Equilibrium reconstruction in RFP SHAx states using Newcomb's equation

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Introduction. The Reversed Field Pinch (RFP) configuration is characterized by the presence of different tearing modes, mostly in the m=0 and m=1 part of MHD spectrum. In the RFX-mod experiment, at relatively low values of plasma current, several modes with similar amplitudes are present: a high level of stochasticity appears in the plasma core, with the consequence of flat density and temperature profiles, in a condition named Multiple Helicity (MH) state. Increasing the plasma current the system achieves the so called Quasi Single Helicity (QSH) state, where only one mode dominates the spectrum, while the energy of the other secondary modes is smaller. In RFX-mod the dominant mode is found to be the m=1/n=7 one. SHAx (Single Helical AXis) states are a special flavour of the QSH condition, achieved in RFX-mod for plasma current beyond 1 MA [1]. These states are considered as improved confinement RFP states because of the appearance of an ordered and spontaneous structure in the plasma core, that dominates on the typical chaos of MH condition: when the dominant mode exceeds a threshold amplitude, the X-point of its magnetic island is expelled and the original axi-symmetric axis is replaced by a helical magnetic axis, that coincides with the O-point of the magnetic island.

We can think of a quantity A inside the plasma as composed of an axi-symmetric part and of a perturbation to it, usually Fourier decomposed: $A = A_0 + \sum_{m,n} a^{m,n}(r) e^{i(m\theta - n\varphi)}$. Tearing mode amplitudes are considered to be small compared to the axi-symmetric equilibrium magnetic field $\mathbf{B}_0$, so we treat them as perturbations superposed to $\mathbf{B}_0$. A complete reconstruction of the tearing modes eigenfunctions within the plasma volume has been done in [2], solving the Newcomb-like equation that arises from the force-free force balance equation. Toroidal geometry is well described by the non-orthogonal and curvilinear coordinate system (r, θ, φ), built as a flux coordinate system for the non-concentric and circular-cross-section magnetic surfaces of $\mathbf{B}_0$. Newcomb equation provides the harmonics of the perturbations $\psi_P^{m,n}$ and $\psi_T^{m,n}$ to the poloidal (ψ_P) and toroidal (ψ_T) flux functions, and a complete reconstruction of the magnetic field over the whole plasma volume arises from the canonical representation of the magnetic field:

$$B = \nabla \psi_T \times \nabla \theta - \nabla \psi_P \times \nabla \varphi . \qquad (1)$$

SHAx states are modelled as pure Single Helicity (SH) states, their magnetic field being composed of the superposition of $\mathbf{B}_0$ and of the perturbed magnetic field that arises from the dominant $\psi_P^{1,7}$ and $\psi_T^{1,7}$ eigenfunctions, neglecting the effect of the residual secondary modes.

With representation (1), valid for every divergence-free field in a toroidal device, magnetic field line equations have the same mathematical form as the canonical equation of motion produced by hamiltonians of one-and-a-half degree of freedom. The functions ψ_P, ψ_T, θ, φ can be identified with the canonical variables in hamiltonian context: φ plays the role of the canonical time, ψ_T of the momentum conjugate to θ, and ψ_P of the field line hamiltonian. The existence of magnetic flux surfaces is assured if there is a function ρ for which $B \cdot \nabla \rho = 0$. In this case ρ will label the flux surfaces, and we can look for action-angle coordinates (also called flux coordinates) with ρ as 'radial' coordinate. The symplectic form (1), with flux coordinates, assures that $\psi_P(\rho)$ and $\psi_T(\rho)$ are flux functions that measure the poloidal and toroidal flux across flux surfaces $\rho=const$, $\psi_T(\rho)$ being the action coordinate. Choosing ψ_T as the radial coordinate, it is manifest that in action-angle variables the hamiltonian $\psi_P \equiv \psi_P(\psi_T)$ is a function of the action alone.

