Comparison of linear and nonlinear ballooning mode stability in a Tokamak equilibrium

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Introduction:

Edge localised modes (ELMs) are mesoscopic instabilites occuring in the edge region of high confinement Tokamak plasmas. The largest type I ELMs can expel up to 10% of the edge particle inventory and thermal energy, resulting in unacceptable levels of heat loads on the divertors for ITER-sized devices. A theoretical model for type I ELMs is that of coupled peeling-ballooning modes within the framework of ideal magnetohydrodynamics (MHD). Ballooning modes are pressure-driven modes, peeling modes are driven by current. Linear ideal MHD is able to predict the correct pressure gradient threshold for ELMs to occur. As it is a linear theory and therefore assumes exponential growth of instabilities over time, it cannot predict the true dynamics. Nonlinear effects lead to saturation of the mode, alter the mode number evolution. So e.g. the size of ELMs cannot be obtained from the peeling-ballooning model.

Focusing on the initial phase of ballooning unstable plasmas, we compare the instability growth rates computed with linear and nonlinear codes. The peeling part is neglected for now. The numerical computations are performed with the linear MHD solver ILSA [1, 2] in MISHKA mode (ideal MHD), calculating the growth rates as a function of the toroidal mode number n as well as the corresponding mode structure. The nonlinear computations are performed with GEM [3], which is a 3D gyrofluid turbulence code with local equations and local geometry (flux tube). It is based on six moment equations and uses self-consistent electromagnetic fields. The flux tube is placed at the position of the maximum pressure gradient.

The equilibrium itself is computed by the high resolution fixed boundary equilibrium solver HELENA [4].

As a first test case, we use a shifted circle model for a circular Tokamak equilibrium. The centers of the circles are shifted towards the low-field side by a small correction term $\Delta(r)$, where r is the radius of the corresponding flux surface. By definition, $\Delta(a) = 0$, with a being the minor radius. Using a parabolic q profile

$$q(r) = q_0 + q_1 \left(\frac{r}{a}\right)^2, \tag{1}$$

with $q_0 = 1.3$ and $q_1 = 5.8$, the profiles read as [7]

$$j_{tor} = j_0 \frac{q_0^2}{q^2} \tag{2}$$

$$p = p_0 \left(\frac{q_0^2}{q^2}\right)^{\frac{4}{3}}, \tag{3}$$

where $j_0 = 1.1 \cdot 10^6 \frac{\mathrm{A}}{\mathrm{m}^2}$. The pressure gradient is increased until above the ballooning stability threshold, but must remain low enough so that the shifted circle model can be applied. The relative Shafranov shift $\Delta(r)/R_0$ is $\sim$ 3.5% - 9% for our set of pressure profiles, where $R_0 = 3.0$m is the major radius. The q factor at the positions of the maximum pressure gradients is $\sim$ 1.5 - 1.55.

Nonlinear gyrofluid computations of edge localised ideal ballooning modes have been performed by the authors of [5]. For general edge nonlinearity, see Ref. [6].

Results:

Varying the value of p_0, we investigate the spectrum and the mode structure of the instabilities with ILSA (fig. 1). For $p_0 < 5 \cdot 10^4$Pa, the equilibrium becomes stable against ballooning modes. For all 4 investigated pressure profiles, the fastest growing mode has the toroidal mode number $n = 18$.

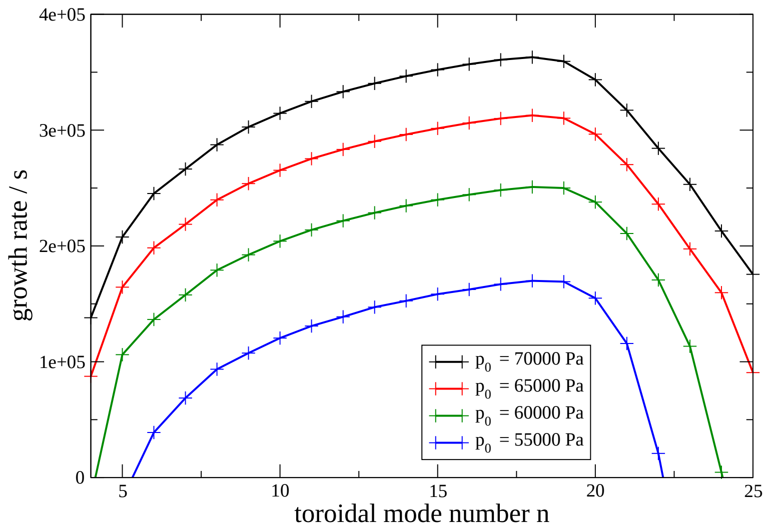

Figure 1: Spectra for several values of p_0

The mode peaks near the position of the maximum pressure gradient, indicated by the thick black line (fig. 2). The radial coordinate s is defined as $\sqrt{\bar{\psi}}$, where $\bar{\psi}$ is the normalized poloidal flux.

