Study of mode coupling coefficient on coaxial gyrotron

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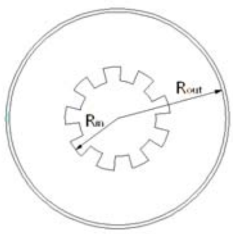
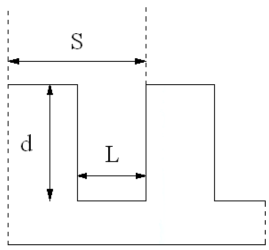
I. INTRODUCTION

Because of its advantages of suppressing mode competition and reducing microwave heat in the cavity wall of gyrotron[1,2], coaxial cavities has been researched generally[3,6,7]. However, no study on the mode coupling coefficients of the second-order transmission line equation of coaxial cavities has been found up now. The most important reason is that it is difficult to derive the mode coupling coefficients due to the complicated structure of coaxial cavities (as shown in Fig.1).To overcome this problem, the surface impedance theory (SIT) is adopted in this paper to deduce the mode coupling coefficients in the transmission line equation.



(a)



(b)

Fig.1 (a) Transverse cross section of a coaxial cavity with a corrugated inner conductor. (b) Unfolded scheme of the corrugated inner conductor.

II. MODE COUPLING COEFFICIENTS OF COAXIAL CAVITIES

The source-free second-order transmission line equation is given as[9]

$$\frac{d^2V_{mn}^{(i)}}{dz^2}=\left[\gamma_{mn}^{(i)}\right]^2V_{mn}^{(i)}+\frac{d\left[\ln\gamma_{mn}^{(i)}Z_{mn}^{(i)}\right]}{dz}\cdot\frac{dV_{mn}^{(i)}}{dz}-\frac{d\left[\ln\gamma_{mn}^{(i)}Z_{mn}^{(i)}\right]}{dz}\cdot\left[\sum_j\sum_{ms}V_{ms}^{(i)}\cdot C_{(s,n)}^{(j,i)}\right]-\sum_j\sum_{ms}\left[\frac{\gamma_{mn}^{(i)}Z_{mn}^{(i)}}{\gamma_{ms}^{(j)}Z_{ms}^{(j)}}\cdot\frac{dV_{ms}^{(i)}}{dz}\right] \tag{1}$$
$$+\sum_j\sum_{ms}V_{ms}^{(j)}\left[\frac{\gamma_{mn}^{(i)}Z_{mn}^{(i)}}{\gamma_{ms}^{(j)}Z_{ms}^{(j)}}\right]\cdot\left[\sum_l\sum_{mt}V_{mt}^{(l)}\cdot C_{(t,s)}^{(l,j)}\right]+\sum_j\sum_{ms}C_{(s,n)}^{(j,i)}\cdot\frac{dV_{ms}^{(i)}}{dz}+\sum_j\sum_{ms}V_{ms}^{(i)}\cdot\frac{dC_{(s,n)}^{(j,i)}}{dz}\quad i=1.or.2$$

where i=1 and 2, it expresses the electric waves and magnetic waves respectively. $Z_{mn}^{(i)}$ is wave impedence, $Z_{mn}^{(1)}=\gamma_{mn}^{(1)}\big/j\omega\varepsilon$ and $Z_{mn}^{(2)}=j\omega\mu\big/\gamma_{mn}^{(2)}$, $\left[\gamma_{mn}^{(i)}\right]^2=\left[k_{mn}^{(i)}\right]^2-k_0^2$, $C_{(s,n)}^{(j,i)}$ is the mode coupling coefficient.

