Equilibrium evolution and energy transport at fast heating in tokamaks

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1. Introduction. The energy balance is considered at the fast transition of the tokamak plasma from one equilibrium state to another. This is an alternative approach to the problems of so-called missing power and ballistic jump of the total heat flux after ECRH switching on in the T-10 tokamak [1]. Also, this includes analysis of the theoretical model [2] for treating the equilibrium response to abrupt change of the heating power. An important element of our study is the exchange of energy between the plasma and magnetic field, which was ignored in [1], though its importance was emphasized in [2]. From [2] our approach is essentially different in treating the boundary conditions determining the flux of energy in/out the system. In [2] this flux could not be found since the boundary conditions were replaced by two constraints: frozen-in magnetic field and fixed plasma boundary. Finally our study is aimed to discussion of the T-10 experimental results [1]. Also, the developed model is applied to interpretation of the results of electron heat transport study during ECRH on the TEXTOR tokamak [3]. Specifically, to explain the discrepancy [3] that the absorbed energy was perfectly confined inside the q=1 surface during the first 5 ms of the heating, while the calculated electron heating rate inside this surface indicated to only one third of the launched EC power.

2. Formulation of the problem. We use the standard force-balance equation for the plasma,

$$\rho \frac{d\mathbf{v}}{dt} + \nabla p = \mathbf{j} \times \mathbf{B} = -\nabla \frac{\mathbf{B}^2}{2} + (\mathbf{B} \cdot \nabla)\mathbf{B}, \tag{1}$$

where ρ is the plasma mass density, $\mathbf{v}$ is its velocity, p the pressure, $\mathbf{B}$ the magnetic field, $\mathbf{j} = \nabla \times \mathbf{B}$ is the current and $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla$. For evolution of $\mathbf{B}$ in the plasma we have

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \tag{2}$$

We consider the integral effects, so the integral consequence of (2) will be used here, which is the conservation of the toroidal magnetic flux Φ_{pl} through the plasma and the magnetic flux Φ_e through the gap between the plasma and the vacuum chamber:

$$\Phi_{pl} \equiv \int_{plasma} \mathbf{B} \cdot d\mathbf{S}_{\perp}, \qquad \Phi_e \equiv \int_{gap} \mathbf{B} \cdot d\mathbf{S}_g. \tag{3}$$

To describe the energy balance, we use the standard equation [4]

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p+\frac{\mathbf{B}^2}{2}\right)+\nabla\cdot\left(\frac{5}{2}p\mathbf{v}+\mathbf{E}\times\mathbf{B}+\mathbf{q}_1\right)=s\,, \tag{4}$$

where $\mathbf{q}_1$ is the diffusive part of the heat flux and s is the heat source.

The conducting wall encircling the plasma at some distance can be considered as ideal at processes faster than the magnetic field diffusion. Then $\mathbf{n}\times\mathbf{E}=0$ at the wall, and integrating (4) over the volume up to the wall and in time from t_1 to t_2 we obtain

$$\Delta E_H = H - \Delta W_m^{pl} - \Delta W_m^{g}\,, \tag{5}$$

where Δ means the increment, E_H is the thermal energy of the plasma, H is the energy deposition from the sources (heating if $H>0$) and W_m^{pl} and W_m^{g} are, respectively, the magnetic energies in the plasma and in the plasma-wall vacuum gap. Precisely,

$$E_H \equiv \frac{3}{2}\int_V p dV\,, \qquad H \equiv \int_{t_1}^{t_2}\int_V s dV dt\,, \qquad W_m \equiv \int_V \frac{\mathbf{B}^2}{2} dV\,, \tag{6}$$

with different V for W_m^{pl} and W_m^{g}. In (5) the "lost" energy is $\Delta W_m^{pl}+\Delta W_m^{g}$. This can be found from the force-balance equation (1) under the constraints $\Phi_{pl}=const$ and $\Phi_e=const$.

3. Energy relations in the cylindrical model. This model is only an intermediate step for the toroidal systems, but it is a step ahead of the transport model used in [1], which is also cylindrical, but with $\mathbf{B}=0$. Compared to [2], the new elements are $\mathbf{v}\cdot\mathbf{n}\neq 0$ and $\Phi_e=const$.

Our goal is evaluation of the total energy ΔE_H absorbed by the plasma. This depends on ΔW_m, which we calculate using the integral consequence of the equilibrium equation (1):

$$\overline{p} = \frac{B_e^2-\overline{B_z^2}}{2}+\frac{B_J^2}{2}\,. \tag{7}$$

Here B_e and B_J are, respectively, the toroidal and poloidal fields at the plasma boundary, and the bar means the averaging over the plasma cross-section $S_\perp$.

