Progress on three-dimensional equilibrium reconstruction

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Abstract

Equilibrium Reconstruction (ER) is an inverse problem, where the signals from experimental diagnostics are used to determine the parameters (current profile, pressure profile, toroidal flux, external currents) that specify an MHD equilibrium. V3FIT [1] is an equilibrium reconstruction code that uses VMEC [2] (a three-dimensional MHD equilibrium code) to solve the forward problem. An important goal for the V3FIT code is to use it routinely for equilibrium reconstructions of stellarators. We report on progress made toward this goal with specific application to the Compact Toroidal Hybrid (CTH) stellarator at Auburn University.

V3FIT was written with magnetic diagnostics as the original source of diagnostic information to determine the equilibrium, but has the possibility of adding further diagnostics. We have started the implementation of using soft x-ray diagnostic information in the V3FIT code, and report preliminary results.

V3FIT uses a quasi-Newton algorithm to converge to a reconstructed equilibrium and in so doing computes a finite difference approximation to the Jacobian matrix. The Jacobian matrix contains a tremendous amount of information about nearby equilibria. A measure of *signal effectiveness*, i.e., how important a given diagnostic is to the determination of one of the reconstruction parameters, can be readily computed from the Jacobian, and we have implemented this computation into the V3FIT code. We present results on the use of the signal effectiveness for the design of new magnetic diagnostics for CTH.

Introduction

In tokamaks, it is common to use an axisymmetric equilibrium reconstruction (ER) code like EFIT [3] to determine accurate MHD equilibria throughout a plasma discharge. The resulting MHD equilibria can then be used as a basis for further computations of plasma behavior, such as stability, RF heating and current drive deposition, and particle and energy transport. The same ER capability is not yet routinely used in stellarators, but is increasingly needed for similar reasons as stellarator discharges attain higher pressures. This paper reports progress in developing an ER code for three-dimensional toroidal equilibria that may be used on stellarators and non-axisymmetrically-perturbed tokamaks and RFPs.

The equilibrium reconstruction process uses the experimental observations available (often from magnetic diagnostics) to determine parameters (current profile, pressure profile, toroidal flux, external currents) that specify an MHD equilibrium. Equilibrium reconstruction is an *inverse problem*, and determines the parameters by minimizing the mismatch between the expected and observed experimental signals.

We have written a three-dimensional MHD equilibrium reconstruction code called V3FIT [1]. The equilibrium in V3FIT is computed with the VMEC [2] code that assumes the existence of closed, nested flux surfaces. VMEC requires a current profile specification, a pressure profile specification, and the total toroidal flux within the plasma as input parameters. The V3FIT code minimizes a χ^2 function that measures the deviation of the model-predicted signals from the experimentally observed signals:

$$\chi^2 = \sum_{i,j} \left(S_i^o - S_i^m(\mathbf{p})\right)\left(\left(\mathbf{C}^S\right)^{-1}\right)_{ij}\left(S_j^o - S_j^m(\mathbf{p})\right) \tag{1}$$

where S_i^o are the observed signals, $S_i^m(\mathbf{p})$ are the model predicted signals, $\mathbf{p}$ is a vector of reconstruction parameters describing the current and pressure profiles, and $\mathbf{C}^S$ is the covariance matrix of the observed signals. For the usual case of uncorrelated Gaussian signal noise, $(\mathbf{C}^S)_{ij} = \sigma_i^S \delta_{ij} \sigma_j^S$, where $(\sigma_i^S)^2$ is the variance of the signal noise. The function $S_i^m(\mathbf{p})$ combines both the MHD equilibrium computation, and the computation of the signal expected in a diagnostic, given the MHD equilibrium. For example, in the case of a magnetic diagnostic, computing the signal requires a Biot-Savart integration over the model plasma current density.

The design of the V3FIT code and its reconstruction algorithm, its performance using simulated magnetic diagnostic data, and some benchmarking results with the EFIT code for an axisymmetric equilibrium is reported in some detail in Ref. [1]. In this paper we report on some recent improvements to V3FIT. In the next section we discuss the addition of soft x-rays as an additional diagnostic for use in reconstruction. Lastly, we discuss the *signal effectiveness*.

Reconstruction with soft x-rays

While ER typically relies predominantly on magnetic diagnostics, soft x-ray observations are also applied to determine the flux surface geometry in tokamaks, particularly in the core [4]. We have recently included soft x-ray diagnostics into the V3FIT code. We take the observed signal for a soft x-ray chord to be the intensity received at an x-ray detector. The model signal is the line-integrated x-ray emissivity,

$$S_i^m = \int_L dl \int dE \left[A(E)\tau(E) \frac{n_e^2(r)}{\sqrt{T_e(r)}} \exp\left(-\frac{E}{T_e(r)} \right) \right] \tag{2}$$

where E is the energy of the x-ray, $A(E)$ is the detector efficiency, $\tau(E)$ is the filter transmission, $n_e(r)$ is the electron density, and $T_e(r)$ is the electron temperature. We assume the density is constant on a flux surface, and from the uniformity of pressure on a flux surface in MHD equilibrium, the temperature is also taken constant on a flux surface.