A. *Coaxial cavities with a smooth-walled tapered inner conductor*

The mode coupling coefficients can be expressed as follows[9]

$$C_{(s,t)}^{(1,1)} = \frac{(\nu_{mt})^2}{(\nu_{mt})^2 - (\nu_{ms})^2} \int_C \frac{\partial \Phi_{mt}^{(1)}}{\partial z} \frac{\partial \Phi_{ms}^{(1)}}{\partial r} dl \tag{2a}$$

$$C_{(t,t)}^{(1,1)} = -\frac{1}{2} \int_C \left[\frac{\partial \Phi_{mt}^{(1)}}{\partial r} \right]^2 \frac{\partial r}{\partial z} dl \tag{2b}$$

$$C_{(s,t)}^{(1,2)} = 0 \tag{2c}$$

$$C_{(s,t)}^{(2,1)} = \int_C \frac{\partial \Phi_{ms}^{(2)}}{\partial l} \frac{\partial \Phi_{mt}^{(1)}}{\partial z} dl \tag{2d}$$

$$C_{(s,t)}^{(2,2)} = \frac{(\mu_{ms})^2}{(\mu_{ms})^2 - (\mu_{mt})^2} \int_C \Phi_{ms}^{(2)} \frac{\partial^2 \Phi_{ms}^{(1)}}{\partial z \partial r} dl \tag{2e}$$

$$C_{(t,t)}^{(2,2)} = -\frac{1}{2} \int_C \left[\frac{\partial \Phi_{mt}^{(2)}}{\partial l} \right]^2 \frac{\partial r}{\partial z} dl \tag{2f}$$

Because $\Phi_{mn}^{(i)}$ of coaxial cavities with a smooth-walled tapered inner conductor in (2) is given[9], we can obtain the mode coupling coefficients

$$C_{(s,t)}^{(1,1)} = \frac{2}{G_{mt} G_{ms}} \cdot \frac{\nu_{mt}^2}{\nu_{mt}^2 - \nu_{ms}^2} \cdot \left\{ \frac{1}{R_{out}} \frac{\partial R_{out}}{\partial z} - \frac{1}{R_{in}} \frac{\partial R_{in}}{\partial z} \frac{J_m(\nu_{mt}) J_m(\nu_{ms})}{J_m\left(\nu_{mt} \frac{R_{in}}{R_{out}}\right) J_m\left(\nu_{ms} \frac{R_{in}}{R_{out}}\right)} \right\} \tag{3a}$$

$$C_{(t,t)}^{(1,1)} = \frac{1}{G_{mt}^2} \cdot \left\{ \frac{1}{R_{in}} \frac{\partial R_{in}}{\partial z} \frac{J_m^2(\nu_{mt})}{J_m^2\left(\nu_{mt} \frac{R_{in}}{R_{out}}\right)} - \frac{1}{R_{out}} \frac{\partial R_{out}}{\partial z} \right\} \tag{3b}$$

$$C_{(s,t)}^{(1,2)} = 0 \tag{3c}$$

$$C_{(s,t)}^{(2,1)} = \frac{2 m R_{out}}{H_{ms} \mu_{ms}} \cdot \frac{1}{G_{mt} \nu_{mt}} \left\{ \frac{1}{R_{out}^2} \frac{\partial R_{out}}{\partial z} - \frac{1}{R_{in}^2} \frac{\partial R_{in}}{\partial z} \frac{J_m(\nu_{mt}) J_m'(\mu_{ms})}{J_m\left(\nu_{mt} \frac{R_{in}}{R_{out}}\right) J_m'\left(\mu_{ms} \frac{R_{in}}{R_{out}}\right)} \right\} \tag{3d}$$

$$C_{(s,t)}^{(2,2)} = 2\mu_{ms} \left\{ \frac{1}{R_{out}} \frac{\partial R_{out}}{\partial z} \left(\frac{m^2}{\mu_{mt}^2} - 1 \right) - \frac{R_{out}}{R_{in}^2} \frac{\partial R_{in}}{\partial z} \left(\frac{m^2}{\mu_{mt}^2} \frac{R_{out}^2}{R_{in}^2} - 1 \right) \frac{J_m'(\mu_{mt}) J_m'(\mu_{ms})}{J_m'\left(\mu_{mt} \frac{R_{in}}{R_{out}}\right) J_m'\left(\mu_{ms} \frac{R_{in}}{R_{out}}\right)} \right\} \left\{ H_{mt} H_{ms} \left(\mu_{ms}^2 - \mu_{mt}^2 \right) \right\}^{-1} \tag{3e}$$

