{tot} Z_o$, the dust density is normalized by $n_{tot} = \sum_{j=1}^{N} n_{jo}$, (total number density of all dust grains), and the electrostatic potential Φ is normalized by $(T_{eff}/Z_o e)$, where $(Z_o^2 n_{tot}/T_{eff}) = [n_{eo}/T_e + n_{co}/T_c + n_{ho}/T_h]$ and $\alpha_i = T_{eff}/Z_o T_i$. Moreover, the dust charge Z_j and mass m_j are normalized by the charge and mass corresponding to the grain of the most probable radius r_o, viz., $Z_o = Z(r_o)$ and $m_o = m(r_o)$. The quasi-neutrality reads then, $\alpha_e N_{eo} + \alpha_e N_{co} + \alpha_h N_{ho} = 1$. The dynamics of the grains is governed by the continuity and momentum equations, closed by Poisson's equation, namely,

$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}\left(u_j n_j\right) = 0$ (5), $\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} = \left(\frac{Z_j}{m_j}\right)\frac{\partial \phi}{\partial x}$ (6), $\frac{\partial^2 \phi}{\partial x^2} = n_e - n_c - n_h + \sum_j Z_j n_j$ (7).

Assuming the physical quantities to depend on $\xi = x - Mt$, where M is the Mach number, the boundary conditions, ϕ, $(d\phi/d\xi)$, $u_j \to 0$ and $n_j \to n_{jo}$, corresponding to unperturbed plasmas at $\xi \to \infty$. The stationary solutions of Eqs.(5,6) are given by, $n_j = \frac{M n_{jo}}{\sqrt{M^2 + 2\phi(Z_j/m_j)}}$ (8).

Substituting for the particle number densities from Eqs.(1–3) and Eq.(8) into Eq.(7), then integrating the resulting equation, we obtain, $\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi, M) = 0$ (9), where,

$$\left.\begin{aligned} V(\phi,M) &= \frac{N_{eo}}{\alpha_e}\left[1-\exp(\alpha_e\phi)\right] + \frac{N_{co}}{\alpha_c}\left[1-\exp(-\alpha_c\phi)\right] + \frac{N_{ho}}{\alpha_h}\left[1-\exp(-\alpha_h\phi)\right] \\ &\quad - C_4 M^2\left(\ln\left[\frac{\mathrm{r_m} + \sqrt{r_m^2 + 2\phi/M^2}}{1+\sqrt{1+2\phi/M^2}}\right] - \frac{\sqrt{\mathrm{r_m^2} + 2\phi/M^2}}{r_m} + \sqrt{1+2\phi/M^2} - \ln(r_m)\right) \end{aligned}\right\} \quad (10)$$

is the Sagdeev potential with size-distributed grains according to a power-law distribution $f(r_d) = C_p\, r^{-p}$, where $p = 4$ (for meteor plasma).

The formation of a double layer, demands[3],

$$\left.\begin{aligned} &a)\ V(0,M) = \partial_\phi V(0,M) = 0 && \text{for all } M. \\ &b)\ V(\phi_m, M) = \partial_\phi V(\phi_m, M) = 0 && \text{for some } \phi_m, M \\ &c)\ V(\phi,M) < 0 && \text{for } M \text{ in } b), \quad \text{and} \quad 0<|\phi|<|\phi_m| \end{aligned}\right\} \quad (11)$$

The condition a) in Eqs.(11.a-11.c) is clearly satisfied by the Sagdeev potential $V(\phi, M)$, as the quasi-neutrality is retrieved, namely,

$\partial_\phi V(\phi,M)\big|_{\phi=0} = -N_{e0} + N_{c0} + N_{h0} - \eta\,(p, r_m) = 0$ (12), where $\eta(p, r_m) = C_p\left(\frac{1-r_m^{-2}}{2}\right)$.

Applying the conditions b) (with p = 4), we obtain:

$$A(\phi_m) + C_4 M^2 \frac{2\phi_m}{H(\phi_m)}\left(\ln\left[\frac{\mathrm{r_m} + \sqrt{r_m^2 + H(\phi_m)}}{1+\sqrt{1+H(\phi_m)}}\right] - \frac{\sqrt{\mathrm{r_m^2} + H(\phi_m)}}{r_m} + \sqrt{1+H(\phi_m)} - \ln(r_m)\right) = 0 \quad (13a)$$

and $$M^2 = \frac{2\phi_m}{H(\phi_m)} \quad (13b)$$

where $$A(\phi_m) = -\left\{\frac{N_{eo}}{\alpha_e}\left[1-\exp(\alpha_e\phi_m)\right] + \frac{N_{co}}{\alpha_c}\left[1-\exp(-\alpha_c\phi_m)\right] + \frac{N_{ho}}{\alpha_h}\left[1-\exp(-\alpha_h\phi_m)\right]\right\} \quad (14a)$$

$$H = H(\phi_m) = \frac{C_4}{r_m^2 B^2(\phi_m)}\left\{C_4 + C_4 r_m^2 + 2 r_m\sqrt{C_4^2 + r_m^2 B^2(\phi_m)}\right\} \quad (14b)$$

and $$B = B(\phi_m) = \left[-N_{eo}\exp(\alpha_e\phi_m) + N_{co}\exp(-\alpha_c\phi_m) + N_{ho}\exp(-\alpha_h\phi_m)\right] \quad (14c)$$