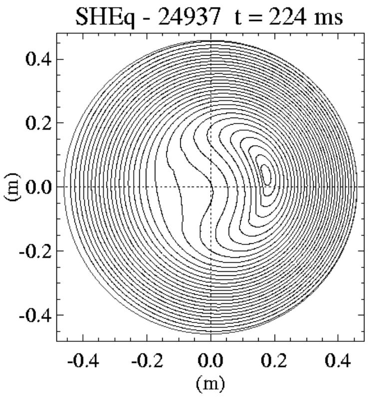

Figure 1: Flux surfaces for SH states, $\Sigma(\chi)$.

In our case we start from flux coordinates for the axi-symmetric field $\mathbf{B}_0$ [2]. Adding a generic perturbation, the circular $r=const$ flux surfaces are deformed and it is not clear a priori if other flux surfaces exist and which may be the function ρ labelling them. In the case of a SH equilibrium, where the perturbation has helical symmetry, ψ_P and ψ_T are functions only of r and of the helical angle $u = \theta - n\varphi$, so it can be shown that conserved flux surfaces exist. A good flux function to label them is the helical flux, defined as $\chi(r,u) = \psi_P - n\psi_T$. It can be shown that $B \cdot \nabla \chi = 0$, thus the helical flux contour gives the shape of flux surfaces $\Sigma(\chi)$ in SHAx states. An example can be seen in *fig.1*. The canonical form of the magnetic field can be written, using the angle u instead of θ and the helical flux χ instead of the poloidal flux ψ_P, as:

$$B = \nabla \psi_T \times \nabla u - \nabla \chi \times \nabla \varphi . \qquad (2)$$

The helical flux is therefore the hamiltonian for SH states and the helical angle u must substitute the poloidal one. We start from (2) to build a new coordinate system describing the SH equilibrium.

SHEq code. A reconstruction of the equilibrium configuration has been implemented in a code named SHEq (Single Helical Axis), where SHAx states are modelled as pure SH states. The coordinate systems constructed on the circular-cross-section magnetic surfaces of the axi-symmetric field $\mathbf{B}_0$ are not appropriate to describe helical flux surfaces, due to the fact that there are points not univocally identified by the poloidal angle θ, as we can see when looking at the inner beam-shaped surfaces in *fig.1*. The problem of determining a new angle defined with respect to the helical axis was solved introducing a coordinate system based on the hamiltonian description of the magnetic field. Choosing ψ_T as the radial coordinate, the hamiltonian χ in equation (2) is function of (ψ_T, u), so (ψ_T, u, φ) are not action-angle coordinates for SH states. Following the standard procedure of hamiltonian mechanics, we can derive action-angle coordinates (ψ_h, u_h), associated to (ψ_T, u), by a canonical coordinate transformation. The function $\chi(\psi_T, u)$ may be locally inverted to yield $\psi_T(\chi, u) \equiv \psi_0$. The action $\psi_h(\chi)$ is nothing but the curvilinear integral of the flux ψ_0, that it turns out to be the toroidal flux inside the flux surface $\Sigma(\chi)$:

$$\psi_h = \frac{1}{2\pi} \oint_{\Sigma(\chi)} \psi_0(\chi, u) du'.$$

Inverting the relation $\psi_h \equiv \psi_h(\chi)$, $\psi_0(\chi, u)$ can be written as a function of the action and of the angle u, namely $\psi_a(\psi_h, u)$. The definition of the generating functions $S(\psi_h, u)$ for the canonical transformation from (ψ_T, u) to (ψ_h, u_u), provides us the angle u_h conjugated to the action. We know that $S = \int_0^u \psi_a(\psi_h, u) du'$ and $u_h = \frac{\partial S}{\partial \psi_h} = \int_0^u \frac{\partial \psi_a}{\partial \psi_h} du'$. In the action-angle variable coordinates, the hamiltonian is still the helical flux $\chi = \chi_0(\psi_h)$, and we choose it as the 'radial' coordinate in the (χ, u_h, φ) coordinate system. In the cylindrical symmetry limit, where the helical deformation vanishes, $\psi_h = \psi_T$ and $u_h = u$.

Results. Flux surface averages are computed In SHEq with the usual formula: $\langle A \rangle = \frac{\iint du_h d\varphi \sqrt{g} A}{\iint du_h d\varphi \sqrt{g}}$, where $\sqrt{g}$ is the Jacobian of the coordinate system.