$$C_{(t,t)}^{(2,2)} = \frac{m^2}{H_{mt}^2 \mu_{mt}^2} \cdot \left\{ \frac{R_{out}^2}{R_{in}^3} \frac{\partial R_{in}}{\partial z} \left[\frac{J_m'(\mu_{mt})}{J_m'\left(\mu_{mt} \frac{R_{in}}{R_{out}}\right)} \right]^2 - \frac{1}{R_{out}} \frac{\partial R_{out}}{\partial z} \right\} \tag{3f}$$

where

$$H_{mn}=\left\{\left[1-\left(\frac{m}{\mu_{mn}}\right)^2\right]-\left[1-\left(\frac{m}{\mu_{mn}\frac{R_{in}}{R_{out}}}\right)^2\right]\left[\frac{J_m^{'}(\mu_{mn})}{J_m^{'}\left(\mu_{mn}\frac{R_{in}}{R_{out}}\right)}\right]^2\right\}^{1/2} \tag{4a}$$

$$G_{mn}=\left\{1-\frac{J_m^2(\nu_{mn})}{J_m^2\left(\nu_{mn}\frac{R_{in}}{R_{out}}\right)}\right\}^{1/2} \tag{4b}$$

B. *Coaxial Cavities With A Corrugated Inner Conductor*

As shown in Fig.1, a coaxial cavitie with a corrugated inner conductor is of complex boundary condition, which can be treated according to SIT[4,5] and the rigorous field matching method[10,12]. Because the expression of $\Phi_{mn}^{(i)}$ can not be available from RFM (in fact, this method is only used for the calculation of eigenvalues and ohmic losses[11,12]), we deduce $\Phi_{mn}^{(i)}$ by SIT

$$\Phi_{mn}^{(1)}=\sqrt{\frac{\pi}{2\varepsilon_m}}\left\{J_m\left(\frac{\nu_{mn}}{R_{out}}r\right)N_m(\nu_{mn})-N_m\left(\frac{\nu_{mn}}{R_{out}}r\right)J_m(\nu_{mn})\right\}\left\{1-\left[\frac{J_m(\nu_{mn})}{J_m\left(\nu_{mn}\frac{R_{in}}{R_{out}}\right)}\right]^2\right\}^{-1/2}\cos m\varphi \tag{5a}$$

$$\Phi_{mn}^{(2)}=\sqrt{\frac{\pi}{2}}\left\{J_m\left(\frac{\mu_{mn}}{R_{out}}r\right)N_m^{'}(\mu_{mn})-N_m\left(\frac{\mu_{mn}}{R_{out}}r\right)J_m^{'}(\mu_{mn})\right\}\left\{1-\left(\frac{m}{\mu_{mn}}\right)^2-\left[\frac{J_m^{'}(\mu_{mn})}{J_m^{'}\left(\mu_{mn}\frac{R_{in}}{R_{out}}\right)+wJ_m\left(\mu_{mn}\frac{R_{in}}{R_{out}}\right)}\right]^2\right\}^{-1/2}\sin m\varphi \tag{5b}$$