For a given set of density and temperature, the resolution of Eqs.[13a, 13b] yields the value of ϕ_m alongwith the associated Mach number M. Furthermore, the limiting condition $\frac{\partial^2 V(\phi,M)}{\partial\phi^2} < 0$ at $\phi = 0$ and $\phi = \phi_m$, imposes a range of acceptable Mach numbers, given by the following inequalities,

$M^2 > \left(\frac{C_4}{4}\right)\frac{1}{\alpha_e N_{e0} + \alpha_c N_{c0} + \alpha_h N_{h0}} \equiv \frac{C_4}{4} = 0{,}75$ (15a) and $M^2 > C_4 \frac{f(H(\phi_m))}{D(\phi_m)}$ (15b) where,

$$f(H(\phi_m)) = \left\{ \frac{1}{(r_m^2 + H)^{3/2}(r_m + \sqrt{r_m^2 + H})} + \frac{1}{(r_m^2 + H)(r_m + \sqrt{r_m^2 + H})^2} - \frac{1}{r_m (r_m^2 + H)^{3/2}} \right. \quad (16a)$$

$$\left. - \frac{1}{(1+H)(1+\sqrt{1+H})^2} - \frac{1}{(1+H)^{3/2}(1+\sqrt{1+H})} + \frac{1}{(1+H)^{3/2}} \right\}$$

and

$$D(\phi_m) = \left[N_{eo}\alpha_e \exp(\alpha_e \phi_m) + N_{co}\alpha_c \exp(-\alpha_c \phi_m) + N_{ho}\alpha_h \exp(-\alpha_h \phi_m) \right] \quad (16b)$$

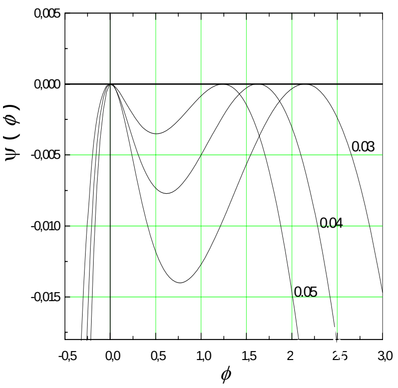

Fig. 1. Sagdeev potential (DL) V(ϕ ,M) versus ϕ for $N_{e0} = 0$ and (N_{c0} / N_{h0})=0,11. The parameter $\alpha = T_c / T_h$ labeling the curves is the ratio of cool to hot ion temperatures.

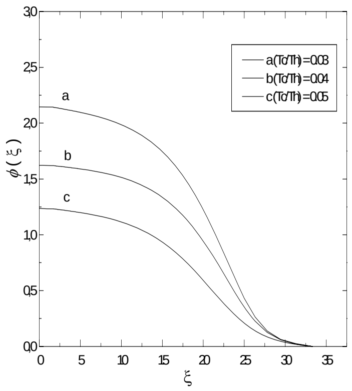

Fig. 2. The DL potential profile $\phi(\xi)$ versus ξ associated with the Sagdeev potential in Fig.1

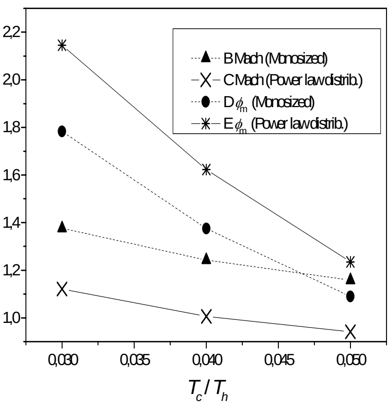

Fig. 3. variation of the DLs amplitude ϕ_m and the corresponding Mach number M versus the ratio $\alpha = T_c / T_h$, for Power-Law size-distribution by opposition to the monosized one.

Conclusion :

In this work, we have presented the possible existence of arbitrary amplitude dust acoustic double layers in an unmagnetised dusty plasma with Boltzmann distributed double species of ions having different temperatures, (T_c and T_h), a cold fluid of dust grains of N different sizes described by a continues Power Law distribution, The variation of the DL amplitude ϕ_m and the Mach number M with the ratio($\alpha = T_c/T_h$) are depicted in Fig.3, where a comparison is made between the monosized case and the power law distribution case. It is shown that subsonic solutions are allowed in this model, whereas in the monosized case only supersonic solution are predicted.

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