# Wave propagation in strongly non-uniform helicon plasma

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**Abstract.** Various aspects of wave propagation being important for helicon-produced plasmas with strong radial density gradient [1] are treated by analytical as well as numerical methods. (I) In agreement with experimental results, the theoretical and numerical analysis of guided helicon wave propagation ($E_z$ = 0) shows for a wide parameter range the quadratic scaling $\omega \approx k_z^2$, which in contrast to the linear dispersion relation, $\omega \approx k_z$, for the usual case of helicon mode propagation in a weakly uniform plasma column (radius $R$) with $k_z R \ll 1$. (II) The eigen mode spectrum is obtained for localized electrostatic modes revealing non-reciprocal behavior with respect to azimuthal propagation (i.e., mode number $m$ > 0). (III) The effects of strong plasma density gradient on mode conversion for oblique wave propagation in helicon plasma experiments are demonstrated.

## I. Helicon mode propagation

The analysis of helicon modes is based on the solutions of the differential equation for the azimuthal component of the electric field intensity of the helicon wave $E_\theta(r,z,\theta,t) = E_\theta(r)\exp(-i\omega t + im\theta + ik_z z)$ reading [2]:

$$\frac{\partial^2}{\partial r^2}E_\theta + \frac{3}{r}\frac{\partial}{\partial r}E_\theta + \frac{1-m^2}{r^2}E_\theta + a^2 E_\theta = 0\,, \quad a^2 = \frac{g}{N_z^2}\left(\frac{m}{r}\frac{\partial \ln n}{\partial r} + \frac{\omega^2}{c^2}\left(g - 2N_z^2\right)\right)E_\theta = 0\,, \quad (1)$$

where $g \equiv \omega_{pe}^2(r)/(\omega\omega_{ce})$, $\omega_{ce} = -eB_0/m_e c$, $\omega_{pe}^2(r) = 4\pi e^2 n(r)/m_e$, $N_z = k_z c/\omega$. Eq.(1) is valid if $m^2 \gg k_z^2 r^2$ and can be obtained from the full system of Maxwell equations for high plasma conductivity along the external magnetic field ($\mathbf{B}_0 = B_0\hat{\mathbf{z}}$) so that $E_z = 0$. We consider the frequency range $|\omega_{ce}| \gg \omega \gg \omega_{LH} \equiv \sqrt{|\omega_{ce}|\omega_{ci}}$ for which the ion motion can be neglected. The effect of density gradient becomes determinant if

$$\left|\frac{m}{r}\frac{\partial \ln n}{\partial r}\right| \gg \frac{\omega^2}{c^2}\left|g - 2N_z^2\right| \quad (2)$$

and the solution of eq.(1) for parabolic density profile $n = n_0(1 - r^2/R^2)$ [3] then becomes

$$E_\theta(r) = CJ_m(ar)/r\,, \quad a^2 = 2\frac{\omega}{|\omega_{ce}|}\frac{\omega_{pe}^2(0)}{c^2 k_z^2}\frac{m}{R^2} \qquad (C = const.) \quad (3)$$

The dispersion relation for helicon modes can be obtained with the boundary condition $E_\theta(r = R) = 0$ assuming the plasma column to be surrounded by a metal wall yielding

$$\omega = |\omega_{ce}| \frac{c^2 \xi_{m,n}^2 k_z^2}{2m\omega_{pe}^2(0)}, \tag{4}$$

where $\xi_{m,n}$=$aR$ are the roots of the equation $J_m(\xi_{m,n}) = 0$. Note that the modes with $m < 0$ cannot propagate.

The analytical findings are compared with numerical calculations for a Gaussian density profile (width $l$ = 0.02 m) based on the helicon wave equation (only assumption $E_z = 0$). Fig.1 shows profiles of the azimuthal electric field component for a wide range of frequencies and densities. It turns out that the profiles are nearly identical at low frequencies while their width decreases with increasing density in the high frequency range. In Fig.2, we compare the analytical dispersion relation (4) with the computations. The quadratic scaling, $\omega \propto k_z^2$, predicted for sufficiently strong gradient (see condition (2)) can also be seen in the computational curves provided that the densities and frequencies are not too high. Only if the (2) is violated, significant deviations from the quadratic scaling occur. It is worth noting that the frequency scales also with $k_z^2$ for helicon waves with $k_z \gg k_\perp \approx R_p^{-1}$ or $k_z R_p \gg 1$ (*whistler dispersion*) whereas, in case of strong density gradient here considered, the opposite condition, $k_z R_p \ll 1$, holds. The above finding is in agreement with experimental investigations of helicon wave dispersion [4].