Substituting (5) into (2), the mode coupling coefficients of coaxial cavities with a corrugated inner conductor can be written as

$$C_{(s,t)}^{(1,1)}=\frac{2}{G_{mt}G_{ms}}\cdot\frac{\nu_{mt}^2}{\nu_{mt}^2-\nu_{ms}^2}\cdot\left\{\frac{1}{R_{out}}\frac{\partial R_{out}}{\partial z}-\frac{1}{R_{in}}\frac{\partial R_{in}}{\partial z}\frac{J_m(\nu_{mt})J_m(\nu_{ms})}{J_m\left(\nu_{mt}\frac{R_{in}}{R_{out}}\right)J_m\left(\nu_{ms}\frac{R_{in}}{R_{out}}\right)}\right\} \tag{6a}$$

$$C_{(t,t)}^{(1,1)}=\frac{1}{G_{mt}^2}\cdot\left\{\frac{1}{R_{in}}\frac{\partial R_{in}}{\partial z}\frac{J_m^2(\nu_{mt})}{J_m^2\left(\nu_{mt}\frac{R_{in}}{R_{out}}\right)}-\frac{1}{R_{out}}\frac{\partial R_{out}}{\partial z}\right\} \tag{6b}$$

$$C_{(s,t)}^{(1,2)}=0 \tag{6c}$$

$$C_{(s,t)}^{(2,1)} = \frac{2m}{G_{ms}\mu_{ms}}\left\{\frac{1}{R_{out}}\frac{\partial R_{out}}{\partial z}\frac{1}{K_{ms}} - \frac{1}{R_{in}}\frac{\partial R_{in}}{\partial z}\frac{R_{out}}{T_{ms}R_{in}}\frac{J_m\left(\nu_{mt}\right)}{J_m\left(\nu_{mt}\frac{R_{in}}{R_{out}}\right)}\right\} \tag{6d}$$

$$C_{(s,t)}^{(2,2)} = \frac{2\mu_{ms}}{\mu_{ms}^2-\mu_{mt}^2}\left\{\frac{\mu_{ms}\left(\frac{m^2}{\mu_{mt}^2}-1\right)}{R_{out}K_{mt}K_{ms}}\frac{\partial R_{out}}{\partial z} - \frac{1}{R_{in}T_{mt}T_{ms}}\frac{\partial R_{in}}{\partial z}\left[\mu_{ms}\left(\frac{m^2R_{out}^2}{\mu_{mt}^2R_{in}^2}+\frac{wR_{out}}{\mu_{mt}R_{in}}-1\right)-\mu_{mt}w^2\right]\right\} \tag{6e}$$

$$C_{(t,t)}^{(2,2)} = \frac{1}{R_{in}}\frac{\partial R_{in}}{\partial z}\frac{1}{T_{mt}^2}\left[w^2+\frac{m^2R_{out}^2}{\mu_{mt}^2R_{in}^2}\right] - \frac{1}{R_{out}}\frac{\partial R_{out}}{\partial z}\frac{m^2}{\mu_{mt}^2K_{mt}^2} \tag{6f}$$

where

$$K_{mn} = \left\{\left[1-\left(\frac{m}{\mu_{mn}}\right)^2\right]-\left[1+w^2-\left(\frac{mR_{out}}{\mu_{mn}R_{in}}\right)^2\right]\left[\frac{J_m'\left(\mu_{mn}\right)}{J_m'\left(\mu_{mn}\frac{R_{in}}{R_{out}}\right)+wJ_m\left(\mu_{mn}\frac{R_{in}}{R_{out}}\right)}\right]^2\right\}^{1/2} \tag{7a}$$

$$K_{mn} = N_m'\left(\mu_{mn}\right)\left\{N_m'\left(\mu_{mn}\frac{R_{in}}{R_{out}}\right)+wN_m\left(\mu_{mn}\frac{R_{in}}{R_{out}}\right)\right\}^{-1}T_{mn} \tag{7b}$$

Equations (3) and (6) are the expression of the mode coupling coefficients of both coaxial cavities with a smooth-walled tapered inner conductor and coaxial cavities with a corrugated inner conductor, and the second-order transmission line equation applicable for the both resonator structures can be achieved after substituting (3) and (6) into the Equation (1).

IV. CONCLUSION

The mode coupling coefficients of coaxial cavities can be deduced according to SIT, and the second-order transmission line equation is established. Making use of the equation, it is convenient to calculate eigenmodes of coaxial cavities.