For smooth pressure and current distributions it is natural to assume that there are no surface currents on the plasma boundary, so that in this case B_e in (7) is the external vacuum toroidal field. In systems with a strong toroidal field the difference between B_z and B_e is small, and the exact equality (7) gives us approximately (for more detail see [5])

$$\overline{B}_z = B_e\left(1-\frac{\beta}{2}+\frac{B_J^2}{2B_e^2}\right), \tag{8}$$

where $\beta = 2\overline{p}/B_e^2$. In this notation, the conservation of the toroidal flux in the plasma means

$$\frac{d}{dt}(\overline{B}_z S_\perp) = 0\,, \tag{9}$$

which must be calculated with account of the plasma deformation, $\mathbf{v}\cdot\mathbf{n}\neq 0$. It is important that this requires variation of $S_\perp$ when $\overline{B}_z$ varies reacting to the β change, see (8).

This strongly affects the magnetic energy in the plasma (in volume V_{pl}),

$$W_m^{pl} = \frac{\overline{B_z^2} + \overline{B_\theta^2}}{2} V_{pl}\,. \tag{10}$$

With simple $B_z^2 = 2B_z B_e - B_e^2 + (B_z - B_e)^2$, where we disregard the last term, this reduces to

$$W_m^{pl} = l\left(\frac{\Phi_{pl}\Phi_e}{S_e} - \frac{\Phi_e^2}{2}\frac{S_\perp}{S_e^2}\right) + \alpha_p\,, \tag{11}$$

where $\Phi_{pl} = \overline{B}_z S_\perp$ and $\Phi_e = B_e S_e$ are introduced as convenient "frozen-in" constants, l is the system length, S_e is the surface of the plasma-wall gap cross-section and

$$\alpha_p \equiv \frac{\overline{B_\theta^2} V_{pl}}{2} \equiv \ell_i \frac{B_J^2 V_{pl}}{2} \tag{12}$$

with ℓ_i the internal inductance. The quantity α_p can give an essential contribution to ΔW_m^{pl} when ℓ_i and B_J are varied substantially. This can be expected during the disruptions. Here we consider the other cases, when $\Delta\alpha_p$ can be disregarded in ΔW_m^{pl}.

With conservation of Φ_{pl} and Φ_e and with natural $\Delta S_e = -\Delta S_\perp$ we obtain from (11)

$$\Delta W_m^{pl} \approx -\frac{B_e^2}{2}\Delta V_{pl}\,. \tag{13}$$

The change of the magnetic energy in the plasma-wall gap ΔW_m^g is determined by the change of the plasma volume, $\Delta V_{pl} = l\Delta S_\perp$, and is given by (with account of $\Delta(B_e S_e) = 0$)

$$\Delta W_m^g \approx \frac{B_e^2}{2}\Delta V_{pl}\,. \tag{14}$$

We show by these formulas that ΔW_m^{pl} and ΔW_m^g are equal by amplitude, but have opposite signs. Therefore, they cancel each other in (5), and we obtain finally

$$\Delta E_H \approx H\,. \tag{15}$$

Note that in [2] the final predictions were $\Delta E_H \approx H/3$ and, instead of (13), $\Delta W_m^{pl} = 0$. These originated from the imposed constraint $\Delta V_{pl} = 0$ and disregard of $\partial/\partial t$ in (9b) in [2].

We can prove, however, that the variations of the magnetic energy in and out of the plasma are a noticeable fraction of H. Representing B_e in (8) as Φ_e / S_e, we obtain from (9)

$$\frac{d}{dt}\frac{S_\perp}{S_e} = \frac{S_\perp}{2S_e}\frac{d}{dt}\left(\beta - \frac{B_J^2}{B_e^2}\right), \tag{16}$$

where only the leading-order terms are retained and $\Phi_e = const$ is also used. This gives us

$$\frac{\Delta V_{pl}}{V_{pl}} = \frac{V_e}{V_e + V_{pl}} \frac{\Delta(2\bar{p} - B_J^2)}{2B_e^2}. \tag{17}$$

Then, if ΔB_J^2 can be disregarded, (13) reduces to

$$\Delta W_m^{pl} = -\frac{\Delta E_H}{3}\frac{V_e}{V_e + V_{pl}}, \tag{18}$$

which gives with (15) $\Delta W_m^{pl} / H = -1/3$ for $V_e >> V_{pl}$. This, formally, corresponds to fixed external field B_e, though with $V_e >> V_{pl}$ the cylindrical approximation may be improper.

4. Conclusion. Our model is based on the ideal MHD equations, which is justified by experimental conditions in [1, 3]. Two states are compared, before and after the fast heating of the plasma. The transition is described by the equation of the energy transfer which is integrated to consider the global changes instead of the transition details. The same geometry is assumed as in [1] and [2]. It is shown that the interaction of the magnetic field with the plasma leads to redistribution of the rapidly injected power without strong effect on the plasma heating efficiency. Our analysis shows (in contrast to predictions in [2]) that all the "missing" power must be perfectly confined in the plasma. This means, in particular, that 400 "missing" kW of the injected 600 kW in the experiments described in [1, 3] must be absorbed, respectively, outside the central core with $\rho = 0.25$ (dimensional radius) in the T-10 tokamak [1] and outside the $q = 1$ surface (safety factor) in the TEXTOR plasma [3].

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