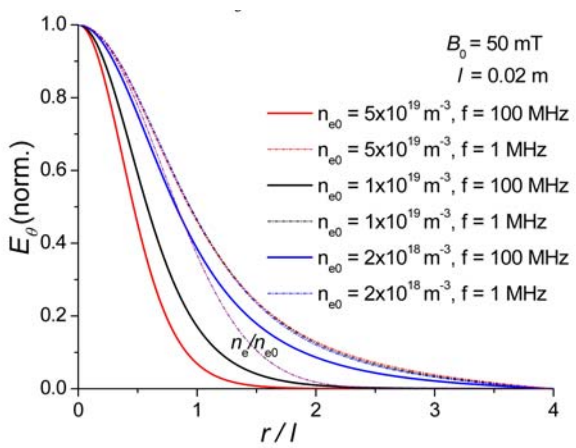


*Fig.1. RF field profiles*

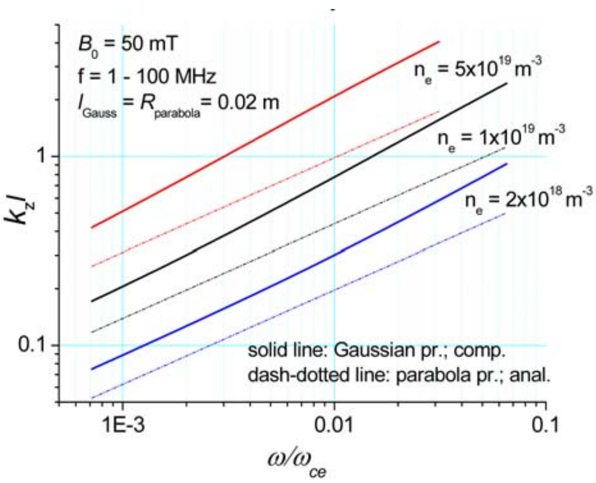


*Fig.2. Helicon dispersion*

## II. Electrostatic mode propagation

Now we analyse the gradient density effect on the propagation of electrostatic (ES) modes; its potential can be presented in the form $\Phi(r) = u(r)/\sqrt{r\varepsilon_\perp(r)}$, where the function $u(r)$ is a solution of the equation

$$\frac{\partial^2}{\partial r^2}u(r) + \left\{ -k_z^2\frac{\varepsilon_{//}}{\varepsilon_\perp} - \frac{1}{2r}L_N^{-1}\left[1 - 2m\frac{|\omega_{ce}|}{\omega}\right]\frac{4m^2-1}{4r^2} - \frac{1}{4}\left[(L_N^{-1})^2 + 2\frac{\partial}{\partial r}L_N^{-1}\right]\right\}u(r) = 0\,, \quad (5)$$

$$\varepsilon_\perp(r) = 1 + \frac{\omega_{pe}^2(r)}{\omega_{ce}^2}, \quad \varepsilon_{//}(r) = 1 - \frac{\omega_{pe}^2(r)}{\omega^2}, L_N^{-1}(r) \equiv \frac{d\ln\varepsilon_\perp(r)}{dr}\,.$$

Assuming dense plasma ($\omega_{pe}^2(0) >> \omega_{ce}^2$) with a Gaussian density profile eq.(5) has a discrete number of localized modes with the eigen values

$$k_z^2 l^2 = 2\left[(2n-1)\frac{\omega^2}{\omega_{ce}^2} + m\frac{\omega}{|\omega_{ce}|}\right], \quad n = 1,2,3... \quad (6)$$

For $m^2 = 1$ eq.(5) has for the lowest localized mode the solution

$$u(x) = x^{3/2}\exp(-x^2/2)\,, \quad (7)$$

and the corresponding eigen value is given by

$$\omega = |\omega_{ce}|\frac{k_z^2 l^2}{2}\,. \quad (8)$$

The fundamental mode (8) can propagate only for positive azimuthal number *m* = 1.