We have incorporated simulated soft x-ray emissivities into V3FIT to solve the forward problem in ER with this diagnostic. Figure 1 shows the viewing chords of the five soft x-ray cameras (at two different toroidal positions) on the CTH stellarator. The simulated brightnesses using a model equilibrium are shown in Fig. 2.

Signal effectiveness

To determine those diagnostics, or signals, most important to determining specific parameters in the equilibrium reconstruction, we introduce a signal effectiveness defined as:

$$E_{ji} \equiv \frac{\sigma_i^S}{\sigma_j^P} \frac{d\sigma_j^P}{d\sigma_i^S} \tag{3}$$

The signal effectiveness is the dimensionless ratio of the fractional reduction in the *j*th parameter variance to the fractional reduction in the *i*th signal variance.

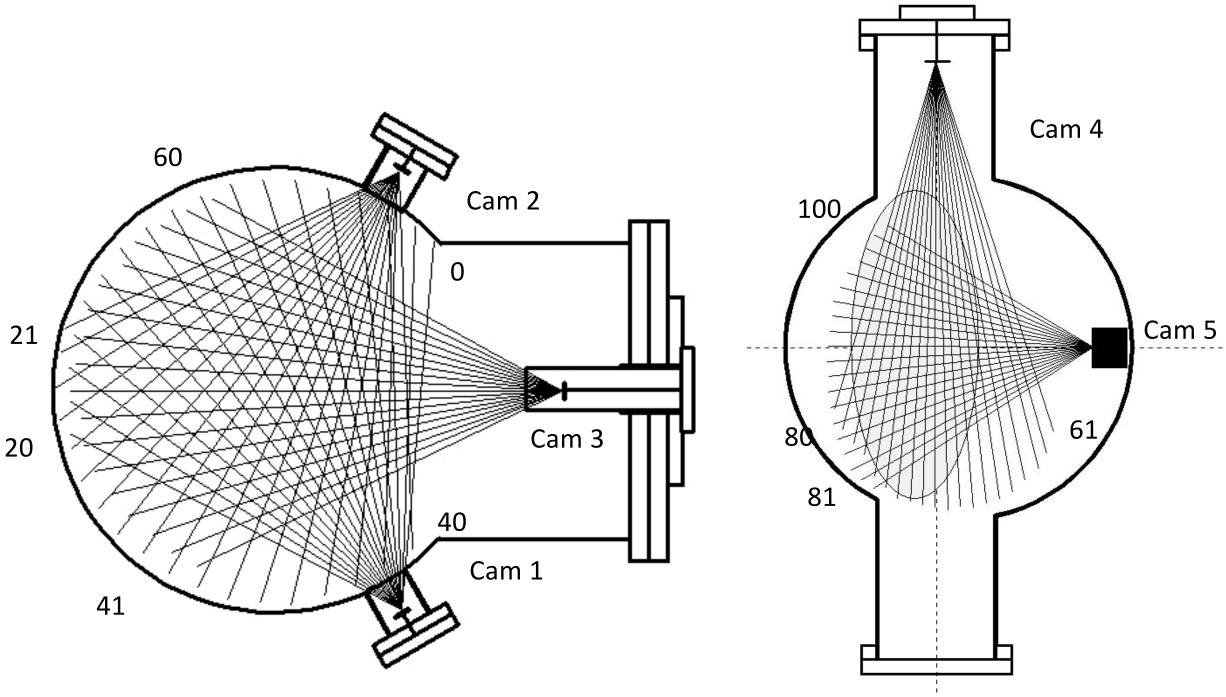

Fig. 1. Viewing chords of soft x-ray cameras on CTH at the two different planes of vertical symmetry. Numbers identify the outermost chords of each 20-channel camera (see Fig. 2).