As an example, the surface-averaged ohmic input power has been computed and plugged into the averaged power balance equation in steady state and for fluid at rest: $\langle \nabla \cdot Q \rangle = \langle \eta J^2 \rangle$. The resistivity η is calculated from the Spitzer formula, assuming a flat Z_{eff} profile with a value adjusted so as to match the total input power, and the current density J is calculated from the

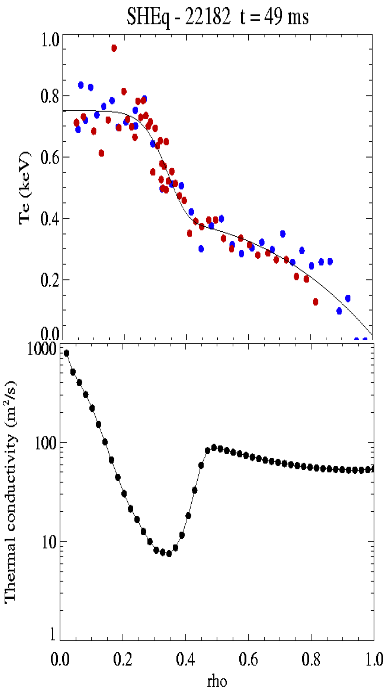

Figure 2. Up: electron temperature profile. Bottom: thermal conductivity profile.

temperature, measured by the Thomson scattering system, is a good flux function, so $T_e=T_e(\chi)$ [1], and we write the heat flow vector Q as $Q=-n\kappa\nabla T_e$, with uniform density profile for n (consistent with measurements, at least up to $r/a=0.8$). This yields an evaluation of the thermal conductivity κ

$$\kappa = -\frac{\int d\chi \langle \eta J^2 \rangle V'}{\frac{dT_e}{d\chi} n \langle g^{11} \rangle V'}$$

using the formula for the average of a divergence. In the final formula for the thermal conductivity κ, $V'=dV/d\chi$ is the specific volume, and $g^{11}=\nabla\chi\cdot\nabla\chi$ is the first metric tensor element. A value around 10 m^2/s (*fig.2* bottom) is found for κ in the region of the internal transport barrier that characterized SHAx states, clearly observable in the steep gradient of the temperature profile (*fig.2* up). Figures are plotted against ρ_h, the normalized helical flux square root.

A large plasma current flows in RFP configurations, and we checked a posteriori if SH equilibria verify the ohmic constraint that arises from the parallel Ohm's law averaged on flux surfaces. The two sides of the ohmic constraint equation $\frac{V_t}{2\pi}\langle B^{\varphi}\rangle = \eta\langle J\cdot B\rangle$ do not coincide, suggesting that the Ohm's law should be includeed in the equilibrium calculation, whereas hitherto α-Θ_0 model is assumed for the zeroth-order parallel current density profile. It is worth noting that, due to the presence of a still finite amplitude of the secondary modes in the RFX-mod SHAx states, the discrepancy found in the Ohmic constraint could be explained by a residual dynamo term.

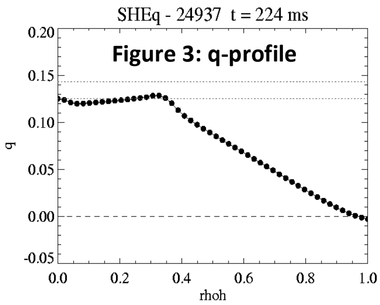

Figure 3: q-profile

We can also use the action-angle variable coordinate system to compute the safety factor q, following its simple definition in flux coordinates: $q_h = \frac{d\varphi}{du_h} = \frac{B^{\varphi}}{B^{u_h}} = \frac{d\psi_h}{d\chi}$. Its relation with the q defined as the ratio between toroidal and poloidal flux is $q=(\iota_h+n)^{-1}$: the resulting q-profile is almost flat in the inner bean-shaped flux surfaces region, with a maximum in correspondence of the internal transport barrier (*fig.3*).

References

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