In Fig.3 and 4 we compare the electric field profile and the dispersion relation of localized electrostatic modes for Gaussian density profile obtained analytically (dashed lines) and numerically (solid lines) from (5). Good agreement between the both descriptions are particularly achieved for the fundamental mode (*n* = 1) concerning the central part of the profile as well as the scaling, $\omega \sim k_z^2$; significant discrepancies arise for higher radial modes.

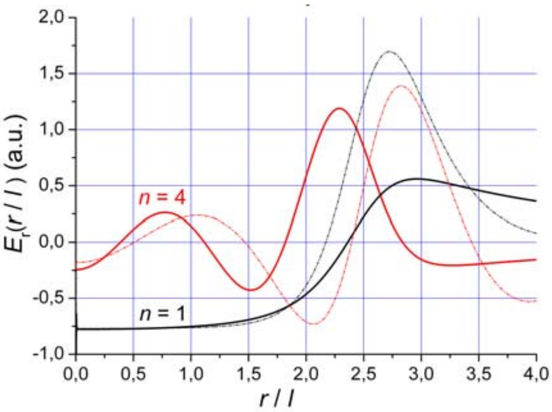


*Fig.3. ES field profiles*

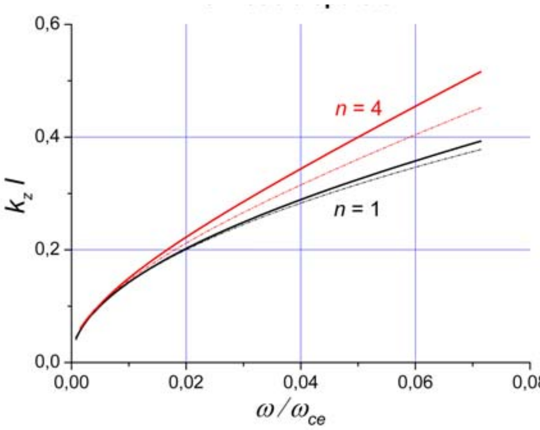


*Fig.4. ES mode dispersion*

### III. Mode conversion

In conclusion, we discuss the influence the strong density gradient on mode conversion assuming however that the density changes slowly over wavelength [5]. In this case, we can present the solutions of the system of Maxwell's equations to the first order in the geometrical-optic approximation. The local dispersion relation for two modes in helicon plasma reads

$$k_{\perp 1,2}^2 = \frac{1}{2}\left(\alpha \pm \sqrt{\alpha^2(x) + 4\beta(x)}\right), \tag{9}$$

where the coefficients $\alpha = \frac{|\omega_{ce}|}{\omega}\left(k_z^2 \frac{|\omega_{ce}|}{\omega} + k_y L_N^{-1}\right)$, $\beta = -\frac{\omega_{pe}^2}{c^2}\left(\frac{\omega_{pe}^2}{c^2} - \frac{|\omega_{ce}|}{\omega} k_y L_N^{-1}\right)$

take into account the density gradient effect. At the position $x_0$ defined by the relation $\alpha^2(x_0) = -4\beta(x_0)$ both modes have the same wavenumbers, and mode conversion takes place. Taking into account the condition previously assumed, $N_z^2 \gg \varepsilon_\perp$**,** we get from this relation that the density gradient in the region of mode conversion is defined by the relation

$$L_N(x_0) \approx -\frac{\omega k_y}{|\omega_{ce}| k_z^2}\,. \tag{10}$$

Note that this condition corresponds to dispersion relation (8) of the electrostatic mode. This holds for a wide range of plasma densities, where $\varepsilon_\perp \ll N_z^2$ and is in contrast to the common theory that neglects density gradient effects and predicts mode conversion at high plasma densities given by $\varepsilon_\perp \approx N_z^2$.

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**References.**

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