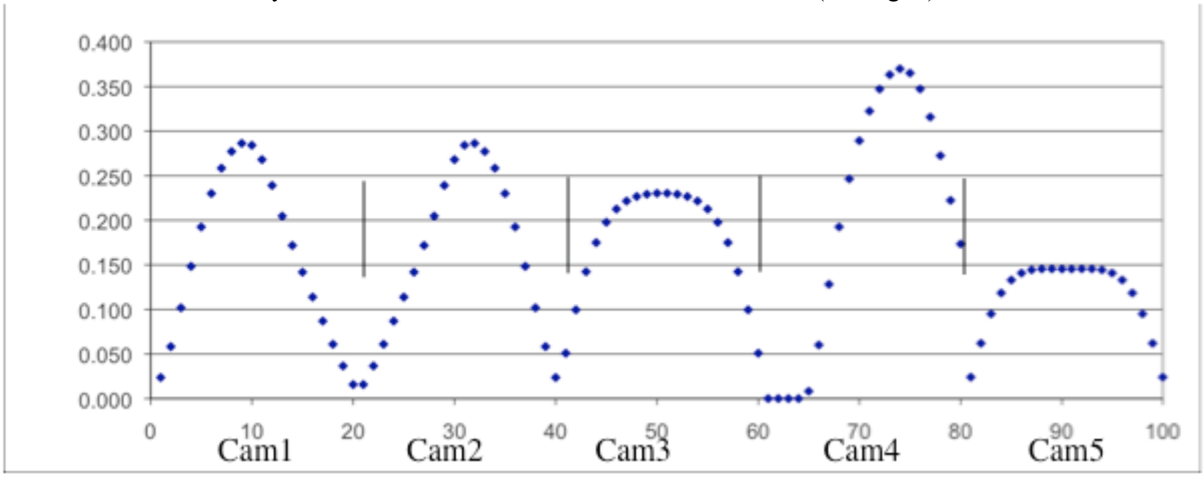

Fig. 2. Computed soft x-ray chordal intensity on each channel from V3FIT using simulated x-ray emissivities. Horizontal axis refers to chord number.

The signal effectiveness (for uncorrelated Gaussian signal noise) can be computed as

$$E_{ji} \equiv \left(\frac{1}{\sigma_i^S \sigma_j^P} \left(J \cdot C^P \right)_{ij} \right)^2 \tag{4}$$

where $\mathbf{J}$ is the Jacobian of the model data with respect to the parameters, and $\mathbf{C}^P$ is the posterior covariance matrix of the parameters.

$$\mathbf{J}_{ij} = \frac{\partial S_i^m(\mathbf{p})}{\partial p_j} \qquad (\mathbf{C}^P)^{-1} = \mathbf{J}^T \cdot \left((\mathbf{C}^S)^{-1} \right) \cdot \mathbf{J} \tag{5a,b}$$

Note that the signal effectiveness is non-negative: decreasing the variance of a signal can *not* increase any posterior parameter variance. Also note that the signal effectiveness is normalized so that

$$\sum_{i\,(Signals)} E_{j\,i} = 1 \tag{9}$$

As an example, the signal effectiveness of each of the soft x-ray signals in determining the current in the vertical field coil of CTH (effectively setting the horizontal position of the plasma centroid) is shown in Fig. 3 for the simulated case illustrated in Fig. 2. The same approach can be taken for more physically significant parameters such as those specifying the current and pressure profiles.

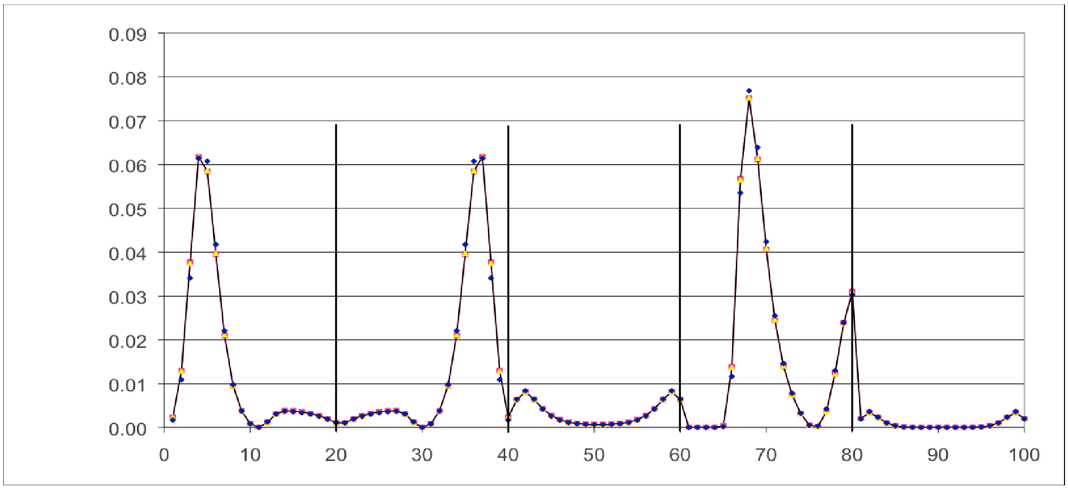

Fig. 3. Computed signal effectiveness for each of the 100 soft x-ray chordal views in determining the current in the vertical field coil.

Conclusion

The development of the V3FIT equilibrium reconstruction code shows significant progress. Soft x-ray chordal intensity has been added to the code as a signal, supplementing the existing magnetic diagnostic signal. Computation of the signal effectiveness has been implemented. The signal effectiveness should be helpful in the interpretation and planning of experimental diagnostics.

Acknowledgments

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