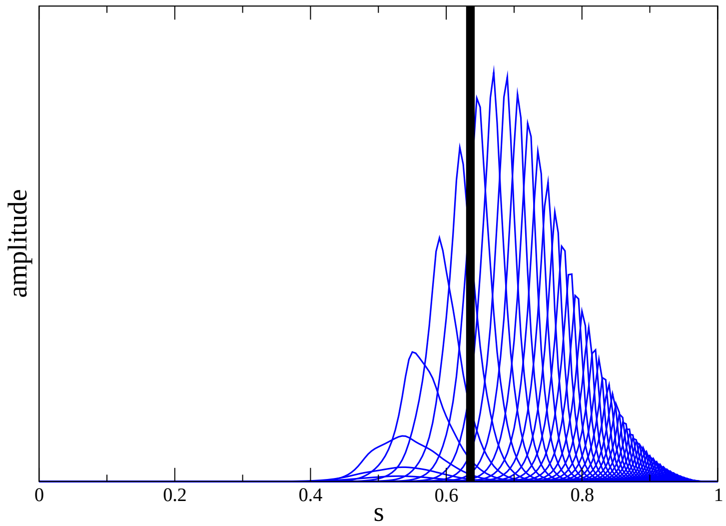

Figure 2: Mode structure for $p_0 = 65000$Pa and $n = 18$

From the time evolution of the total energy E_{tot} computed by GEM (fig. 3), the growth rates for the initial exponential phase can be calculated (fig. 4).

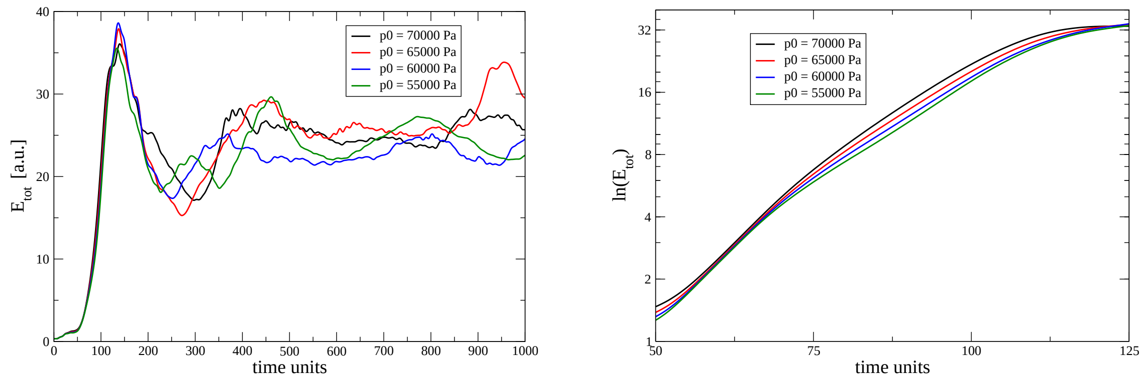

Figure 3: Time evolution of E_{tot} (left), zoom onto linear phase (right)

The time units are normalized to $\frac{L_\perp}{c_s} = 5.0523 \cdot 10^{-7}$s.

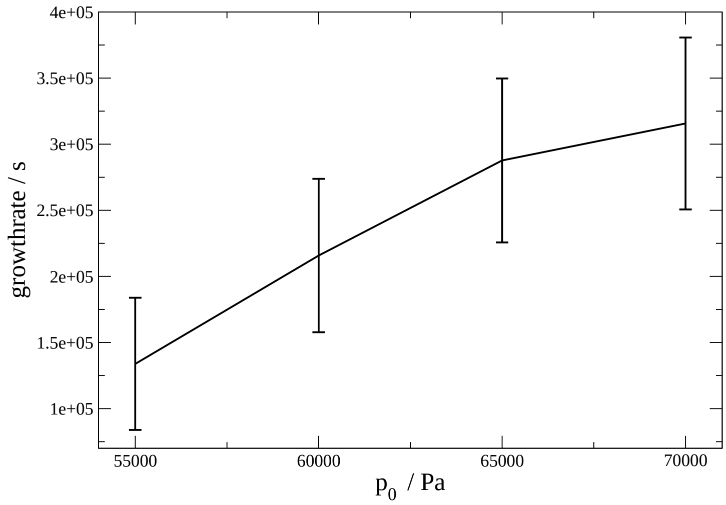

Figure 4: Growth rates for linear phase

Within the error bars, the growth rates calculated by GEM agree with those of the fastest growing modes computed with ILSA.

Outlook:

The next step is to use the global gyrofluid code GEMR [3]. We want to compare the growth rates computed by GEMR with the ones obtained from ILSA and also GEM, so we can see the differences between flux tube and global treatment. The maximum pressure gradient position will be shifted towards the edge, the final goal being a pressure profile that resembles an experimental H mode pressure profile. Furthermore, shaping of the plasma will be taken into account.

References

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